

## Analysis of Systematic Errors in Climate Products

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### Introduction

Remote sensing of the Earth's atmosphere and surface properties using observations from operational satellites has yielded significant advances in both weather prediction and, more recently, climate studies. In order for observations from operational satellites to be useful in climate studies, considerable attention must be paid to minimizing any systematic errors in the time series of observations from multiple satellites. Systematic errors can arise from several different causes: the intercalibration of one satellite to another in order to 'stitch together' a long time series, instrument and satellite health issues, and in the inversion of the radiative transfer equation. There has been considerable discussion of the first two causes, relatively little attention has been paid to the latter and it is thus, the focus of this discussion.

The MSU measurements from NOAA's polar orbiter sense broad atmospheric layer and have a long period of record (since 1979). For this reason, there have been several global change studies based on MSU channel measurement to infer tropospheric temperature change and trends. Unfortunately, many studies incorporate empirical vertical weighting functions and their combinations to retrieve lower tropospheric temperature trends. The use of an empirical weighting function lacks understanding of radiative properties in the atmosphere, hence we propose to use a radiative transfer model (RTM) used in numerical weather prediction data assimilation (e.g., RTTOV-v.8). Using a rigorous analytical approach, we test the hypothesis of combinations of MSU channel to correlate with mid-tropospheric temperature change and evaluate the systematic error that arises from this type of retrieval. Using the radiative transfer model, we can simulate MSU channel radiance (or brightness temperature) given profiles in a physically-consistent way.

### Analytical Derivation

Following the work Eyre (1987), we view the direct inversion solution to the radiative transfer equation as fundamentally ill-conditioned and, instead, seek to optimally solve an alternative problem. Consider a measurement system, in this case a satellite observing system, that provides observation information (with error) constrained with prior information (also with error). We then seek to find the optimal combination of the two via:

$$\hat{x} - x_0 = W \bullet (y_m - y_c \{x_0\}) \quad (1)$$

where,

$\hat{x}$  is the vector of retrieved atmospheric parameters

$x_0$  is the first-guess value of the vector

$y_m$  is the vector of multi-channel radiance measurements

$y_c \{x_0\}$  is the corresponding vector appropriate to the first guess

$W$  is the ‘inverse matrix’

The ‘inverse matrix’  $W$  is an operator that projects increment of measured radiances (differences of measured minus simulated) onto temperature increment from the first guess. In linear regression formulation,  $W$  is a minimum variance solution under the assumption of normal errors of both measurement and prior information. We follow an approach adopted in data assimilation whose solution for linear problem as such;

$$W = (K \bullet C)^T \bullet (K \bullet C \bullet K^T + E)^{-1} \quad (2)$$

where,

$C$  is the error covariance of the first guess,  $x_0$

$E$  is the error covariance of the measurements,  $y_m$

$K$  contains the partial derivatives of the measurements with respect to the profile evaluated at  $x_0$ , superscripts T and  $-1$  denote matrix transpose and inverse

The linear approximation to the forward radiative transfer problem is:

$$y_m - y_c \{x_0\} = K \bullet (x_T - x_0) + \varepsilon_m \quad (3)$$

where,

$x_T$  is the vector of true geophysical parameters

$\varepsilon_m$  is the vector of measurement errors, assumed to be random, Gaussian, unbiased and includes unbiased errors in the forward radiative transfer model.

Combining the forward and inverse radiative transfer equations and rewriting it in terms of the retrieval, first-guess and measurement errors yields,

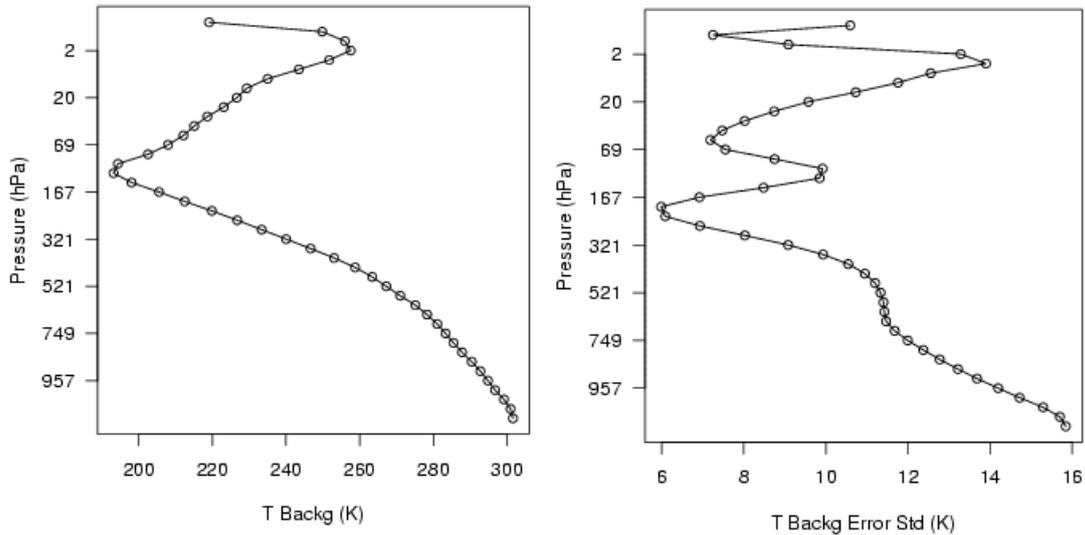
$$\hat{x} - x_T = (I - R) \bullet (x_0 - x_T) + W \bullet \varepsilon_m \quad (4)$$

