

A LINEAR SIMULTANEOUS SOLUTION  
FOR TEMPERATURE AND ABSORBING CONSTITUENT PROFILES  
FROM RADIANCE SPECTRA

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## 1. INTRODUCTION

A linear form of the radiative transfer equation (RTE) is formulated for the direct and simultaneous estimation of temperature and absorbing constituent profiles (e.g., water vapor, ozone, methane, etc.) from observations of spectral radiances. This unique linear form of the RTE results from a definition for the deviation of the true gas concentration profiles from an initial specification in terms of the deviation of their "effective temperature" profiles from the true atmospheric temperature profile. The "effective temperature" profile for any absorbing constituent is that temperature profile which satisfies the observed radiance spectra under the assumption that the initial absorber concentration profile is correct. Differences between the "effective temperature", derived for each absorbing constituent, and the true atmospheric temperature is proportional to the error of the initial specification of the gas concentration profiles. The gas concentration profiles are thus specified after inversion of the linearized RTE from the retrieved "effective temperature" profiles assuming that one of the assumed concentration profiles is known (e.g., CO<sub>2</sub>). Because the solution is linear and simultaneous, the solution is computationally efficient. This efficiency is important for dealing with radiance spectra containing several thousand radiance observations as achieved from current airborne<sup>1</sup> and planned future spaceborne interferometer spectrometer sounders<sup>2,3</sup>. Here the solution is applied to spectral radiance observations simulated for current filter radiometers and planned interferometer spectrometers to demonstrate the anticipated improvement in future satellite sounding performance as a result of improved instrumentation and associated sounding retrieval methodology.

It has been shown from simulation studies<sup>4,5,6</sup> and airborne experiments that atmospheric temperature and water vapor profiles can be achieved with drastically improved vertical resolution and accuracy from remotely sensed infrared radiance observations which have a spectral resolution and spectral observation density which is one to two orders of magnitude better than current satellite systems. As a result of these studies, future polar orbiting and geostationary satellite sounding instrumentation is being specified to possess the high spectral resolution and spectral observation density properties required to achieve the sounding quality improvement. However, a practical application of this new instrument technology to the real-time operational production of soundings from their data requires the utilization of an extremely efficient profile retrieval algorithm. Such an algorithm is defined here and tested using spectral radiances simulated for both the currently operational High-resolution Infrared Sounder (HIRS) filter radiometer on the TIROS-N series of polar orbiting satellites and the planned GOES Platform High-resolution Interferometer Spectrometer (GPHIS).

## 2. ANALYTICAL DEVELOPMENT

In this section, we develop a linearized form of the radiative transfer equation (RTE) and formulate its inverse solution for the temperature and absorbing gas profiles from spectral radiance observations. In the equations which follow:

$R_\nu$  = spectral radiance with subscript  $\nu$  denoting spectral frequency

$B_\nu(p)$  = Planck radiance at pressure level  $p$

$p$  = pressure with subscript  $s$  denoting the surface

$\tau_\nu(p)$  = the total transmittance of the atmosphere above atmospheric pressure level  $p$

$\delta()$  = the difference between the true quantity and the initial value denoted by a superscript  $o$

$\Pi()_i$  = the product of  $i$  quantities

$\Sigma()_i$  = the summation of  $i$  quantities

$\tau_i$  = the transmittance of the atmosphere for the  $i$ th absorbing constituent

$k_{\nu_i}$  = absorption coefficient for the  $i$ th absorbing constituent

$q_i$  = mixing ratio of the  $i$ th absorbing constituent

$U_i$  = path length of the  $i$ th absorbing constituent

$$(u_i = - \int_g^p q_i(p') dp')$$

$T(p)$  = the true atmospheric temperature profile

$T_i(p)$  = the "effective temperature" of the  $i$ th absorbing constituent where the "effective temperature" profile is that atmospheric temperature profile which would give rise to the radiance observed in spectral regions where the  $i$ th absorbing constituent dominates the absorption/emission process assuming that the initial gas concentration profile is correct.

$T_{B\nu}$  the brightness temperature at spectral frequency  $\nu$

The true spectrum of monochromatic radiance exiting the earth-atmosphere system is

$$R_V = B_V(p_s)\tau_V(p_s) - \int_0^{p_s} B_V(p)d\tau_V(p) .$$

The radiance spectrum corresponding to an assumed initial temperature and absorbing gas distribution is

$$R_V^0 = B_V^0(p_s)\tau_V^0(p_s) - \int_0^{p_s} B_V^0(p)d\tau_V^0(p) .$$

Thus, the difference between the true and initial radiance spectrum is

$$\delta R_V = R_V - R_V^0 = B_V(p_s)\tau_V(p_s) - B_V^0(p_s)\tau_V^0(p_s) - \int_0^{p_s} B_V(p)d\tau_V(p) + \int_0^{p_s} B_V^0(p)d\tau_V^0(p) . \quad (1)$$

Employing the linear perturbation definition:

$$\delta B_V(p) = B_V(p) - B_V^0(p)$$

$$\delta \tau_V(p) = \tau_V(p) - \tau_V^0(p)$$

(1) can be written as

$$\delta R_V = B_V(p_s)\delta\tau_V(p_s) + \delta B_V(p_s)\tau_V^0(p_s) - \int_0^{p_s} B_V(p)d[\delta\tau_V(p)] - \int_0^{p_s} \delta B_V(p)d\tau_V^0(p) . \quad (2)$$

Since

$$\int_0^{p_s} B_V(p)d[\delta\tau_V(p)] = B_V(p_s)\delta\tau_V(p_s) - \int_0^{p_s} \delta\tau_V(p)dB_V(p)$$

as a result of integration by parts, the fully non-linear equation becomes

$$\delta R_{\nu} = \tau_{\nu}^0(p_s) \delta B_{\nu}(p_s) - \int_0^{p_s} \delta B_{\nu}(p) d\tau_{\nu}^0(p) + \int_0^{p_s} \delta \tau_{\nu}(p) dB_{\nu}(p) \quad (3)$$

The difference from a completely linearized equation is that  $dB_{\nu}(p)$  is not taken along the initial state.

