# An Alternate Algorithm to Evaluate the Reflected Downward Flux Term for a Fast Forward Model

D.S. Turner
Meteorological Service of Canada
Downsview, Ontario, Canada

## Introduction

In order to assimilate vast amounts of satellite data received daily, fast and efficient radiative transfer models are favoured. Many approximations are used by these models in order to minimize their computational time. An appropriate approximation is one where observed radiances can be simulated by the model with zero bias and little error. Inappropriate approximations are ones where significant non-zero biases are produced. Significant non-zero biases result in observations having little impact on numerical weather prediction schemes unless an empirical bias correction scheme is employed. It is preferable to avoid approximations that produce non-zero biases, or at least find approximations that minimize the magnitude of the bias.

Some approximations only work over a limited range of situations. Within this range the biases are negligible, however once outside this range the biases may become significant. The current approximations of the attenuated reflected downward flux (ARDF) term of the top of the atmosphere (TOA) radiance equation, as seen by a satellite, are examples of limited applicability. Current fast forward models (FFM) approximate the downward flux by the downward radiance evaluated using the TOA transmission function defined by either the satellite zenith angle, model M1, (eg RTTOVS, Saunders et al, 1999) or a constant angle, model M2, (eg MSCFAST, Garand et al, 1999). The latter is more realistic as it precludes an explicit dependency on zenith angle, but is computationally slower as it requires two passes through a transmittance model. Earlier studies with the High-resolution Infrared Radiation Sounder, HIRS, and the Atmospheric Infrared Sounder, AIRS, (Turner, 2001) indicate that the bias of the difference between a line-by-line radiative transfer model (LBL) and these models are satellite zenith angle and surface pressure dependent, and that in many instances have a small bias only for a small range of relatively high emissivities and surface pressures. For example, assuming a target bias of less than .1K the useful range of emissivities in M1 is from 1 to .9 at low altitudes. Consequently this model is inappropriate for some frequencies over land where emissivities may be as low as .6 in sandy soil.

This article proposes an alternate method of approximating the reflected downward flux that attempts to reduce the bias over a wider range of emissivity and surface pressure whilst using less computational time than the two pass method of M2. The method modifies the surface to TOA transmittance function by raising it to the power $\kappa$ which is a function of the secant of the satellite zenith angle and surface pressure. A comparison of the proposed model, M3, is made with the other two models across the AIRS channels.

## Alternate Algorithm

The downward flux is approximated by replacing the angular integration of the transmission function by a transmission function evaluated at a representative zenith angle. The optimum diffusivity factor, the secant of this angle, is usually taken to be 1.66. The diffusivity factor varies with the optical depth (Liu, 1988), thus the assumption of a diffusivity factor that is not related to the atmosphere as in M1, or a constant diffusivity factor regardless of atmosphere, as in M2, invariably leads to errors. Consequently any improvement to the ARDF model should allow for a diffusivity factor that varies with optical depth.

The ARDF term is the product of the upward return transmittance and the downward flux, $<\Im_s(\theta)\, F^{\downarrow}>$, and is approximated by

$$\frac{(1-\varepsilon)}{\pi} <\Im_s(\theta)> \left[\left(\sum_{k=1}^{s} \frac{<\Im_{k-1}(\varphi)> - <\Im_k(\varphi)>}{<\Im_{k-1}(\varphi)> <\Im_k(\varphi)>} <\bar{B}_k>\right) <\Im_s(\varphi)>\right] \qquad (1)$$

where the term enclosed in [] is the approximation to $F^{\downarrow}$, $\Im_k$ is the level k to TOA transmittance, $\theta$ is the satellite zenith angle, $\varphi$ defines the path of the transmittance function approximating the downward flux, B is the Planck function, $\varepsilon$ is the surface emissivity (Note: the reflection is assumed to be isotropic) and the subscript **s** denotes the surface, which can be a topographical or cloud top surface. $\Re$, $\Im$ and B are functions of wavenumber and $<f>$ signifies that $f$ has been convoluted with a response function and integrated over wavenumber. The emissivity is considered to be constant over a response function. For models M1 and M2, $\varphi=\theta$ and $\sec\varphi=1.66$ respectively.

