

# The retrieval error analysis of atmospheric temperature profile from Satellite Data

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## Abstract

A theoretical analysis is performed to evaluate the retrieval precision of atmospheric temperature profiles obtained with the HIRS/3 data. This method is based on singular value decomposition and empirical orthogonal function technology. The theoretical results showed that the absolute errors in low latitude were less than that in the middle-high latitude and the geography distribution of relative errors is opposite to absolute errors in general. For vertical distribution, the errors are biggish in the upper and lower atmosphere; lesser from 500hpa to 850hpa. The sensibilities of retrieval to observation are the lowest in middle atmosphere.

## Introduction

The meteorological satellite can detect the global. The functions of satellite data used in weather and climate prediction are always regarded very important. Recently, high performance vertical detectors are put into the launched meteorological satellite one after the other. It has been proved that using three- and four-dimension assimilation method can assimilate the satellite radiation data directly and improve the global weather forecast distinctly. But as an information source of atmospheric parameter, the information provided by satellite data is narrow. In other words, the retrieval precision of temperature and humidity is limited. So the estimation of retrieval precision is important to the exertion of retrieval results and the assimilation of satellite data. Theoretically speaking, retrieval errors come from three aspects: radiance observation error, vertical resolution error and error of the fast radiative transfer model. Many scholars have theoretically analyzed the retrieval vertical resolution and errors on temperature and humidity profiles since 1970s (Huang et al., 1992; Rodgers, 1988; Thompson et al., 1986). These analyses were based on radiative transfer equation (RTE). Because the retrieval errors in theory vary with regions and seasons, a more detailed image of error distribution is needed.

In this paper, an analytic method of retrieval error is proposed. It is based on 'generalized linear inverses theory'. The retrieved atmospheric parameter modal can be separated under the hypothesis of RTE'S linearization. At the same time, the basal vertical structure of temperature can be picked-up by using EOF technique.

## The method of error analysis

On the condition of linear approximation, the issue of retrieving temperature profile can be come down to calculate the integral equation:

$$\delta I_{\nu} = \int_0^{p_s} K_{\nu}(p) \delta T(p) dp \quad (1)$$

where  $I_{\nu}$  is the radiation dose reached satellite inductor at frequency  $\nu$ ,  $T(p)$  is the air

temperature at pressure  $p$ ,  $p_s$  is the surface pressure,  $K_v(p)$  is variation kernel function.  $\delta I_v = I_v - I_v^0$ ,  $\delta T = T - T^0$ , superscript 0 denote reference value. For practical application, when the number of available observation channels is  $N$  and the layers of the profile to be retrieved is set to  $M$ ,  $N$  linear equations can be constructed and expressed by:  $\delta \mathbf{I} = \mathbf{K} \delta \mathbf{T}$ .

As mentioned in introduction, the characters of kernel function often make the retrieval ill-posed and cause the solution nonunique or sensitive to the observation errors. The properties of the solution can be examined by generalized inverse theory (Wiggins, 1972). This theory is based on singular vector decomposition (SVD). From the theory, matrix  $\mathbf{K}$  can be decomposed as:

$$\mathbf{K} = \mathbf{U}_p \mathbf{\Lambda}_p \mathbf{V}_p^T, \quad (2)$$

where  $\mathbf{\Lambda}_p$  is  $P \times P$  matrix containing  $P$  nonzero eigenvalues along the diagonal;  $\mathbf{U}_p$ ,  $\mathbf{V}_p$  is constructed by the left vectors  $u_i$  and right vectors  $v_i$ , respectively.  $u_i, v_i$  satisfy the orthogonal relation  $u_i^T u_j = \delta_{i,j}$  and  $v_i^T v_j = \delta_{i,j}$ . The decomposition implies that  $\delta \mathbf{T}$ 's projection in  $\mathbf{V}_p$  have relationship with the observation data and can be retrieved from observation. While the projection of  $\delta \mathbf{T}$  in subspace  $\mathbf{V}_0$ , is independent of the observation. That is the so-called resolution error. Assume the observation errors are independent and have the same variance  $\sigma_d^2$ . Then the variance of observation-caused error is obtained by:

$$\sigma_{rd}^2 = \frac{\sigma_d^2}{M} \text{tr}(\mathbf{V}_p \mathbf{\Lambda}_p^{-2} \mathbf{V}_p^T) = R \sigma_d^2, \quad (3)$$

where  $\text{tr}(\mathbf{A})$  is the trace of matrix,  $R$  is error amplification factor which is mainly determined by the minimum of eigenvalue  $\lambda_p^2$ . In actual calculation,  $\lambda^2$  is set to zero if  $\lambda^2$  is less than a specified value (Chou, 1986; Wunsch, 1978).