Using the chain rule;  $dB_{\nu}(p^0) = \beta_{\nu}^0(p) dT(p)$ , and the Taylor approximation;  $\delta B_{\nu}(p) = \beta_{\nu}^0(p) \delta T(p)$ , where  $\beta_{\nu}^0 = \partial B_{\nu}(T^0)/\partial T$ , the linear temperature dependence can be separated in the first integral term and the lapse rate dependence becomes explicit in the second integral term.

$$\delta R_{\nu} = \beta_{\nu}^0(p_s) \tau_{\nu}^0(p_s) \delta T_s - \int_0^{p_s} \beta_{\nu}^0(p) \delta T(p) \frac{d\tau_{\nu}^0(p)}{dp} dp + \int_0^{p_s} \beta_{\nu}^0(p) \delta \tau_{\nu}(p) \frac{dT(p)}{dp} dp \quad (4)$$

Now for monochromatic radiation

$$\tau_{\nu}(p) = \exp\left[-\frac{1}{g} \sum_i \int_0^p k_{\nu i} q_i(p') dp'\right].$$

Because of the exponential form, the total atmospheric transmittance can be defined as the product of the transmittance of all the individual absorbing gases; thus,

$$\tau^0 = \prod_i \tau_i^0$$

and

$$d\tau^0 = \tau^0 \sum_{i=1}^N d \ln \tau_i^0, \text{ where } N = \text{number of absorbing constituents} \quad (5)$$

Relation (6) leads to the finite difference approximation

$$\delta \tau = \tau^0 \sum_{i=1}^N \delta \ln \tau_i \quad (6)$$

Also, for monochromatic radiation it can be shown that

$$\tau_{vi}(p) = \exp\left[-\frac{1}{g} \int_0^p k_{vi} q_i(p) dp\right] = \exp\left[-\int_0^{U_i} k_{vi} dU_i\right].$$

so that

$$\ln \tau_{vi} = -\int_0^{U_i} k_{vi} dU_i = -\int_0^p k_{vi} \frac{dU_i}{dp} dp.$$

Using this relation, the following linear expansion holds:

$$\delta \ln \tau_{vi} = -\int_0^p k_{vi} \frac{d\delta U_i}{dp} dp = \delta U_i \frac{d \ln \tau_{vi}^0}{dU_i^0} - \int_0^p \delta U_i \frac{d}{dp} \left( \frac{d \ln \tau_{vi}^0}{dU_i^0} \right) dp \quad (7)$$

where integration by parts was used and  $k_{vi} \equiv d \ln \tau_{vi}^0 / dU_i^0$ .

Thus, from (6) it follows that

$$\delta \tau_v = \tau_v^0 \sum_i \delta U_i \frac{d \ln \tau_{vi}^0}{dU_i^0} - \tau_v^0 \sum_i \int_0^p \delta U_i \frac{d}{dp} \left( \frac{d \ln \tau_{vi}^0}{dU_i^0} \right) dp \quad (8)$$

where the temperature dependence of  $\tau$  is ignored. A fully linearized solution can be achieved by neglecting the second term on the right of (8) which is much smaller than the first term. Neglecting this second order term of (8) and substituting the remainder into (4) gives

$$\begin{aligned} \delta R_v &= \beta_v^0(p_s) \tau_v^0(p_s) \delta T_s - \int_0^{p_s} \beta_v^0(p) \delta T(p) \frac{d\tau_v^0(p)}{dp} dp \\ &+ \sum_i \int_0^{p_s} \delta U_i(p) \tau_v^0 \left( \frac{d \ln \tau_{vi}^0}{dU_i^0} \right) \beta_v^0(p) \frac{dT(p)}{dp} dp \end{aligned} \quad (9)$$

We note from (9) that there is no information about the gas concentration profile,  $\delta U_i(p)$ , in regions where the lapse rate ( $dT(p)/dp$ ) is zero. Now (9) can be rewritten in the form

$$\delta R_{\nu} = \beta_{\nu}^{\circ}(p_s) \tau_{\nu}^{\circ}(p_s) \delta T_s - \sum_i \{ \delta T(p) [dU_i^{\circ}(p)/dp] - \delta U_i(p) [dT(p)/dp] \} W_{\nu i}(p) dp \quad (10)$$

where

$$W_{\nu i}(p) = \beta_{\nu}^{\circ}(p) \tau_{\nu}^{\circ}(p) [d \ln \tau_{\nu i}^{\circ}(p) / dU_i^{\circ}(p)]$$

If we define

$$\delta T_i(p) \equiv \delta T(p) - \delta U_i(p) \left[ \frac{dT(p)}{dU_i^{\circ}(p)} \right] \quad (10a)$$

Then (10) becomes

$$\delta R_{\nu} = \beta_{\nu}^{\circ}(p_s) \tau_{\nu}^{\circ}(p_s) \delta T_s - \sum_{i=1}^N \int_0^{p_s} \beta_{\nu}^{\circ}(p) \delta T_i(p) \tau_{\nu}^{\circ}(p) d \ln \tau_{\nu i}^{\circ}(p) \quad (11)$$

which is a general and fully linearized perturbation form of the radiative transfer equation.