Frequently it is easier to consider differences between equivalent brightness temperature, BT, instead of radiance when comparing models. BT($f$) signifies that the radiance $f$ has been converted to a brightness temperature. Conversions are made using the band correction coefficient method outlined in Planet (1988).

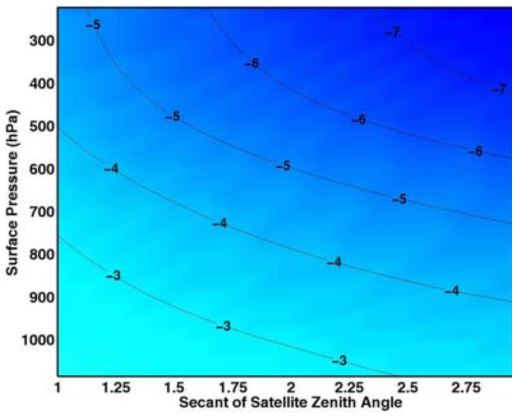


Fig 1: Comparison of the bias (K) across 52 atmospheres between BT($<\Im_s F^{\downarrow}>$) and BT($<\Im_s>$ $<F^{\downarrow}>$) for AIRS channel 1018.

Past experience shows that a term <ab> may not necessarily be approximated by <a><b>. There are many such terms in Eqn 1, particularly in the downward flux term. Figure 1 illustrates the mean of the difference between BT($<\Im_s F^{\downarrow}>$) and BT($<\Im_s><F^{\downarrow}>$), or bias, across a set of representative atmospheres. Even under this idealistic situation the biases are large. As there are many AIRS channels that exhibit similar behaviour this approximation cannot be relied on. Therefore any scheme to improve the ARDF model should also incorporate a correction to account for the decomposition of $<\Im_s F^{\downarrow}>$ in Eqn 1.

A simple approximation to the ARDF term was found that involves a simple transformation applied to the upward transmittance profile. The method makes the supposition that, on average, for a given surface pressure and zenith angle there exists a value $\varkappa$ which modifies $\Im(\theta)$ such that replacing $\Im(\varphi)$ with

$\Im(\theta)^{\varkappa}$ provides a good estimate of the RDF term; ie,

$$\frac{(1-\varepsilon)}{\pi} <\Im_s(\theta)> \left[ \left( \sum_{k=1}^{s} \frac{<\Im_{k-1}(\theta)>^{\kappa(p_s,\theta)} - <\Im_k(\theta)>^{\kappa(p_s,\theta)}}{<\Im_{k-1}(\theta)>^{\kappa(p_s,\theta)} <\Im_k(\theta)>^{\kappa(p_s,\theta)}} <\bar{B}_k> \right) <\Im_s(\theta)>^{\kappa(p_s,\theta)} \right] \quad (2)$$

$\kappa$ is interpreted as a combination of a variable diffusivity factor which depends on the optical depth above the surface and a correction that takes into account the decomposition of $<\Im_s F^{\downarrow}>$.

The evaluation of $\varkappa$ is straightforward. First, the fast mean transmittance and Planck models are constructed. Then for each surface pressure and zenith angle combination a search for the optimal $\kappa$ that minimizes

$$\left| \langle \Im(\theta, p_s) F^{\downarrow}(p_s) \rangle - \{\Im_s(\theta)\} \left( \sum_{k=1}^{s} \frac{\{\Im_{k-1}(\theta)\}^{\varkappa(p_s,\theta)} - \{\Im_k(\theta)\}^{\varkappa(p_s,\theta)}}{\{\Im_{k-1}(\theta)\}^{\varkappa(p_s,\theta)} \{\Im_k(\theta)\}^{\varkappa(p_s,\theta)}} \{\bar{B}_k\} \right) \{\Im_s(\theta)\}^{\varkappa(p_s,\theta)} \right| \le \delta \quad (3)$$

is implemented, where $\delta$ is an error criterion (ideally zero). The quantities enclosed by {} are obtained from the FFM. The procedure is repeated for a set of surface pressures and secants for each channel and the results set in a table.

For the creation of the new model, M3, $\{\Im_s(\theta)\}$ is first evaluated, followed by a determination of the relevant $\kappa(p_s,\theta)$ via bi-cubic interpolation within the $\kappa$-table, followed by the modification of $\{\Im_k(\theta)\}$ by $\kappa$ which is used by the ARDF calculation (Eqn 1). It should be noted that since the values of $\kappa$ are obtained by a minimization involving the fast model under consideration, they may not be directly applicable to a different model. For example, a set of $\kappa$ derived for RTTOV may not necessarily be optimal for MSCFAST.