where I is the identity matrix and  $R = W \bullet K$

### Application to MSU Tropospheric Temperature Trends

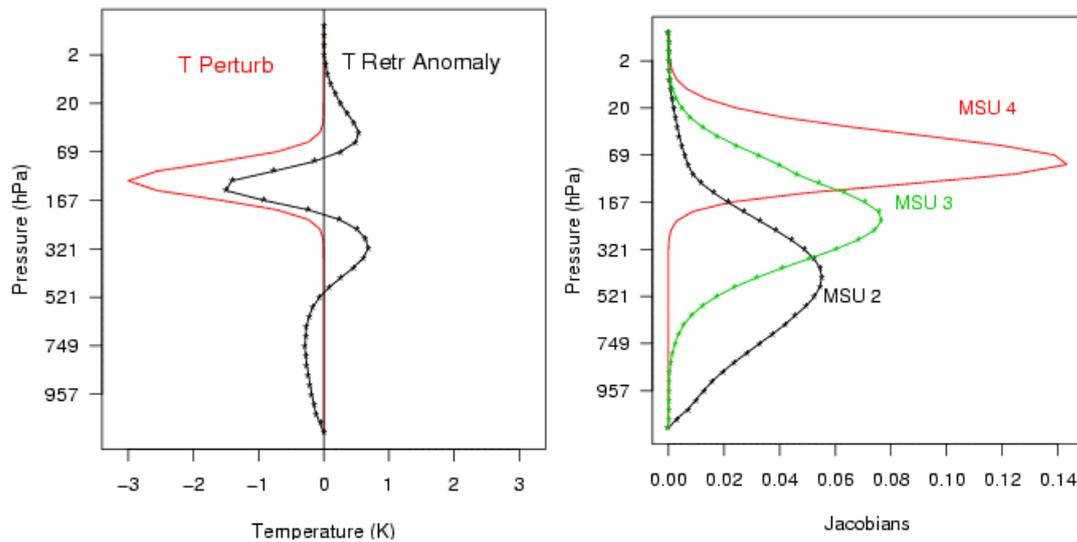
We use reference temperature profile of tropics (Fig.1) from the dataset in RTTOV package, and standard deviations (Fig. 2) are from sampled profiles of ERA-40 analysis

(Chevallier, 2002). In the tropics, the tropopause is near 100 hPa and the standard deviations of the reference profiles are large near the surface, at the tropopause and at the stratopause.



**Figure 1 (left).** Reference temperature profile used in simulations,  $\mathbf{x}_T$  of Eq. 4.

**Figure 2 (right).** Temperature error standard deviations used in the computation of retrieval, that is diagonal component of matrix  $\mathbf{C}$ .

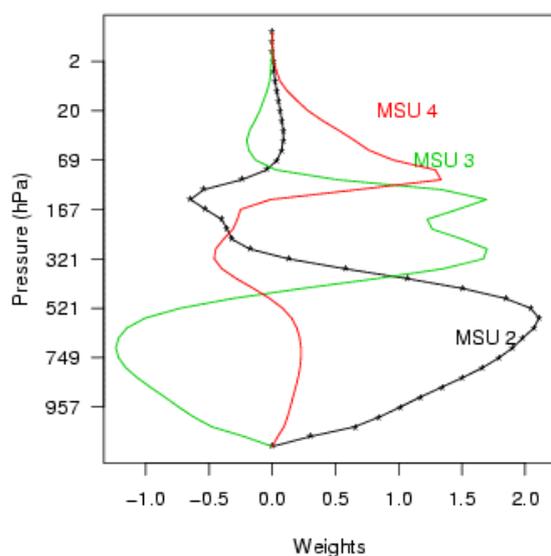


**Figure 3 (left).** Perturbed temperature, that is,  $\mathbf{x}_o - \mathbf{x}_T$  of Eq. 4, and increment of retrieved temperature profiles, that is  $\hat{\mathbf{x}} - \mathbf{x}_T$  of Eq. 4. This example is at the 3 K cooling at the 100 hPa, and the retrieval anomaly is the result of applying  $(\mathbf{I} - \mathbf{R})$  matrix in Eq. (4)