In (11),  $N$  is the number of optically active atmospheric constituents,

$$\beta_{\nu}^{\circ}(p) = \partial B_{\nu}(T^{\circ}) / \partial T \quad ,$$

and

$$\delta T_i(p) = T_i(p) - T^{\circ}(p) \quad .$$

### 3. INVERSE SOLUTION

The general strategy for solving (11) for atmospheric temperature and absorbing constituent profiles from a satellite observed spectral radiance distribution involves the following steps:

- (1) solve for  $\delta T_i(p)$  for all constituents using a direct linear simultaneous matrix inverse solution of (11), given a spectrum of radiance,  $R_{\nu}$
- (2) let  $T(p) = T^{\circ}(p) + \delta T_k(p)$  where  $k$  denotes a constituent whose concentration, and therefore atmospheric transmittance profile (i.e.,  $\tau_i(p)$ ) is known a priori (e.g.,  $CO_2$ )
- (3) use (10a) to solve for the concentration of all remaining constituents, i.e.,

$$U_i(p) = U_i^{\circ}(p) + \frac{dU_i^{\circ}(p)}{dT(p)} [T(p) - T_i(p)]$$

A. Statistical Matrix Inverse Solution of (11)

It is convenient to write (11) in terms of brightness temperature by assuming

$$\delta R_{\nu} = \frac{\partial B_{\nu}(T_B^0)}{\partial T} \delta T_{B\nu}$$

or

$$\delta T_{B\nu} = \tilde{\beta}_{\nu}^0(p_s) \tau_{\nu}^0(p_s) \delta T_s - \sum_{i=1}^N \int_0^{p_s} \tilde{\beta}_{\nu}^0(p) \delta T_i(p) \tau_{\nu}^0(p) d \ln \tau_{\nu i}^0(p)$$

where

$$\tilde{\beta}_{\nu}^0(p) = (\partial B_{\nu}(T^0)/\partial T) / (\partial B_{\nu}(T_B^0)/\partial T)$$

In matrix form

$$t_b = At \tag{12}$$

The elements of A are

$$A_{\nu,0} = \tilde{\beta}_{\nu}^0(p_s) \tau_{\nu}^0(p_s)$$

$$A_{\nu,j} = -\tilde{\beta}_{\nu}^0(p_{i,j}) \tau_{\nu}^0(p_{i,j}) d \ln \tau_{\nu i}^0(p_{i,j})$$

$$j = 1, 2, \dots, N * M_i$$

where N is the number of constituents and  $M_i$  is the number of quadrature pressure levels devoted to each constituent.

The generalized statistical/physical solution of (12) is given by the Maximum Likelihood Retrieval<sup>4,5</sup>

$$t = S_d A^T (A S_d A^T + E_b)^{-1} t_b \tag{13}$$

$S_d$  is a dependent sample statistical covariance matrix and  $E_b$  is the covariance of the brightness temperature error. The elements of the temperature covariance matrix, s, are defined

$$S_{j,k} = (1/N_s) \sum_{n=1}^N [T_i^n(p_j) - T^o(p_j)] [T_i^n(p_k) - T^o(p_k)]$$

$$j,k = 1, \dots, N \cdot M_i \\ i = 1, \dots, N$$

where  $T_i^n(p_j)$  is the temperature of the  $i$ th constituent at the  $j$ th constituent pressure level, the superscript  $n$  represents the  $n$ th profile in the sample set, and  $N$  is the total number of atmospheric profiles in the statistical sample, and note that  $T^o(p_j)$  is used repeatedly  $N$  times in the equation.

Here,

$$T_i(p_j) = T(p_j) - \frac{dT(p_j)}{dU_i^o(p_j)} [U_i(p_j) - U_i^o(p_j)] \quad (14)$$

The linear dimension of the matrix to be inverted in (13) is the number of spectral channels which in our case is the largest of the dimensions considered. In order to reduce the dimensionality of the matrix inverse, we can use the matrix identity

$$S_d^T (A S_d^T + E_b)^{-1} = (A^T E_b^{-1} A + S_d^{-1})^{-1} A^T E_b^{-1}$$

to achieve

$$t = (A^T E_b^{-1} A + S_d^{-1})^{-1} A^T E_b^{-1} t_b = C t_b \quad (15)$$

Now the linear dimension of the matrix to be inverted is  $(N \cdot M_i + 1)$ . In (14), the elements of  $E_b$  are

$$E_{b(i,j)} = E_{b(i,j)} = \tau_{\epsilon}^2(T) (v_i) \quad i=j$$

$$E_{b(i,j)} = 0 \quad i \neq j$$

where  $\sigma_{\epsilon}$  = expected brightness temperature noise for a given spectral wave number, and it is assumed that the errors are totally random.

It is noted that  $C$  in (15) is a constant matrix if the temperature dependence of the atmospheric transmittance is small. In principle, the  $C$



matrix is dependent only on the temperature and absorbing constituent profile statistics utilized (i.e., their means and covariances) and not on the actual profiles to be retrieved. Of course,  $C$  also is dependent upon the matrix  $A$ , which is evaluated for the initial condition  $T^0$  and  $\tau^0$ .

## B. Error Analysis

The derivation below follows closely the work of Fleming<sup>6</sup> for efficiently estimating the profile errors to be expected from the application of (15) to actual radiance observations.