Another primary source of retrieval error is the resolution error. In order to analysis it, EOF technique is used. Consider a time series of temperature profiles represented by a  $L \times M$  matrix  $\mathbf{A}$ . Solve  $\mathbf{A}^T \mathbf{A} \mathbf{q}_i = \gamma_i \mathbf{q}_i$ , we get  $M$  eigenvalues  $\gamma_i$  and eigenvectors  $\mathbf{q}_i$ .  $\delta T_i$  can be expanded according to the orthogonal primary function. After the projection of  $\mathbf{q}_i$  in space  $\mathbf{V}_p$  had been counted, the total resolution error can be obtained by:

$$\sigma_{s,k}^2 = \sum_{i=1}^{M'} \langle c_i \rangle^2 e_{i,k}^2 = \sum_{i=1}^{M'} \langle c_i \rangle^2 (q_i - \sum_{j=1}^P a_{ij} v_j)^2, \quad (4)$$

where  $\langle c_i \rangle^2$  can be directly calculated from sample data. When the model error didn't be considered, retrieval error  $\sigma_{r,k}$  is the square root of the sum of observation-caused error, resolution error and truncation error of EOF.

## The retrieval error analysis

Utilizing the error estimate method given in last section, the global retrieval error distribution of temperature profile can be calculated with HIRS/3 data. The air parameter samples are the  $1^\circ \times 1^\circ$  NCEP data in Jan and Jul 1999-2003. The reference values are the average temperature profiles at each spot. The vertical structure of temperature increment in most regions can be represented by the first 7 truncated EOF vectors.

## Optimum truncation order in SVD

It's seen from last section that resolution errors decrease when the truncation order in SVD

increases, while the observation-caused errors increase. A good truncate order should be selected by considering both stability and resolution. When the observation error  $\sigma_d^2$  and atmosphere parameter samples were given, the optimum truncate order can be confirmed according to the least error in whole layer. Fig.1 is the distribution of optimum truncation order obtained with HIRS/3 data in Jan and Jul. Here the assumption that brightness temperature error is 0.25K is applied. It was shown in fig.1 that P is small near Tibetan Plateau, low latitude region and some regions of Antarctica. In Jan, P is 2 on some equator regions of Indian Ocean. In Jul, the regions corresponding to small P extend. In most regions of middle-low latitude of Northern Hemisphere P is smaller than or equal to 4. In Southern Hemisphere, the values of P are similar to that in Jan. The magnitude of P represents the information of temperature profiles retained in the satellite data. The larger P, the more information can be retrieved from satellite data.

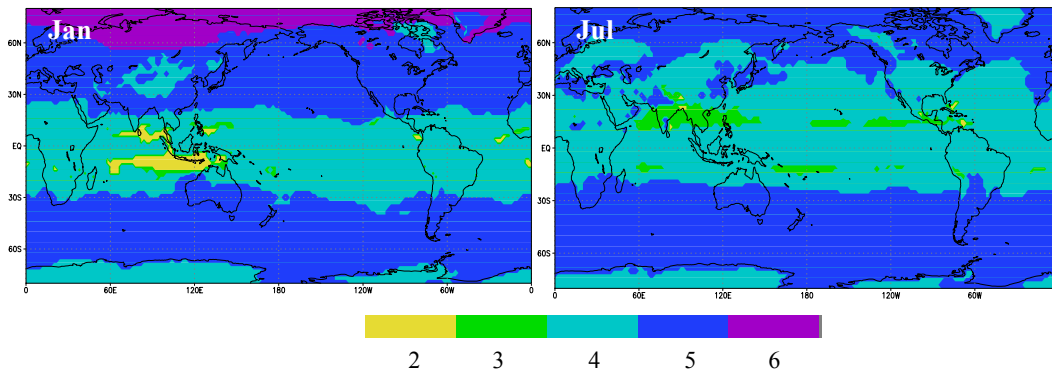


Fig.1 The optimum truncation order  $P$  in Jan and Jul

In fact, the distribution of optimum truncation order is dependent on the distribution of temperature mean square deviation. Fig.2 is the distribution of averaged mean square deviation of temperature over the whole layer. It shows that the value in low latitude region has minimum and it is larger in winter than that in summer in mid-high latitude region (Northern Hemisphere in Jan and Southern Hemisphere in July). This explains that high (or low) mean square deviation of temperature is corresponding to high (or low) optimum truncation order.