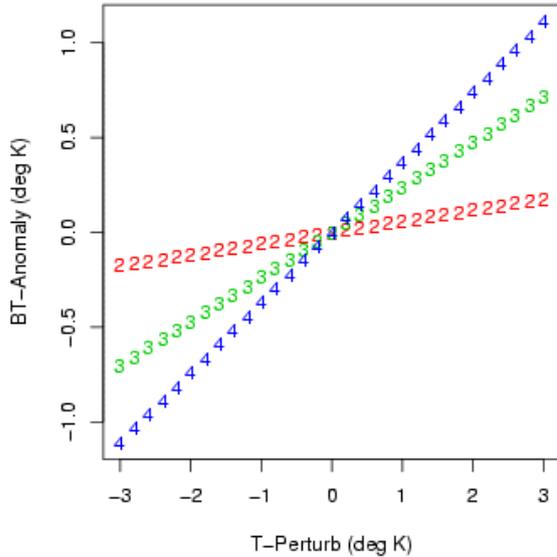
**Figure 4 (right).**  $\mathbf{K}$  matrices (Jacobian), whose elements are partial derivatives of MSU radiances with respect to state vectors.

We have perturbed temperature profiles at 100 hPa level by adding an increment of 0.2 K varying between +3 K and -3 K. Figure 3 is an example of -3 K cooling perturbation with no random error component. There are real world analogies to the perturbations; positive perturbations of stratospheric temperature have occurred in response to volcanic aerosol following eruptions, long-term stratospheric cooling has been observed in response to ozone loss. Jacobians (K matrices) are computed for three MSU channels (Fig.4). It is important to note that the Jacobians are not the same as the vertical weighting functions or empirical weighting functions used by other MSU researchers. The Jacobians are, however, more useful when considering incremental changes in the sensitivity of the MSU channels to perturbations in temperature.

We computed retrieval anomalies, Eq. (4) by explicitly computing matrices  $W$  in Eq. (2). Figure 5 shows the vertical distributions of  $W$  for three MSU channels. The structure of  $W$  is complex since it is a function of the background error, Jacobian, and measurement error. Figure 6 shows relationship between simulated radiance anomaly (difference from the no-perturbation) and amplitudes of perturbations. They are linear since we used a linear system. As expected, MSU channel 4 is most sensitive to the imposed perturbation since we imposed the perturbation in the stratosphere



**Figure 5.**  $W$  matrices corresponding to perturbed profile in Fig.3. The  $W$  matrix is function of background matrix  $C$ , Jacobian  $K$ , and measurement error  $E$  as seen in Eq. (2).



**Figure 6.** Anomaly of three MSU simulated radiances (brightness temperatures) from the base state with respect to temperature perturbations. Numbers stand for MSU channels. Channels 3 and 4 have higher sensitivities than channel 2 as perturbation is made at the stratosphere.

In regression solutions to the radiative transfer equation, the minimization technique forces all the systematic error to be reflected in the coefficients of the regression. As noted by Eyre, we would like the R matrix to approach the identity matrix which, in the case of regression, means all coefficients close to 1. Implicitly, regression solutions contain assumptions about the radiative properties of the atmosphere and channels being used. Such approaches can be accurate solutions to the radiative transfer equation and are computationally easy to apply to large amounts of data. Unfortunately, lack of careful application of systematic error analysis to the assumptions implicit in regression algorithms can lead to the propagation of systematic errors larger than the climate phenomena under study. This behavior is not well appreciated by the climate community.