## Simulations of Radiances

All quantities of the form $<f>$ are calculated with the fast LBL radiative transfer model (FLBL, Turner, 1995) assuming a non-scattering plane parallel atmosphere. These quantities were calculated for 52 diverse ECMWF model atmospheres (Chevallier, 2001) on 48 levels (.005 to 1085 hPa). The calculations were repeated for 17 zenith angles (sec$\theta$ = 1 to 3. in .125 steps), 21 emissivities (0 to 1. in .05 steps), and 24 surface pressures (223 to 1085 hPa). The wide range of surface pressures accounts for a wide range of topographical surfaces and cloud tops. These calculations are repeated for all 2378 AIRS channels.

Quantities of the form $\{f\}$ are evaluated by a fast model. The basic FFM is that of M1 (RTTOV). M1 uses the same predictors and methodology described in Saunders et al (1999). The regression coefficients of the RTTOV fast transmittance model are generated from the FLBL calculations applied to the atmospheres and conditions described above. Model M2 is a modified version of M1 in which a second pass is made through the fast transmittance model for sec$\varphi$=1.66. These transmittances are used to evaluate M2's downward flux, $\{F^{\downarrow}\}$. The new model, M3, is also a modified version of M1 in which the surface to space transmittance is modified by $\kappa$ to form its effective value of $\{F^{\downarrow}\}$. The $\kappa$-table is constructed using Eqn 3 with M1 supplying the quantities $\{f\}$. The minimization is applied to 24 surface pressures (223 to 1085 hPa) and 6 secants (1, 1.25, 1.5, 1.75, 2., 2.25) for each member of the 52 atmospheres. $\kappa(p_s,\theta)$ is the mean value of $\kappa(p_s,\theta)$ across the 52 atmospheres. In general the values of $\kappa(p_s,\theta)$ range between 0 and 2. They tend to decrease towards 0 with increasing optical depth; ie, with increasing surface pressure and increasing $\sec\theta$.

## Results

The FLBL, M1, M2 and M3 brightness temperature for each atmosphere are evaluated for 6 secants, 21 emissivities and 24 surface pressures and the bias and standard deviation (stdv) of differences between the FLBL and the FFMs are evaluated. These statistics represent the FFM model error.

On average M3 is a faster algorithm than M2. It takes about 1.25 times more CPU time than M1, whereas M2 takes about 1.6 times longer than M1.

A sample of the comparison between the 3 models for a specific ($\varepsilon$, $p_s$, $\theta$) as a function of AIRS channels is illustrated in Figs 2 and 3 along with the case where $\varepsilon$=1 (no ARDF). These figures only provide a snap shot of the total range of possibilities considered. Figures 4 through 7 illustrate the full variation of ($\varepsilon$, $p_s$, $\theta$) for a couple of channels.

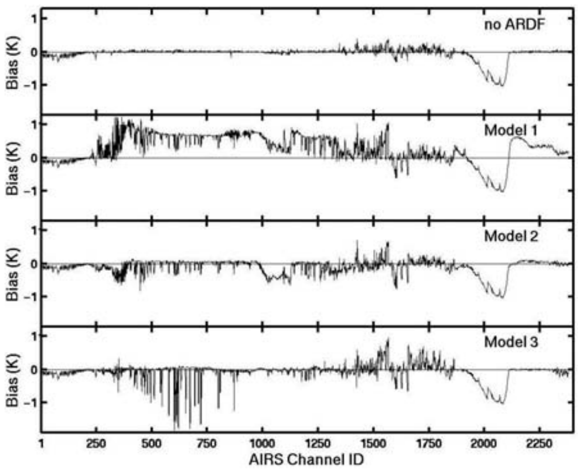


Fig 2: Bias of the differences between the FLBL and the 3 FFMs for $\varepsilon$=.7 and $P_s$=1013.25 hPa for a nadir view.

