Robust Variational Inversion: with simulated ATOVS radiances

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Motivation

Actually most technique for retrieval is based upon the least-square principal traced back to Gauss in 1809. The advantage of the least-square method is that the algorithm is easy to be implemented computationally. However, the least-square method is known to be non-robust. That is, the method is very sensitive to the outliers (data with the gross error), and generally needs a strict procedure of quality control to reject the outliers. In fact, some robust methods have been developed in the statistics community since 1960's. For example, a so-called "M-estimators" method from Peter Huber is well known in the "robust statistics". In this study, we will examine a "Robust Variational Inversion (RVI)" methods based upon the Huber-estimators. In one side, the method is of variational due to the use of the adjoint technique. In another side, the method is robust by taking into account the Non-Gaussian feature of data. The method is implemented in the framework of the 1DVAR retrieval with ATOVS data.

Methodology

In remote-sensing retrieval and data assimilation, one generally minimizes a cost function to obtain the best state estimation. For our robust variational inversion algorithm, the cost function can be written as

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b) + \frac{1}{2}[H(\mathbf{x})-\mathbf{y}]^T \mathbf{W(r)}\mathbf{R}^{-1}[H(\mathbf{x})-\mathbf{y}]$$

where
- $\mathbf{B}, \mathbf{R}$ are respectively the background and observation error covariance matrices.
- $\mathbf{X}_b, \mathbf{y}$ are respectively the background and observation vectors .
- H the observation operator which transforms from x to y space.
- $\mathbf{W(r)}$ A diagonal weight matrix with the elements $w(r_i)$ and the normalised departure $r_i = [y_i - H_i(x)]/\sigma_i$.

Weight function $w(x) = \psi(x)/x$ with $\psi(x) = dF(x)/dx$ called "influence function" (Huber, 1981).
$F(x)$ is the cost function of observation term. Obviously, for the square cost function $F(x) = x^2/2$, $w(x) = 1$.
For non-square cost functions which can be derived from Maximum-Likelihood Principal, $w(x)$ will not be 1 any more.

A descent algorithm generally requires to compute the gradient of the cost function at each iteration, namely

$$\nabla_x J(\mathbf{x}) = \mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b) + \mathbf{H}^T[\mathbf{W(r)}\mathbf{R}^{-1}(H(\mathbf{x})-\mathbf{y}) + \frac{1}{2}(H(\mathbf{x})-\mathbf{y})^T \frac{W(r)}{\sigma_o}\mathbf{R}^{-1}(H(\mathbf{x})-\mathbf{y})]$$

Note that the weight matrix W(r) should be recomputed with the updated residual r at each iteration, so the algorithm is also called "Re-weighted Least-Square" method.

A Robust 1DVAR Inversion framework

For testing the above algorithm, a Robust 1DVAR Inversion system originated from the 1DVAR code of ECMWF (Chevallier etal, 2002) is implemented to apply to the ATOVS radiances retrieval problem. A few new developments of the original code were done for this study: (1) the RTM interface is updated from RTTOV6 to RTTOV7; (2) the analysis variables are changed from the original 3 cloud variables profile to the free-cloud temperature T and the specific humidity q profile; (3) adding the robust weight function interface.
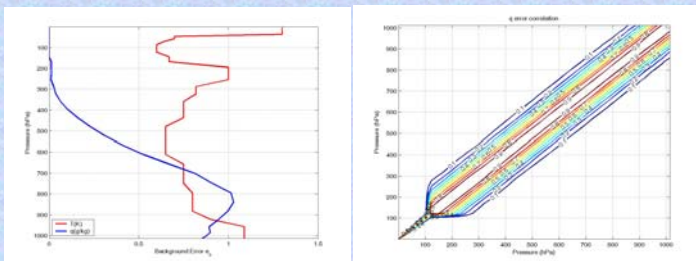

Fig. 1: The background error standard deviation for T and q (left panel) and the error correlation for q (right panel). The temperature error values are taken from Eyre etal. (1993), and the specific humidity error follows the scheme of Rabier etal. (1998). The error correlation is the same for T and q beside that the error correlation of q above 100hPa is removed.
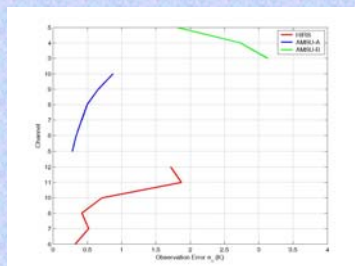

For observations, we take ATOVS HIRS channels 6,7,8,10,11,12 (sensitive to humidity), AMSU-A channels 5-10 (sensitive to temperature) and AMSU-B channels 3-5 (humidity channels). The observation error covariance matrix R is supposed to be diagonal, and equal to the diagonal elements of HBH^T which is explicitly computed using K matrix model of RTTOV. That is, the weight of the background and observations is considered to be equal. The error standard deviation of different channels is shown in Fig. 2. We can see that the error of humidity channels is much larger than that of temperature channels as also shown by Andersson etal. (2000).