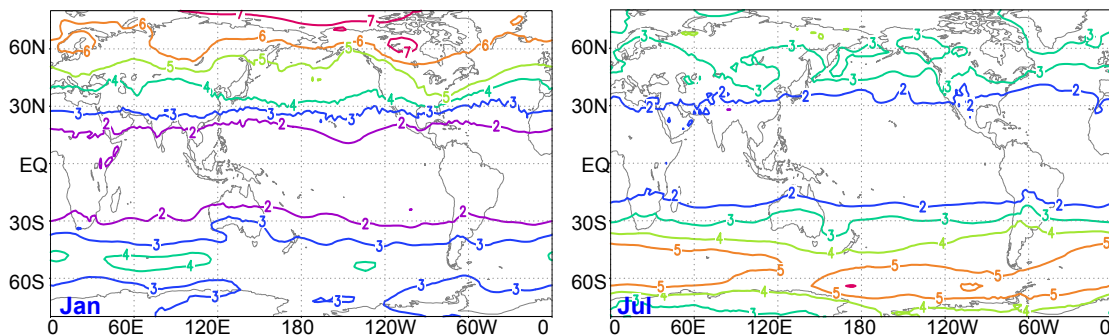


Fig.2 The rms (K) of temperature in whole level in Jan and Jul

## Result analysis

The retrieval errors can be calculated according to the optimal truncation order in each net point. Figs.3 is the retrieval errors at 500hpa, 850hpa in Jan and Jul. At 500hpa, the high error regions in northern hemisphere lie in the region from north Pacific to North American.

Qinghai-Xizang Plateau is also a high error zone; the error is about 2.0K currently. The high error zone in the southern hemisphere is still in the high latitude region, the error is about 2.0K. The error in low latitude is less than 1.0K. Error distribution in Jul is similar to Jan. But the values are less than Jan, about 0.5K. The retrieval errors in 850hpa tie to landform obviously. Whether Jan or Jul, the errors in mainland are higher than that of ocean regions clearly. The errors in mainland are about 2.0K (Jan) or 1.5K (July). The errors in ocean region are about 0.5-1.5K.

