

Updated Progress Report, September, 1993

Some Mathematical Methods for Variational Objective Analysis and Accuracy Estimation

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and

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1. A new GCV-like method has been developed for estimating the relative accuracy between two different sources of information to be assimilated in a numerical weather prediction model, simultaneously with one or more 'penalty' parameters. The results appear in University of Wisconsin Statistics Department Technical Reports 894 and 902 by Feng Gao and a manuscript in preparation by Gao, Wahba and Johnson.

2. Bayesian 'Confidence Intervals' for the forecast based on the GCV-like relative accuracy estimates in 1. have been obtained and some tests demonstrating their accuracy also appear in TR 902 and in preparation.

3. Practical numerical methods for the implementation of Generalized Cross Validation (GCV) and Unbiased Risk (UBR) estimates of certain tuning parameters has been established, in the context of a very large scale variational objective analysis, where the cost function is minimized by an iterative descent algorithm. It is shown theoretically and in some 'toy' examples that the number of iterations can be considered a form of filtering, and that early stopping of the iteration can be very effective or even optimal, especially if certain appropriate preconditioners are used. A demonstration experiment for these practical numerical methods, (called randomized GCV and randomized UBR) to simultaneously estimate an optimal number of iterations, along with with the estimation of a tuning parameter, has been demonstrated in a toy two dimensional problem. Further demonstration work in more complicated problems is continuing.

4. Theoretical results which suggest how the method can be extended to four dimensional variational assimilation with the model as either a strong or a weak constraint have been established. Design of a 'toy' experiment to demonstrate the theory and practice is in progress.

5. A general theory of the estimability of multiple smoothing, weighting, and tuning parameters is under development. A number of examples which are relevant to the practical choice of which parameters may be estimated by GCV, UBR and MLE methods in three and four dimensional mesoscale and global scale numerical weather prediction have been worked out.

6. A test for spatial "white noise" vs. spatial "low frequency" structure for observations minus predictions, has been developed. This has potential for use as a diagnostic tool for certain systematic model errors, local phase errors and/or improper tuning.

7. A simple method for outlier (bad data) detection based on probability residual plots after spline smoothing and tuning by GCV has been demonstrated.

Work Plan, October 1, 1993-September 30, 1994

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1. Complete the preparation for publication of results reported in the progress report, sections 1. - 7.
2. Continue the development of 4 dimensional tuning methods. We are presently designing a toy experiment to test and demonstrate the use of randomized GCV and UBR in the context of a four dimensional assimilation. We expect to use a very simple toy dynamical system with simulated model and observational errors to establish the feasibility and suggest the validity of these methods to tune a four dimensional analysis, particularly one with model errors of a partly unknown nature. In addition we expect to provide reasonable, robust accuracy statements in this context. Aside from their application to forecasting, some of these methods have a large potential for improving reanalysis of historical data. We will continue this effort during the coming budget period.

Wahba/Johnson

1 October 1993 - 30 September 1994

I.	Labor and Fringe Benefits	<u>Hours</u>	<u>Rate</u>	<u>Cost</u>
	a) Principal Investigator GW	265	83.87	\$ 22,226
	b) Co-Investigator DJ	115	78.37	9,013
	c) Research Assistant	942	18.93	17,832
	d) Programmer FR	155	30.97	4,800
	e) Secretary	26	18.00	468
	Subtotal			<u>\$54,339</u>
II.	Travel			
	a) 5trips/1person/3days/Wash.DC			4,977
	(air fare \$500 per trip - lodg., meals, etc./day \$132 - car rental \$34/day)			
III.	Computer system maintenance			2,000
IV.	SSEC Indirect Cost at 40%			24,526
V.	Tuition Remission			
	26% of I.c. less f.b.			<u>4,158</u>
	TOTAL			<u><u>\$90,000</u></u>

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Meryl E. Gest
Meryl E. Gest, Admin. Officer
Research Administration-Financial