### 1. Temperature Errors

Consider an independent sample of soundings for which we wish to apply (15) and evaluate the errors in retrieval of  $T_i$  and  $U_i$ . If we let  $D$  denote the difference between the actual soundings and the retrieved soundings (i.e.,  $D = T - T_r$ ), then it follows from (15) and (12) that

$$D = T - (A^T E_b^{-1} A + S_d^{-1})^{-1} A^T E_b^{-1} A T - (A^T E_b^{-1} A + S_d^{-1})^{-1} A^T E_b^{-1} \tilde{E}_b \quad (16)$$

where  $T$  is the matrix of constituent temperatures and  $\tilde{E}_b$  is a matrix of radiant brightness temperature errors for the independent statistical sample and the superscript  $-1$  indicates the matrix inverse.

It follows that

$$D = (I - CA)T - C\tilde{E}_b$$

where  $I$  is the identity matrix and  $C = (A^T E_b^{-1} A + S_d^{-1})^{-1} A^T E_b^{-1}$ . Thus, the retrieval error covariance matrix,  $D^*$ , is

$$D^* = DD^T = (I - CA)TT^T(I - A^T C^T) + C\tilde{E}_b\tilde{E}_b^T C^T = (I - CA)S_I(I - A^T C^T) + C\tilde{E}_b\tilde{E}_b^T C^T$$

where  $S_I = TT^T$  of the independent sample set and the covariances between temperature and random measurement errors are assumed to be zero and where the same radiance error covariance statistics for the independent and dependent sets are assumed. Thus,

$$D^* = [I - (A^T E_b^{-1} A + S_d^{-1})^{-1} A^T E_b^{-1} A] S_I [I - A^T E_b^{-1} A (A^T E_b^{-1} A + S_d^{-1})^{-1}] + C\tilde{E}_b\tilde{E}_b^T C^T$$

Using the matrix identity

$$[I - (X+Y)^{-1} X] = (X+Y)^{-1} Y$$

then

$$D^* = (A^T E_b^{-1} A + S_d^{-1})^{-1} (A^T E_b^{-1} A + S_d^{-1} S_I S_d^{-1}) (A^T E_b^{-1} A + S_d^{-1})^{-1} \quad (17)$$

A similar result was obtained by Crosby and Weinreb.<sup>9</sup>

It is interesting to note that if  $S_I = S_d$ , then

$$D^* = (A^T E_b^{-1} A + S_d^{-1})^{-1}$$

which is the form originally derived by Fleming.<sup>6</sup>

The RMS retrieval error is simply the square root of the diagonal elements of  $D^*$ .

## 2: Constituent Concentration Errors

It follows from (9) that

$$U = U^0 + F[T_a - T] \quad (18)$$

where  $U$  is the vector of gas concentrations for each constituent and pressure level,  $U^0$  is the initial approximation to  $U$ ,  $F$  is a diagonal matrix whose elements are the vertical deviation of the mean gas concentration with respect to atmospheric temperature, and  $T_a$  is the atmospheric temperature (i.e., that portion of  $t$  for which the gas concentration profile is known).

In matrix form, it follows that the retrieval error of  $U$ , i.e.,  $E_u$  is

$$E_u = U - \hat{U} = \alpha [E_T^a - E_T^j]$$

or

$$E^* = E_u^T E_u = F [D_{T_a}^* - D_{T_a, T_u}^* - D_{T_u, T_a}^* + D_{T_j, T_u}^*] F \quad (19)$$

where  $D^*$  is given by (17);  $D_{T_a}^*$  is that portion of the matrix referring to the covariance of the error of temperature for the "known" constituent,  $D_{T_a, T_u}^*$  is that portion of the matrix referring to the covariance of the error of  $T_a$  with the error of the temperature of the "unknown" constituent ( $T_u$ ), and  $D_{T_j, T_u}^*$  is that portion of the matrix referring to error covariance for the unknown constituent.

The RMS retrieval error is simply the square root of the diagonal of  $E^*$ .

## 4. RESULTS OF RETRIEVAL ERROR ANALYSIS

An error analysis was performed for the Advanced Infrared Radiation Sounder (AIRS) which will be a facility instrument on the Polar Platform. The AIRS will provide a continuous spectral measurement of the earth-atmosphere upwelling radiance from  $590\text{ cm}^{-1}$  to  $2940\text{ cm}^{-1}$  with a spectral resolution ( $\nu/\Delta\nu$ ) of 1200/1. The AIRS radiometer accuracy will be equivalent to a brightness temperature performance of better than  $0.25^\circ\text{C}$  for a scene temperature of  $260^\circ\text{K}$ . The AIRS instrument performance specification is based on the airborne High resolution Interferometer Sounder (HIS)<sup>10</sup> which has demonstrated high vertical sounding resolution and sounding accuracy from NASA high altitude aircraft.<sup>3,8</sup>

The error analysis was performed by a numerical solution of equations (17) and (19) using a global climatological data set<sup>11</sup> to define the independent and dependent sample covariance matrices ( $S_I$  and  $S_D$ ) and FASCODE<sup>11</sup> to calculate the spectrum of atmospheric transmittance profiles for the instruments considered. The absorbing constituents considered were water vapor, ozone, and the uniformly mixed constituents ( $\text{CO}_2$ ,  $\text{N}_2\text{O}$ ,  $\text{CH}_4$ ,  $\text{SO}_2$ ,  $\text{CO}$ ). The uniformly mixed constituents were treated as a unit so that the number of atmospheric transmittance components was three. Forty pressure levels ranging between 0.1 mb and 1000 mb were considered for each of the three constituents so that the linear dimension of the matrix inverse was 121. The error matrix,  $E_b$ , was constructed assuming that the errors were random (i.e., uncorrelated from wavelength to wavelength) and equivalent to a brightness temperature error of  $0.25^\circ\text{C}$  for a scene temperature of  $260^\circ\text{K}$ . Retrieval errors were estimated for five cases: (I) full AIRS spectral resolution and coverage ( $590\text{-}2940\text{ cm}^{-1}$  with 1200/1 spectral resolution), (II) partial AIRS spectral coverage ( $590\text{-}1930\text{ cm}^{-1}$ ), (III) limited AIRS spectral coverage ( $590\text{-}1100\text{ cm}^{-1}$ ), (IV) the 19 low spectral resolution ( $15\text{ cm}^{-1}$ ) channels of the HIRS (High resolution Infrared Radiation Sounder) now flying on operational polar orbiting satellites,<sup>12</sup> and (V) a selection of 115 AIRS spectral channels listed in the AIRS Basic Information Package (BIP) distributed by NASA.<sup>9</sup>