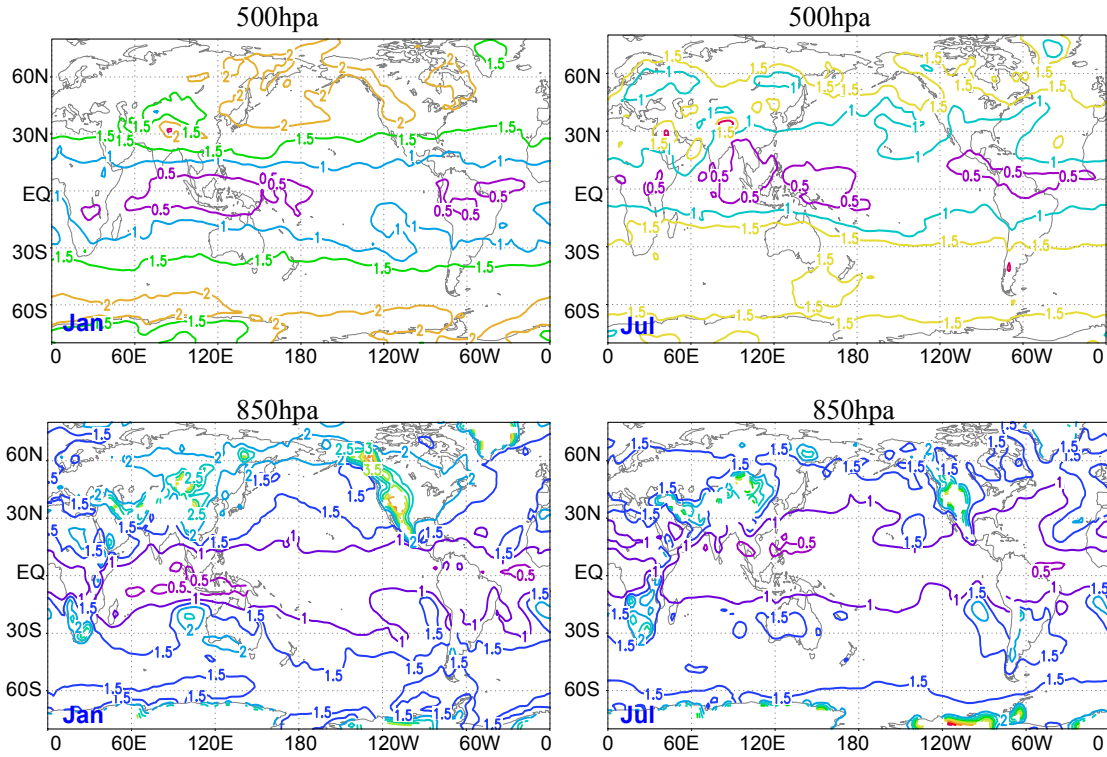


Fig.3 The absolute errors at 500hpa and 800hpa

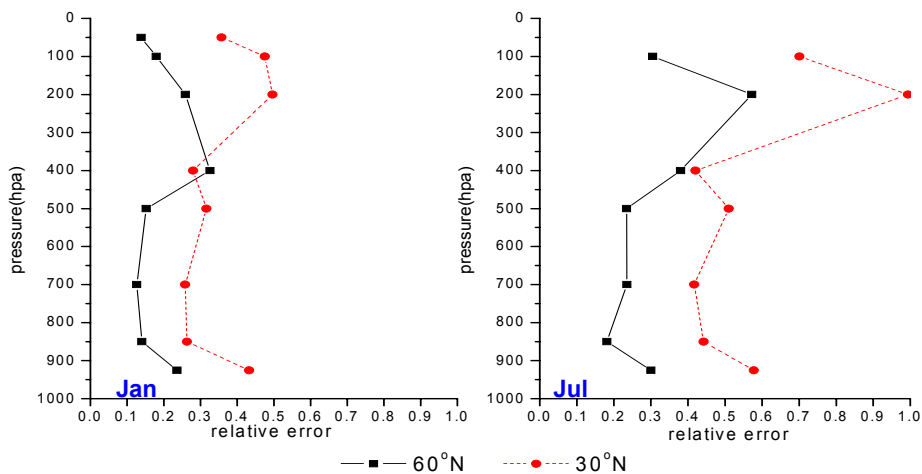


Fig. 4 Mean relative error at 60°N and 30°N

The total conclusion gained from above analysis is that the retrieval errors at ocean and Torrid Zone were lesser. But that didn't represent the retrieval ability, because the temperature

time variability in these regions is lesser. In order to show it, the relative errors that are defined as the quotient of absolute errors and the rms of temperature have been calculated. Figs.4 is the vertical profiles of relative errors averaged in two latitudes. The results showed that the character of vertical distribution at 60°N, 30°N is similar in Jan and Jul. That is, the relative errors are small in middle atmosphere; high in upper and lower atmosphere. The character of concrete values is that the higher (lower) latitude is corresponding to smaller (bigger) relative error. The variation trend at latitude is different to absolute errors completely. The comparison of relative error in Jan and Jul showed that the value in Jan was smaller than that in Jul. Especially the difference between Jan and Jul at 30°N is relatively big. These distribution characters of error in latitude relate to the time variability of the atmospheric temperature.

### Summary and discussion

The method of counting resolution errors and observation-caused errors with satellite data is based on 'generalized linear inverses theory'. When the observation error is given, the retrieval errors at each altitude can be calculated easily. The main modal of atmospheric vertical structure can be gained by EOF. Using NCEP temperature data as the samples, this method can calculate the optimal truncation order in generalized linear inverses and calculate the error distribution of temperature profile at each equipressure surface with HIRS/3 data in Jan and Jul. The basic characters are as follows:

(1)The optimum truncation order decides the effective information offered by satellite measurements. The values obtained with HIRS/3 data are between 3 and 7. The results show that the optimum truncation order is small in low latitude regions and high in mid-high latitude areas. This character relates to the time variability of temperature profile.

(2)For vertical distribution, the retrieval errors of temperature profile are more at upper and lower atmosphere. The errors between 400hpa and 850hpa are relatively small.

(3)For geography distribution, the absolute errors in middle-high latitude region, especially in the northern hemisphere, are the most; in equator zone are the least. While the distribution character of relative errors is reverse to absolute errors. This indicates that satellite data offered more effective information in the regions of biggish temperature variability.

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# Proceedings of the Fourteenth International TOVS Study Conference

Beijing, China  
25-31 May 2005