Fig. 2: Specified observation error standard deviation for chosen ATOVS channels. They are obtained by explicitly computing HBH^T.

Summary

· A robust 1DVAR framework is implemented for retrieval of (T, q) profile from ATOVS radiances.
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· Preliminary test of robust algorithm with Huber weight function indicates potential of the method.
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· Further study in detail is need to compare with other weight function and to apply to real data.
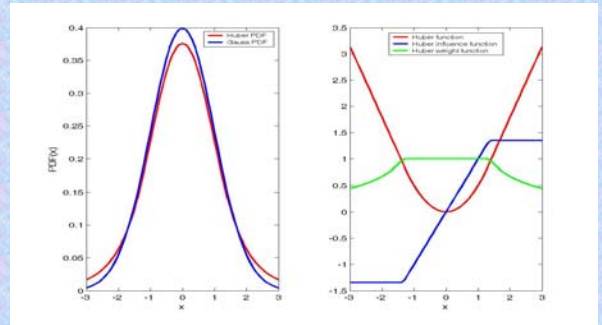
Fig.3: left panel: the Gaussian pdf and Huber's pdf (Huber, 1981); right panel: the cost function, influence function and weight function related to Huber's pdf which is used for the robust algorithm in this study.

Preliminary results with simulated ATOVS radiances

· **Truth**: an ECMWF profile from the original 1DVAR code.
· **Background**: Truth + Gaussian random error (100 realisations) which holds statistically the matrix B.
· **Observation**: RTM(Truth) + Gaussian or Laplacian (long tail) random error (100 realisations) with the same standard deviation as shown in Fig. 2.
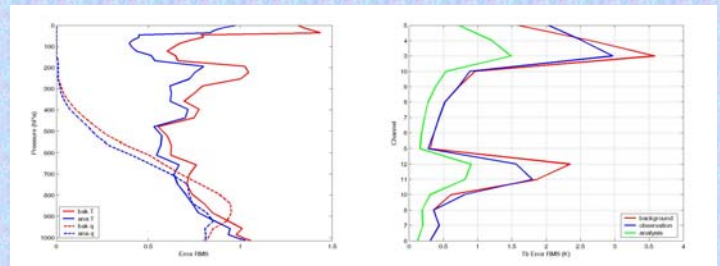

Fig 4: left panel: the averaged background and analysis error RMS of 100 realizations with Gaussian observation error and the least-square method. right panel: the background, observation and analysis error RMS in observation space.
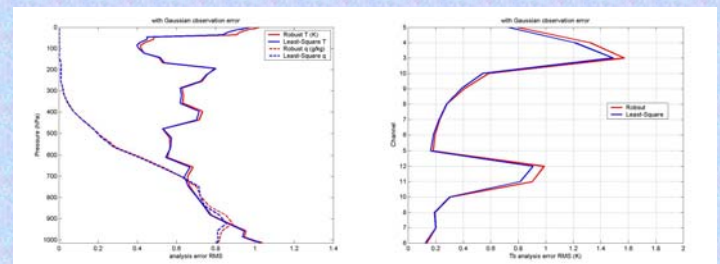

Fig 5: Comparison of the analysis error RMS for the robust and least-square algorithms with the Gaussian observation error, respectively in analysis-variable space (left panel) and observation space (right panel). Least-square is better than robust one.
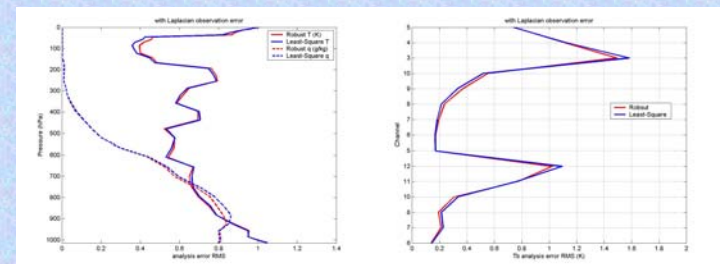

Fig 6: Same as Fig 5, but with the Laplacian observation error (long tail distribution). Robust result is modestly better than least-square result, particularly for specific humidity q.

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