Figures 1a and 1b show the anticipated AIRS retrieval performance when using all or portions of the specified spectral coverage. It is shown that most of the atmospheric temperature and lower level moisture information is contained in the  $590\text{-}1100\text{ cm}^{-1}$  region of the spectrum. This results from the temperature profile sensitive  $15\mu\text{m}$   $\text{CO}_2$  radiance spectrum and the lower tropospheric water vapor radiance producing rotational water vapor lines in this spectral region. The inclusion of the  $1100\text{-}1930\text{ cm}^{-1}$  region does not induce a significant improvement in temperature profiling accuracy, but does provide a very significant improvement in middle and upper tropospheric water vapor profiling accuracy due to the inclusion of radiance from the moderately strong vibrational water vapor lines in this spectral region. Inclusion of the  $2160\text{-}2940\text{ cm}^{-1}$  region produces further significant improvements in the temperature and water vapor profiling accuracy in the lower troposphere ( $p > 750\text{ mb}$ ) due to the water vapor lines and mixed  $\text{N}_2\text{O}$  and  $\text{CO}_2$  emission lines coupled with the strong Planck radiance dependence upon temperature in this short wavelength region. The lower tropospheric temperature accuracy improvement of  $0.5^\circ\text{C}$  due to the inclusion of  $4.3\mu\text{m}$  region  $\text{N}_2\text{O}/\text{CO}_2$  radiances is particularly noteworthy for accurate profiling of the meteorologically important thermal structure of the boundary layer.

Figure 2 shows the expected accuracy of the AIRS temperature and water vapor retrievals relative to that estimated for the currently operational HIRS radiometer (curve IV). It is noteworthy that the HIRS estimate achieved here is very consistent with that shown from comparisons of actual HIRS statistical retrievals with collocated radiosonde observations.<sup>12,13,14,15</sup> This result gives credibility to the expected errors calculated for the future AIRS instrument. As can be seen from Fig. 2, the AIRS represents a major improvement over the HIRS in terms of sounding capability with the temperature and moisture profiling accuracy being improved by a factor of two or better at all levels between the surface and ten millibars. It is also noteworthy that although the majority of this improvement can be achieved with the minimum set of 115 channels given in the AIRS "BIP", significant added accuracy improvement ( $\approx 0.5^\circ\text{C}$  and 5-10% for temperature and moisture, respectively) is achieved by using the entire spectrum for the sounding retrieval.

The reason one might want to limit the number of spectral channels sent to the ground for profile retrievals, is to minimize the telemetry band width requirements. However, it follows from equation (15) that a more efficient means of limiting the data rate without sacrificing profiling accuracy could be achieved by computing the term  $(A^T E_b^{-1}) t_b$  on board the spacecraft and transmitting the elements of this vector which is of the dimension the number of atmospheric retrieval levels (e.g., 40 as used here) times the number of profile constituents (e.g., three; water vapor, ozone, and the uniformly mixed gases) plus one (for surface temperature). Since  $(A^T E_b^{-1})$  is a constant matrix, it can be stored on-board and updated when necessary. The data volume for transmission could then be reduced to about 200 data words from the otherwise required 4000. The 200 data words could also include many specific channel radiances for "windows" and in minor constituent absorbing regions which are useful for purposes other than profiling temperature, water vapor and ozone.

Finally, Fig. 3 shows the ozone profile retrieval accuracy estimated from a solution of equation (19). Shown is the accuracy of the "effective ozone temperature" retrieval (obtained from equation (17)) and the accuracy of the integrated ozone concentration. It can be seen that the accuracy of the vertically integrated ozone concentration is nearly constant at about 2.5%, but this is due to the fact that most of the ozone is in the stratosphere where the effective ozone temperature retrieval accuracy is better than  $2.0^\circ\text{C}$ . As can be seen, the effective ozone temperature retrieval in the troposphere is very poor indicating that there is little tropospheric ozone mixing ratio profile information in the AIRS data. However, the 1-3% accuracy shown for the stratospheric concentration profile is very good.

## 5. SUMMARY

An efficient linear sounding retrieval methodology has been formulated to deal with large volume radiance spectra to be achieved with future advanced infrared sounding instruments. The sounding solutions were used to estimate profile retrieval errors associated with the future Polar Platform AIRS facility instrument and the currently operational HIRS sounding radiometer. The results show that the AIRS will improve our currently achievable temperature and water vapor profiling accuracy by at least a factor of two and enable us to observe the concentration profile of stratospheric ozone with an accuracy of 3% or better.

The linear algorithm also enables the satellite radiance spectral data to be condensed through simple on-board processing by at least an order of magnitude, prior to telemetry to the ground, without the loss of any of the profile information content of the spectrum. This feature is important if data transmission bandwidth becomes a limitation of high spectral resolution. Large spectral coverage approaches.