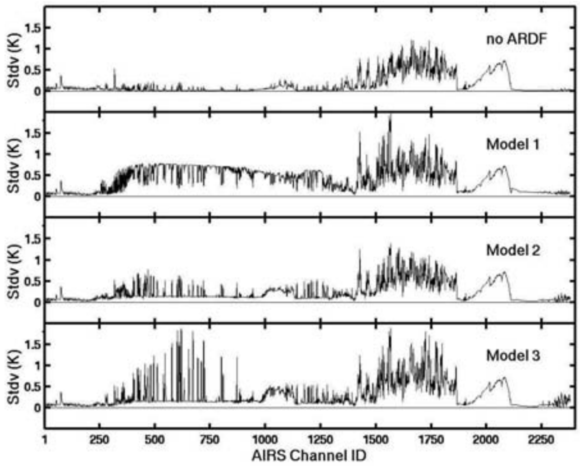


Fig 3: Stdv of the differences between the FLBL and the 3 FFMs for $\varepsilon$=.7 and $P_s$=1013.25 hPa for a nadir view.

In channels where the weighting function peaks well above the surface there is no difference between the models. In all virtually all other channels M1's performance is poor, which is mostly due to the explicit angular dependency in $F^{\downarrow}$.

M2 and M3 perform well over a wide range of $\varepsilon$ and $p_s$ in many channels, but does poorly in others. Both channels exhibit, at worst, a very small angular dependancy due to the explicit angular dependence of the return path, however the dependency is considerably smaller than that observed in M1. Both extend the range of $\varepsilon$ and $p_s$ where the biases are acceptablly small. In some channels M3 performs very well as seen in Fig.4, where the bias is acceptable over the entire range of $\varepsilon$ and $p_s$. In other channels M2 performs better than M3 as in Fig. 6, but only in a few channels does M2 achieve the very low bias across the full spread of $\varepsilon$ and $p_s$ as M3 does. In many instances the biases are acceptably low for emissivities greater than .5 and surface pressures greater than 850hPa which is the sufficient for much of the global surface, ocean and land.

