

# A NON-LINEAR APPROACH FOR DERIVATION OF TEMPERATURE PROFILES FROM TOVS-DATA

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## 1. Introduction

The calculation of temperature profiles from radiances received from TIROS Operational Vertical Sounder (TOVS) includes many problem areas. Some of these are cloud contamination, surface effects, angular effects and inversion methods. This paper will primarily deal with inversion methods.

Most inversion methods for derivation of temperature profiles from TOVS data are based on statistical techniques (Smith and Woolf, 1976). This paper will present a physical, non-linear inversion method, based on the integral form of the radiative transfer equation. The test data were chosen for the ALPEX Outer Region, March 4-5, 1982.

## 2. Theory

The radiative transfer equation for satellite based radiances is

$$\gamma(\nu, \theta) = \epsilon_{\nu} \cdot B(\nu, T_s) \cdot \tau(\nu, \theta, p_s) - \int_0^p B(\nu, T(p)) \cdot \frac{\partial \tau}{\partial p}(\nu, \theta, p) \cdot dp \quad (1)$$

where

- $\gamma(\nu, \theta)$  = radiance,
- $\nu$  = frequency,
- $\theta$  = zenith angle,
- $p$  = pressure ( $p_s$  = surface pressure),
- $T(p)$  = temperature ( $T_s$  = surface temperature),
- $\epsilon_{\nu}$  = emissivity,
- $\tau(\nu, \theta, p)$  = transmittance,
- $B(\nu, T)$  = Planck function.

Calculation of transmittances is described in the NOAA Technical Report NESS 85, Transmittances for the TIROS Operational Vertical Sounder. These calculations are made available through a NOAA software package for 40 different levels. Transmittance depends not only on frequency, angle and level, but also on water content and ozone content between the pressure level of interest and the satellite.

Numerical integration of (1) gives

$$\gamma_i = \sum_{j=0}^L a_{ij} B_i(T_j) + \epsilon_i \cdot b_i \cdot B_i(T_0)$$

for frequency  $i$  and level  $j$  ( $j = 0$  is surface,  $L$  is highest level). The coefficients  $a_{ij}$  and  $b_i$  depend on  $\tau$ .

Suppose we have an initial guess of temperature  $T_j^{(0)}$ , and the exact temperature is

$$T_j^{(1)} = T_j^{(0)} + \Delta T_j^{(1)}, \quad j=0,1,\dots,L,$$

then

$$\begin{aligned} \gamma_i &= \sum_{j=0}^L a_{ij} \cdot B_i(T_j^{(1)}) + \epsilon_i \cdot b_i \cdot B_i(T_0^{(1)}) \\ &= \sum_{j=0}^L a_{ij} \cdot B_i(T_j^{(0)} + \Delta T_j^{(1)}) + \epsilon_i \cdot b_i \cdot B_i(T_0^{(0)} + \Delta T_0^{(1)}) \\ &\approx \sum_{j=0}^L (a_{ij} \cdot B_i(T_j^{(0)}) + a_{ij} \cdot \Delta T_j^{(1)} \cdot \frac{\partial B_i}{\partial T}(T_j^{(0)})) + \epsilon_i \cdot b_i \cdot B_i(T_0^{(0)}) \\ &\quad + \epsilon_i \cdot b_i \cdot \Delta T_0^{(1)} \cdot \frac{\partial B_i}{\partial T}(T_0^{(0)}). \end{aligned}$$

After subtraction this becomes:

$$\begin{aligned} Y_i^{(1)} &= \gamma_i - \sum_{j=0}^L a_{ij} \cdot B_i(T_j^{(0)}) - \epsilon_i \cdot b_i \cdot B_i(T_0^{(0)}) \\ &= \sum_{j=0}^L a_{ij} \cdot \frac{\partial B_i}{\partial T}(T_j^{(0)}) \cdot \Delta T_j^{(1)} + \epsilon_i \cdot b_i \cdot \frac{\partial B_i}{\partial T}(T_0^{(0)}) \cdot \Delta T_0^{(1)} \end{aligned}$$

or a linear equation system

$$\overline{Y}^{(1)} = A^{(1)} \cdot \overline{\Delta T}^{(1)}.$$

From this linear equation system  $\overline{\Delta T}^{(1)}$  can be calculated, which will give  $\overline{T}^{(1)} = \overline{T}^{(0)} + \overline{\Delta T}^{(1)}$ . This could be generalized to an iterative scheme:

$$\begin{aligned} T^{(0)} &= \text{initial temperature,} \\ \overline{T}^{(k+1)} &= \overline{T}^{(k)} + \overline{\Delta T}^{(k+1)} \quad \text{with } k=0,1,2,\dots, \end{aligned}$$

where  $\overline{\Delta T}^{(k+1)}$  is solved from the linear equation system

$$\overline{Y}^{(k+1)} = A^{(k+1)} \cdot \overline{\Delta T}^{(k+1)}.$$

In order to weight all equations equally, each equation  $i$  should be multiplied by

$$1/\frac{\partial B_i}{\partial T}(B_i^{-1}(\gamma_i)).$$

Note that matrix  $A$  will be changed every iteration, because of its dependence on transmission coefficients, which in turn are dependent on the temperature profile. However, since temperature dependence is weak,  $A$  does not have to be recalculated for each iteration. Temperature profiles may be approximated with empirical orthogonal functions (Smith and Woolf, 1976). To make a stable system, tests have shown that more than three empirical orthogonal functions must not be used, if standard methods for solving linear equation systems are used.

### 3. Tests

Test data are taken from the data set of ALPEX Outer Region, March 5, 1982. The corrected radiances are used.

The following frequencies are used: HIRS channels 3, 4, 5, 6, 7, 15, 16 and 18; MSU channels 3 and 4. Water content data, used for calculation of transmission, are taken from radiosonde data at Copenhagen, Denmark. This is used for the whole area. This is a crude approximation, but because of the low water content in this period, it applies rather well. Empirical orthogonal functions are calculated from a set of radiosonde data at Gothenburg, Sweden, from March during period 1969-74. Emissivity is equal to unity for the MSU-channels, because correction for surface effects is already done in the NOAA test data.

The emissivity for HIRS channels could be handled in different ways. In Susskind and Rosenberg (1979), emissivity is taken at 0.85 or 0.95 for land and water, respectively at wavelengths greater than  $10 \mu\text{m}$ , and 0.95 or 0.98 for wavelengths less than  $5 \mu\text{m}$ . The non-linear system is solved for  $\epsilon = 0.86(0.02)1.00$ . The emissivity which gives minimum residual norm  $\|\bar{Y}\|_2$  is chosen.

Only cloud-free areas are dealt with. The subroutines from the Cooperative Institute for Meteorological Satellite Studies for initial guess temperature profiles are used to start the iterative scheme. Temperatures above 100mb are not allowed to change from initial guess, this means that 21 different levels (1000-100mb) are used. Five iterations have been made for each point, transmission coefficients have been calculated before the first and the fourth iteration.

### 4. Further development

Development of the method will continue. The ideas from this conference will greatly influence further work. A few points which seem important are:

- use of EOF-profiles with levels above 100mb;

- calculation of water content (this could be done with HIRS-channels 10,11, 12 with an iterative method, similar to the one used for temperature, or a statistical method);
- how to choose an appropriate emissivity coefficient must be analyzed further;
- only the first three empirical orthogonal functions are now used, but at least eight profiles ought to be used for a sufficiently good approximation (this results in an ill-conditioned system, which must be solved by some regularization method).

### References

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