In conclusion, it is noted that the algorithm developed here has been successfully applied to both airborne and surface-based radiance spectra achieved with the HIS instrument.<sup>1</sup> The results of these real data applications of the "linear simultaneous solution" will be reported in forthcoming publications.

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## FIGURE CAPTIONS

Figure 1: (a) Temperature profile retrieval errors for: I. Full AIRS spectral resolution ( $\lambda/\Delta\lambda=1200$ ) and coverage ( $590-2940\text{ cm}^{-1}$ ), II. Partial AIRS spectral coverage ( $590-1930\text{ cm}^{-1}$ ), and III. Limited AIRS spectral coverage ( $590-1100\text{ cm}^{-1}$ ). (b) Water vapor profile retrieval errors for I, II, III spectral coverages listed above.

Figure 2: (a) Temperature profile retrieval errors for: I. Full AIRS spectral resolution ( $\lambda/\Delta\lambda=1200$ ) and coverage ( $590-2940\text{ cm}^{-1}$ ), IV. The 19 low spectral resolution ( $\lambda/\Delta\lambda=50$ ) HIRS infrared channels, V. A selection of 115 AIRS spectral channels listed in the NASA Basic Information Package (BIP). (b) Water vapor profile retrieval errors for the I, IV, and V spectral characteristics listed above.

Figure 3: Retrieval error of "effective ozone temperature" and vertically integrated ozone concentration. The temperature scale ranges from 0 to  $10^\circ\text{K}$  whereas the relative concentration scale ranges from 0 to 4%.



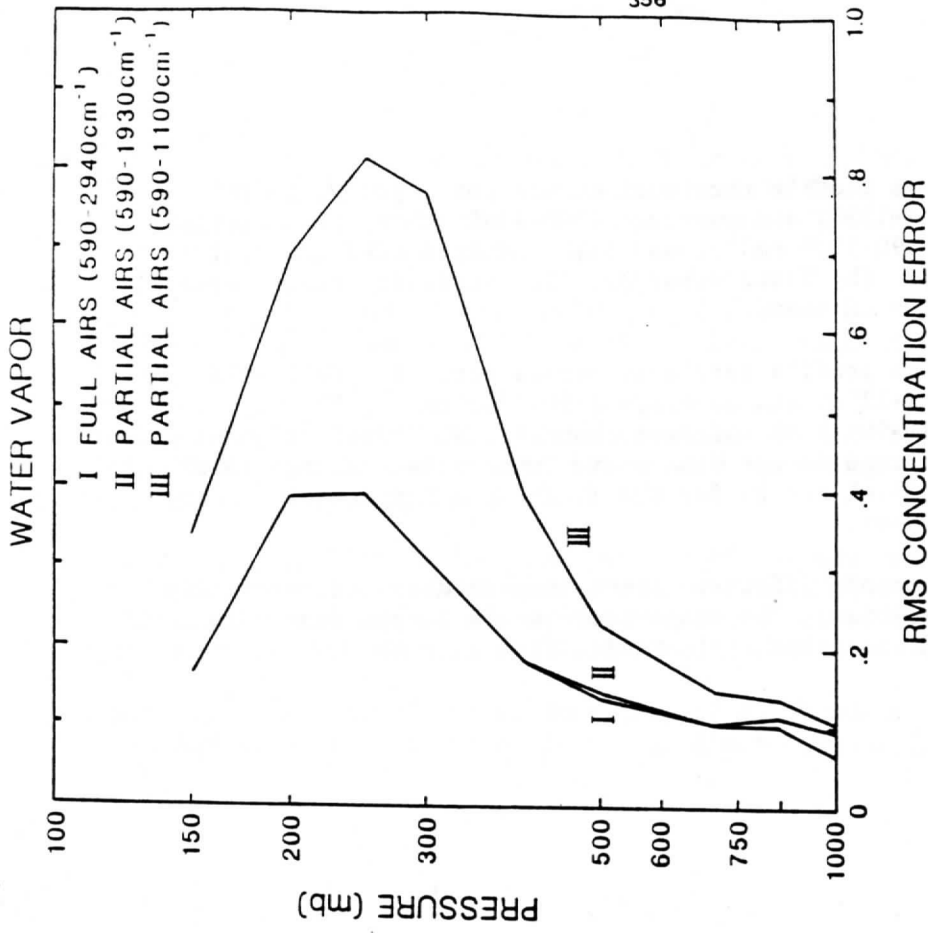
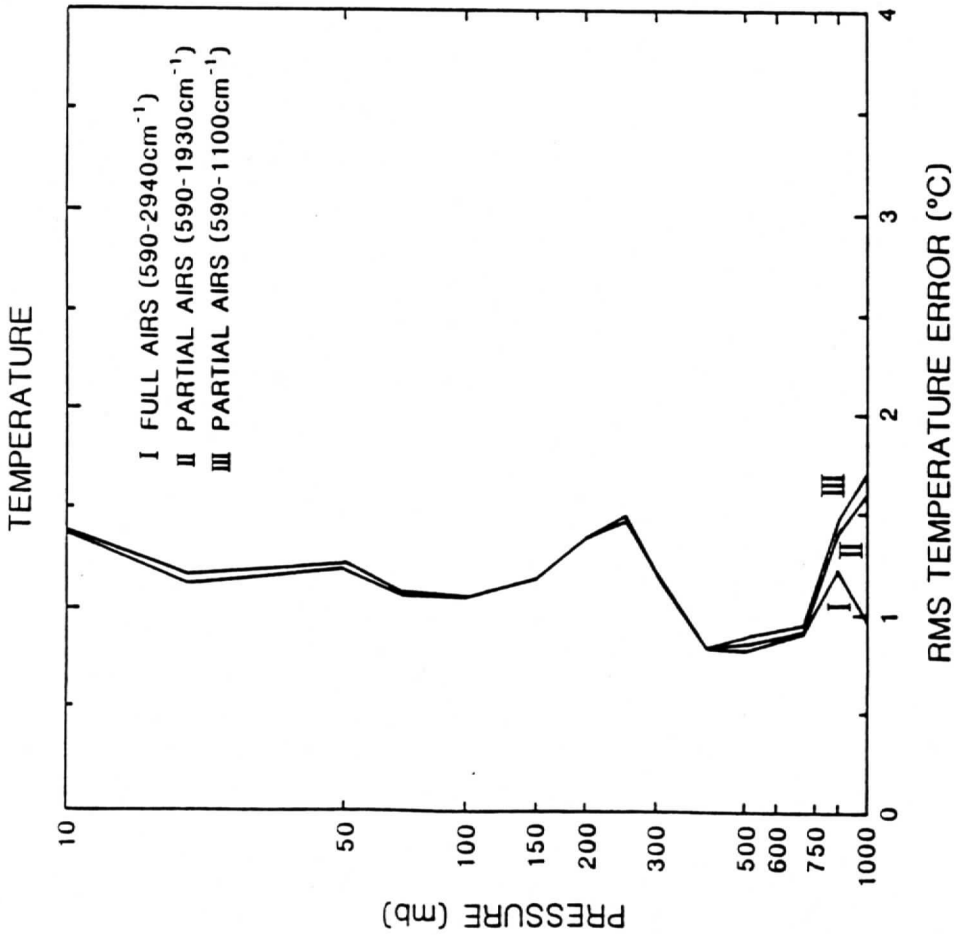