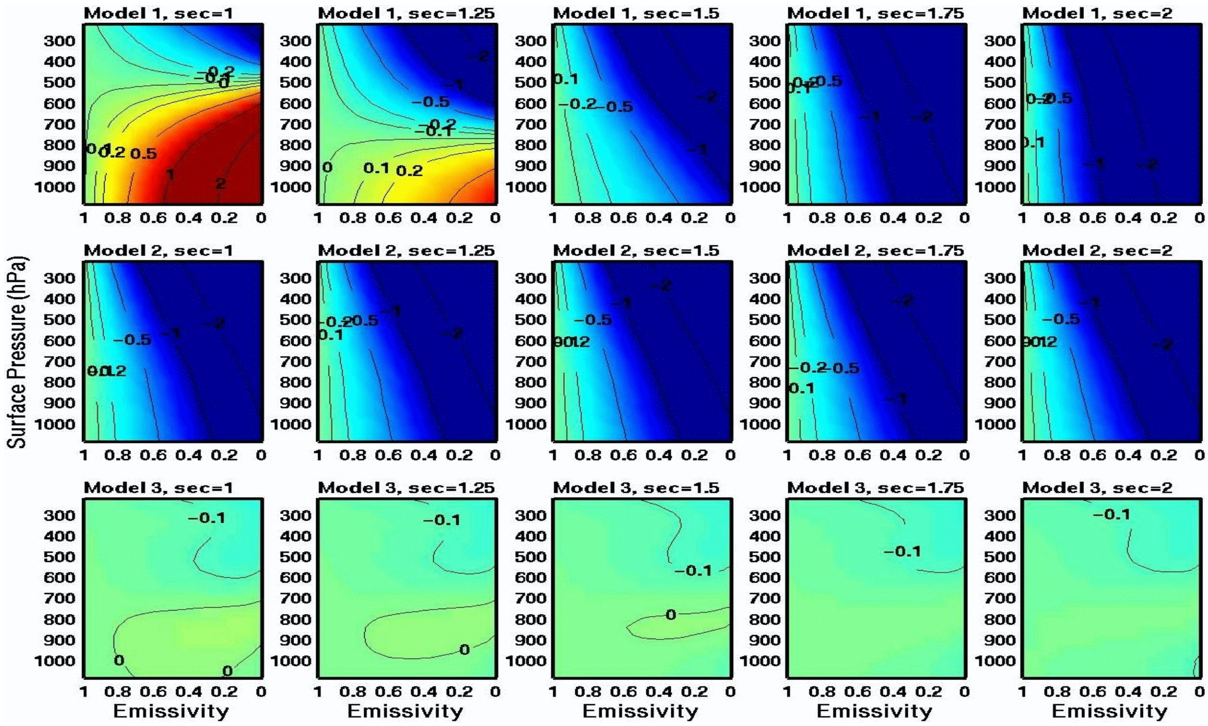


Fig 4: M1, M2 & M3 bias (K) as a function of surface emissivity, surface pressure and zenith angle secant for AIRS channel 1018 (1007.86 ($cm^{-1}$)).

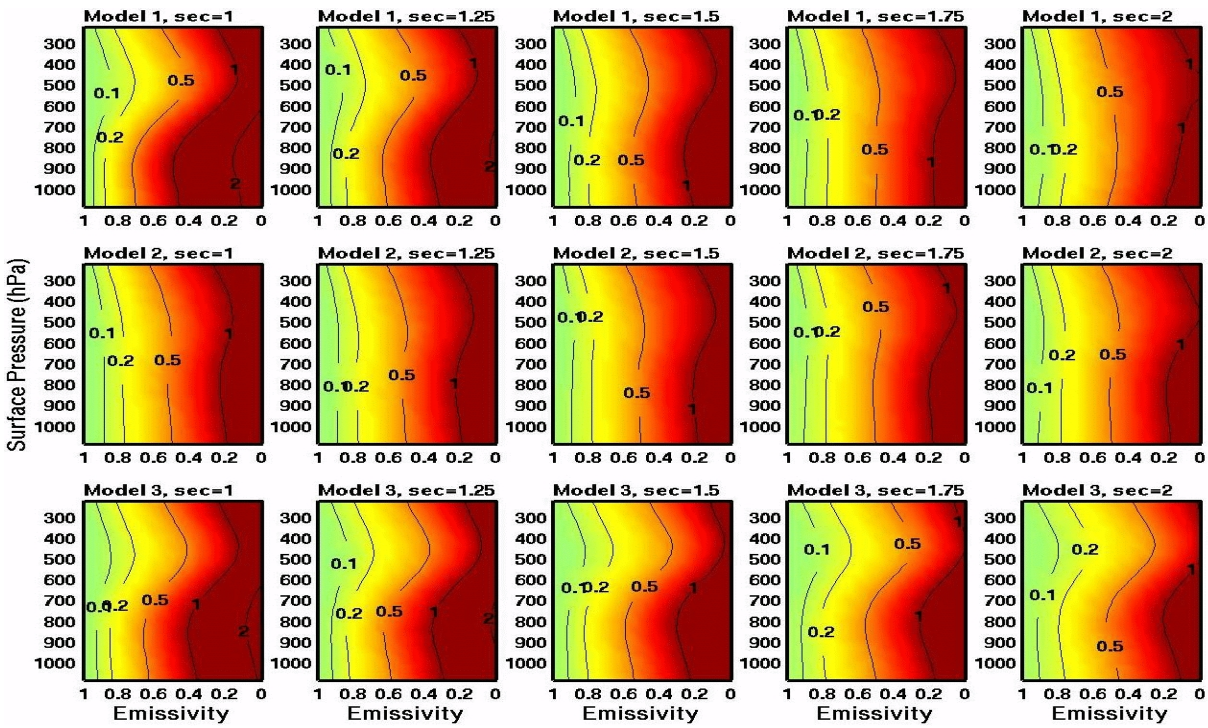


Fig 5: M1, M2 & M3 stdv (K) as a function of surface emissivity, surface pressure and zenith angle secant for AIRS channel 1018 (1007.86 ($cm^{-1}$)).