In contrast to the simple impact of the stratospheric perturbation upon brightness temperature, the impact on the retrieved temperature is not straightforward because of vertical structure of errors (Fig. 2). Because of such vertical variability, the defining vertical layer for climate trend analysis becomes very important. We follow definition of Fu and Johanson (2005) in deep layer;

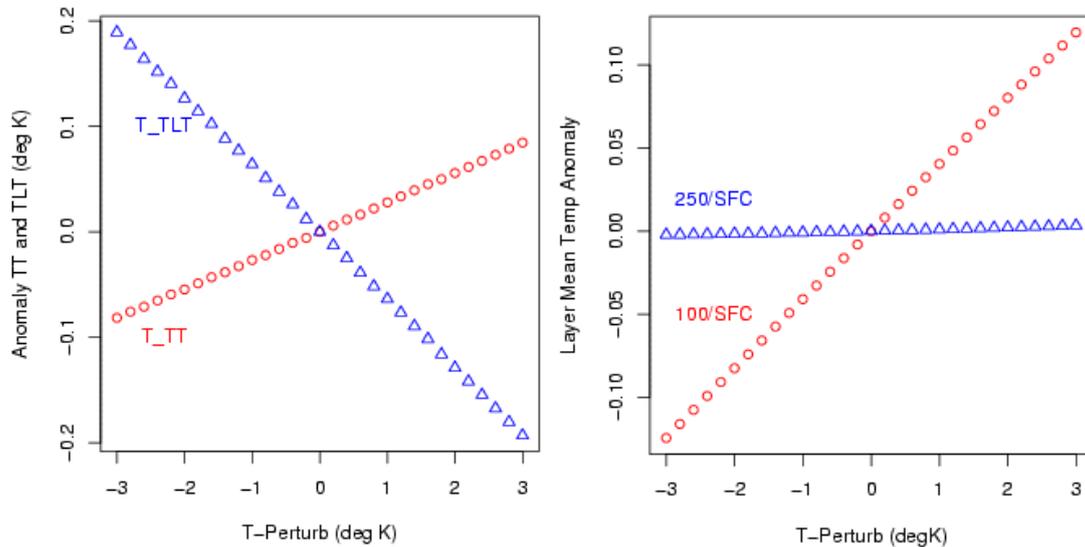
$$T_{TLT} = a_{23} T_2 + (1 - a_{23}) T_3, \quad (5a)$$

$$T_{TT} = a_{24} T_2 + (1 - a_{24}) T_4, \quad (5b)$$

where  $a_{23} = 1.69$ ,  $a_{24} = 1.61$  and  $T_2, T_3, T_4$  are simulated MSU radiances.

Figure 3 clearly shows that stratospheric perturbation spreads throughout the atmosphere. For example, when MSU channel 2 has 0.175 K cooling, channel 3 with 0.703 K cooling, and channel 4 with 1.109 K cooling from the base state of tropical atmosphere, the temperature must give rise two maxima (at 35.5 hPa, 286.6 hPa) and two minima (at 122.04

hPa, 702.7hPa). Thus, the amplification of systematic error by the first guess information arising from the mean of the profiles used in the derivation of the regression coefficients can lead to an anomaly in layer-mean temperature substantially different from the actual known imposed and known perturbation.



**Figure 7 (left).** Empirically retrieved layer temperatures according to Equations 5a and 5b.

**Figure 8 (right).** Layer mean temperature anomalies for layer of 100/SFC and 250/SFC with respect to stratospheric perturbations. The layer of 250 hPa – SFC is an equivalent parameter to compare with Fu and Johanson’s parameter  $T_{TLT}$ .

While  $T_{TLT}$  in Figure 7 supports Fu and Johanson’s motivation to account for stratospheric contamination in the MSU channel 2, the actual layer mean of retrieved temperature in 250/sfc (Figure 8) is opposite to  $T_{TLT}$  and much smaller sensitivity. The change of  $T_{TLT}$  is -0.3825 K while retrieved mean layer changed +0.0055 K, which imply that the trend estimated by  $T_{TLT}$  is 87 times the actual retrieval in the physically consistent dataset.

## Conclusion

All sources of systematic error must be rigorously evaluated when evaluating long-term trends. Analytical analysis of the radiative transfer equation using typical values for the background error covariance shows that amplification of systematic errors is to be expected. This method should be applied to all methods for retrieval of geophysical parameters for trend analysis. Then, by assuming a reasonable value for the value of the systematic error perturbation, one can make an estimate of the value of the systematic error.

In the case of this example, the long-term trend in stratospheric brightness temperature of MSU 4 can be used as an estimate. Estimates of this trend range from -0.323 K/decade to -0.446 K/decade over the last 25 years. To compare to the Fu and Johanson trend for the 16 year period for  $T_{TLT}$  using the lower MSU 4 trend value, we have  $-0.3825 \cdot -0.5168 = 0.1976$

K/decade. Using equation 5a we get a trend of 0.19 K/decade. The magnitude of the systematic error is found to be on the same order of the computed trend. This trend is thus not accurate but simply is due to the imposition of first guess information through the regression coefficients and the negative trend in MSU 4.

## References

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