Figure 1: (a) Temperature profile retrieval errors for: (I) Full AIRS spectral resolution ( $\Delta/\Delta\lambda=1200$ ) and coverage (590-2940  $\text{cm}^{-1}$ ), (II) Partial AIRS spectral coverage (590-1930  $\text{cm}^{-1}$ ), and (III) limited AIRS spectral coverage (590-1100  $\text{cm}^{-1}$ ). (b) Water vapor profile retrieval errors for I, II, and III spectral coverages listed above.

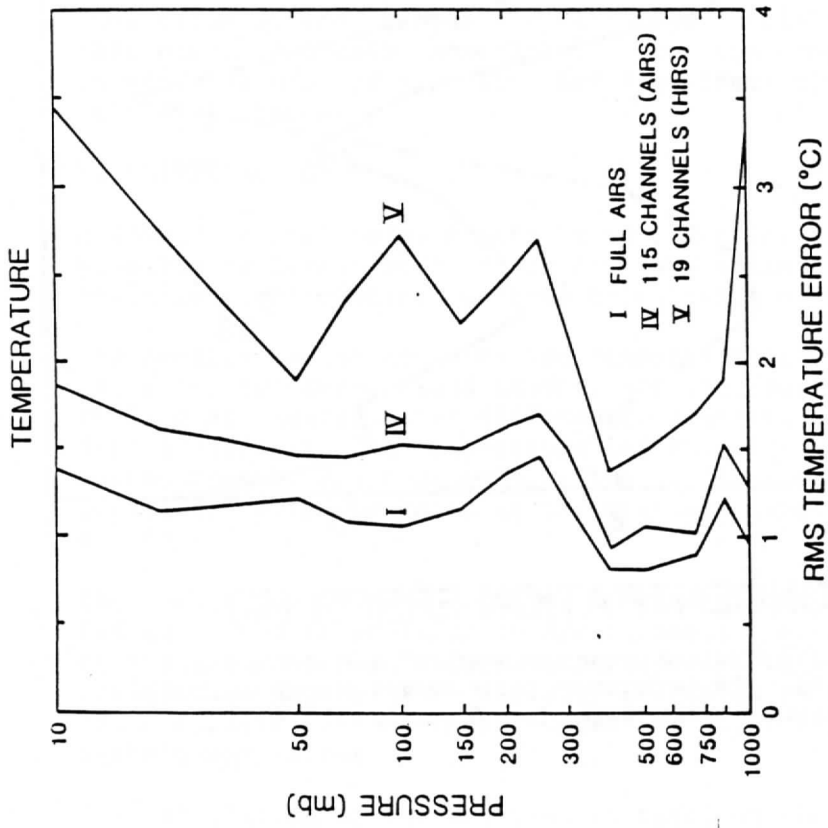
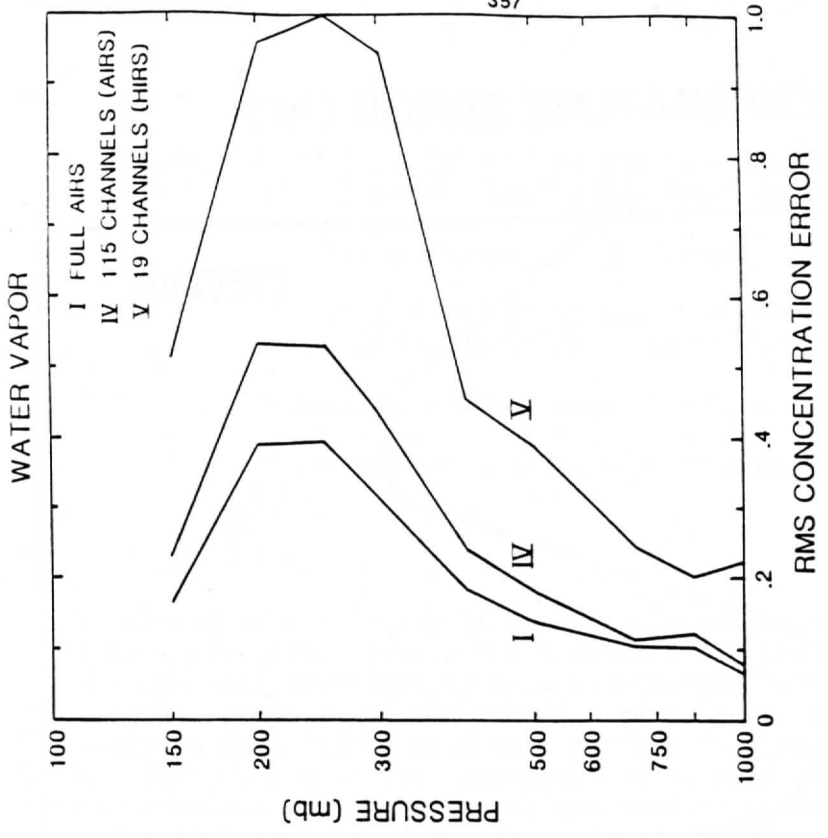