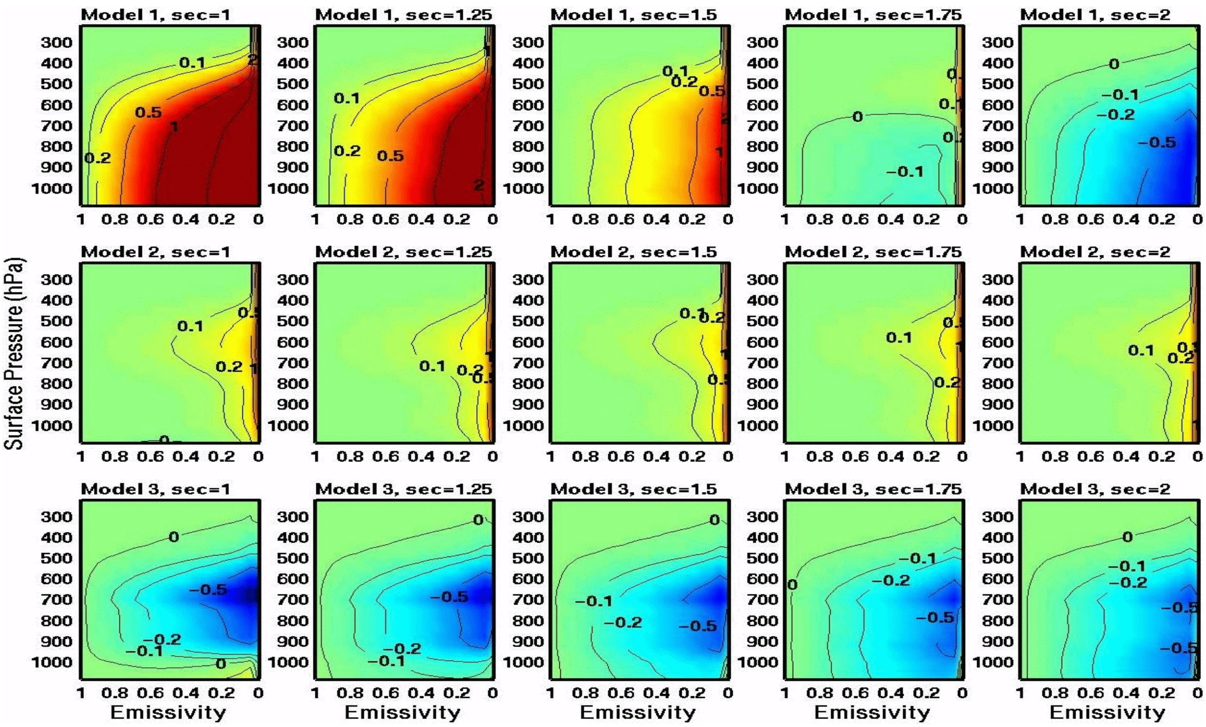


Fig 6: M1, M2 and M3 bias (K) as a function of surface emissivity, surface pressure and zenith angle secant for AIRS channel 610 (851.80 ($cm^{-1}$))

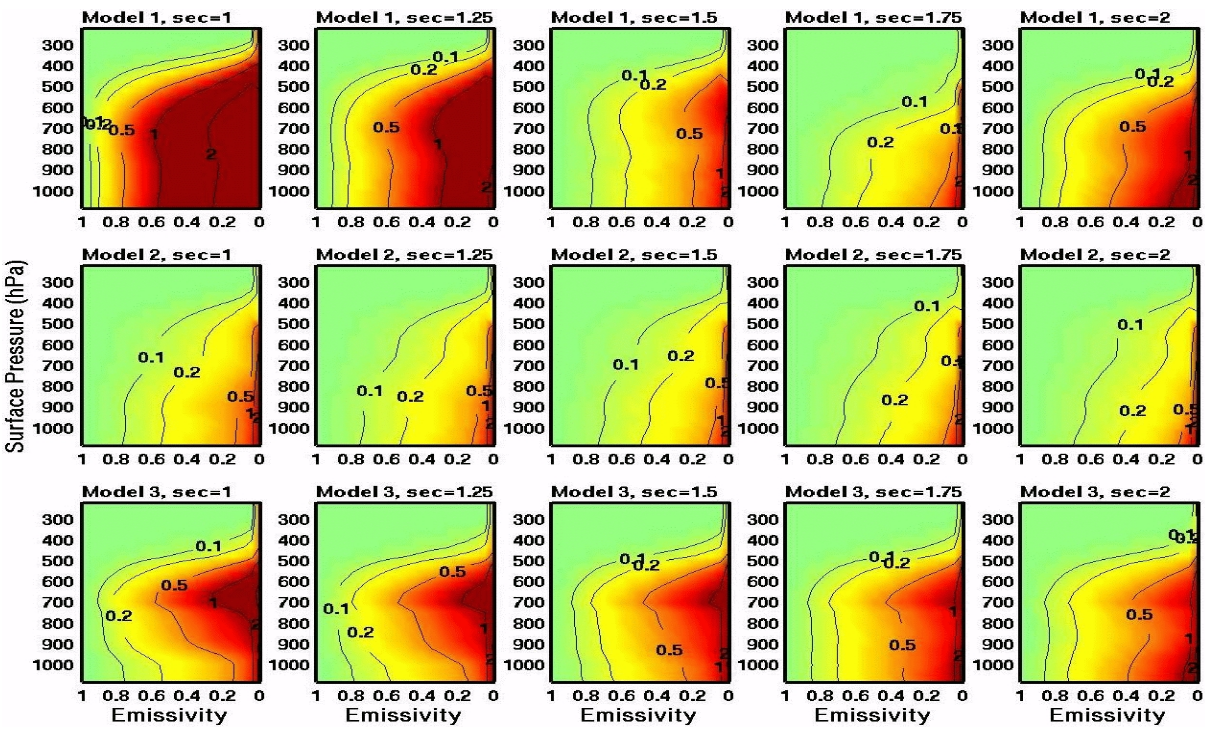


Fig 7: M1, M2 and M3 stdv (K) as a function of surface emissivity, surface pressure and zenith angle secant for AIRS channel 610 (851.80 ($cm^{-1}$)).

The angle dependency of the M1 standard deviations is weaker than the bias. The patterns of stdv for M2 are generally similar to those of the average pattern (over sec$\theta$) of M1 and those of the new model are similar to M2 but occasionally better. Generally the stdv increases with decreasing emissivity. Except for the angle dependency, the stdvs are not very different over the three models.

## Conclusions

An alternate algorithm has been developed which is more accurate than M1. Under many conditions it is as good as or better than the double pass model M2 and significantly faster in its execution compared with M2. Although demonstrated with the assumption of isotropic reflection, the scheme is actually independent of the angular distribution of the reflected energy, requiring only that the reflectivity be constant over the response function.

Unfortunately the new scheme is not yet consistently as good as the double pass method. Neither M2 nor M3 could be used exclusively over the entire AIRS spectrum without a penalty for many channels.. However, depending on the level of desired accuracy M3 could be used exclusively if the range of $\varepsilon$ and $p_s$ is constrained to emissivities and surface pressures greater than about .5 and 850hPa respectively. Exclusive use of M2 is possible under a somewhat more restricted regime. It is also possible that either model could be used exclusively under a carefully chosen subset of channels.

In principle one would expect that M3 would be the better model since it correctly allows for a variable diffusivity factor, however this is not always the case. There are many "spikes" in the bias and stdv