Figure 2: (a) Temperature profile retrieval errors for: (I) Full AIRS spectral resolution ( $\lambda/\Delta\lambda=1200$ ) and coverage ( $590-2940\text{ cm}^{-1}$ ), (IV) The 19 low spectral resolution ( $\lambda/\Delta\lambda=50$ ) HIRS infrared channels, (V) A selection of 115 AIRS spectral channels listed in the NASA Basic Information Package (BIP). (b) Water vapor profile retrieval errors for the I, IV, and V spectral characteristics listed above.

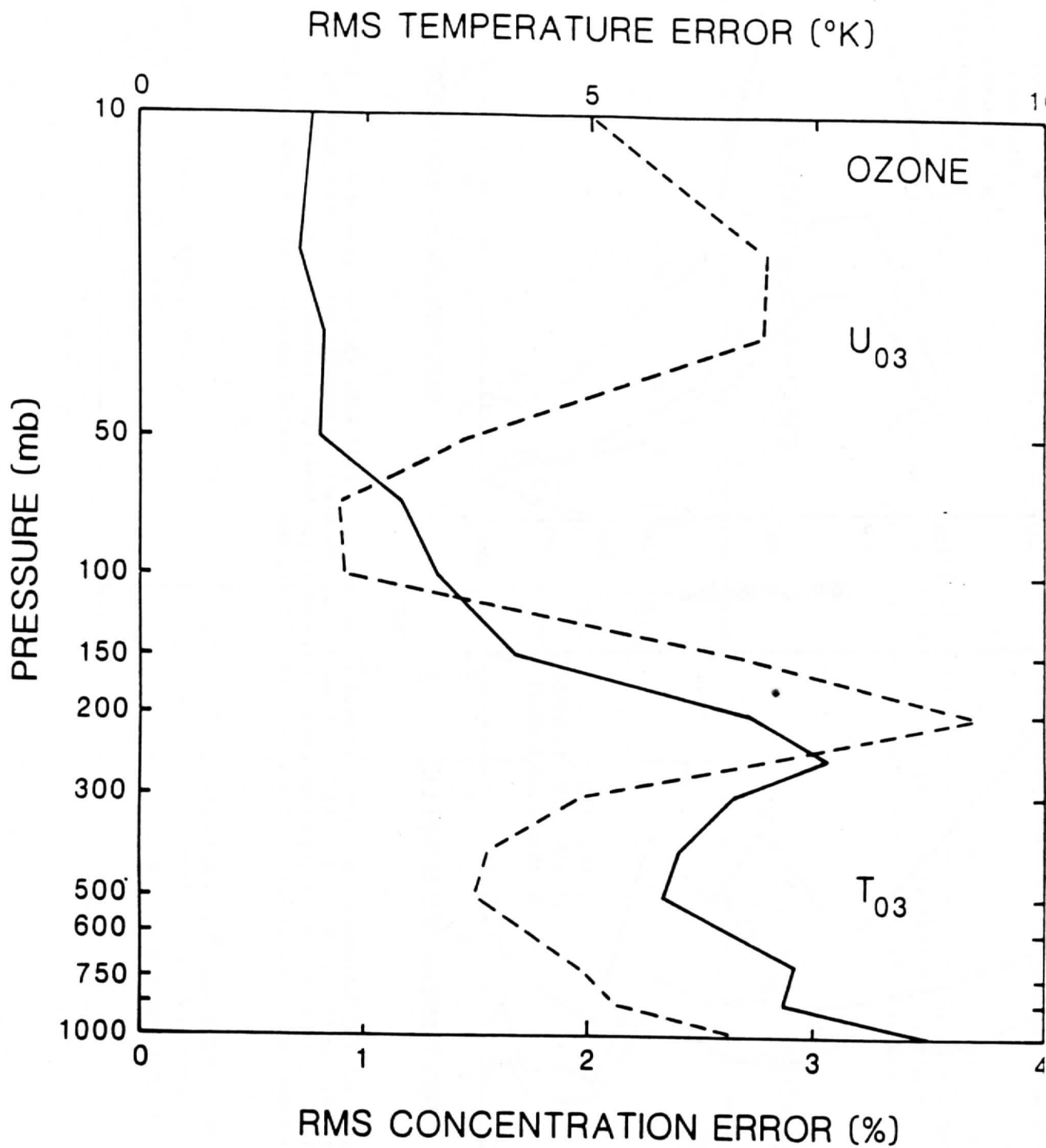


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