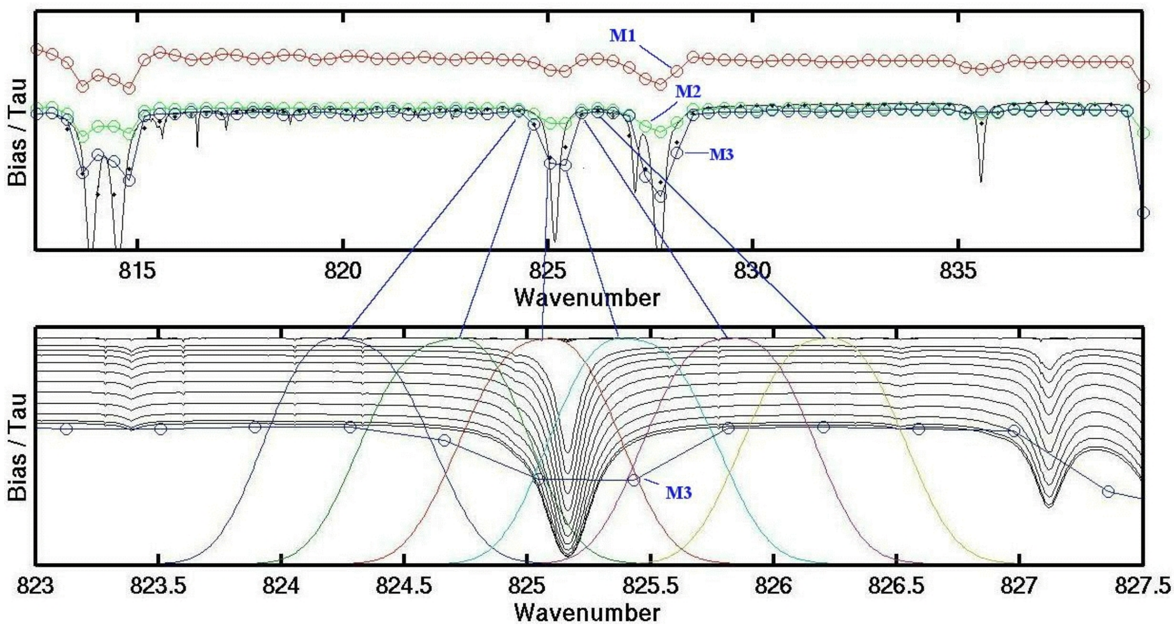


Fig 8: The upper box is an expanded view about channel 539 (825.046(cm$^{-1}$)) of the bias curves of Fig 2. The dark line with no circles is the TOA transmittance spectrum from 1013hPA for a standard atmosphere. The lower box is a further expansion of the spectrum. In addition to some additional TOA transmittance spectra from various pressures, six AIRS response functions are superimposed.

curves of Figs. 2 and 3. It is in these regions where M2 does better than M3. Figure 8 is a high resolution examination of Fig 2 about channel 539. Here it can be seen that the bias of all three models 'spike', M1 improves and the others do not (the stdv also 'spikes' at these locations). When overlayed with a typical TOA transmittance spectra and AIRS response functions it can be seen that these 'spikes' collocate with moderate to strong transmission lines, specifically the near-wing and core regions where the transmission function is highly non-linear. Most of the spikes observed in Fig. 2 are collocated to water vapour lines

More study is required to determine if the spikes can be eliminated. The problem appears to be in the coefficient generation scheme since the bias and stdv have 'spikes' when the ARDF is excluded. Until the problem of the spikes can be resolved the only advantage M3 has over M2 is computational speed, otherwise either model can be used, or specific models for specific AIRS channels.

## References

Chevallier, F., Sampled Databases of 60-level Atmospheric Profiles from ECMWF Analyses, SAF programme Research Report No. 4, EUMETSAT/ECMWF, 28pp, 2001.

Garand, L., D.S. Turner, C. Chouinard, and J. Hallé 1999. A physical formulation of atmospheric Transmittances for the massive assimilation of satellite infrared radiances, *J. Appl. Meteorol.* **38**, 541-544.

Liu, Quanhua and Johannes Schmetz, 1988. On the problem of an Analytical Solution to the Diffusivity Factor. *Beitr. Phys. Atmosph.*, **61**, 23-29.

Planet, W.G., Data Extraction and Calibration of TIROS-N/NOAA Radiometers. NOAA Tech. Memo. Ness 107-Rev. 1, U.S. Dept. of Commerce, Washington, D.C. U.S.A., 58pp, 1988

Saunders, R., M. Matricardi and P. Brunel, 1999. An improved fast radiative transfer model for assimilation of satellite radiance observations. *Appl. Opt.* **34**, 8396-8399.

Turner, D.S., 1995. Absorption Coeficient Estimation Using a Two Dimensional Interpolation Procedure, *JSQRT*, **53**(6) 633-637.

Turner, D.S., Revisiting the Attenuated Reflected Downward Flux Term of the Radiative Transfer Equation, in Technical Proceedings of the Twelfth International TOVS Study Conference, Lorne, Victoria, Australia, 27 Feb- 5 Mar 2002

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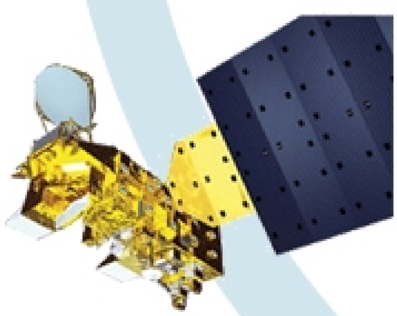

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