INVERTING INTERFEROGRAM SIGNALS TO RETRIEVE ATMOSPHERIC TEMPERATURE PROFILES.

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1. INTRODUCTION

High resolution infrared sounders for measuring the Earth emission spectrum from satellite are now recognized fundamental for improving accuracy and vertical resolution of the retrieved atmospheric structure. Research is presently under development concerning the use of interferometer spectrometers (e.g. IASI) and the related processing of interferogram signals. In the present communication the technique which is under development at our research centre is shown. Examples are illustrated by simulating synthetic interferogram signals corresponding to spectra with a spectral resolution of 0.25 cm^{-1}. Only the spectral range $600 \div 800$ cm^{-1} has been considered and only the problem of retrieving temperature profiles has been fully analysed. Aim of the present paper is mostly to evaluate the effect of apodization or lag windows upon the final product, i.e., the temperature profiles. The tool of apodization is briefly summarized in section 2, while section 3 deals with the inversion algorithm which has been implemented and section 4 illustrates the results.

2. THE TOOL OF APODIZATION

It is commonly believed that to achieve the objective of retrieving temperature profiles with an accuracy of 1 degree, high resolution infrared radiance measurements (spectral resolution of about $0.25 \div 0.5$ cm^{-1}) are needed with a radiometric accuracy of about 0.25 K at a scene temperature of about 260-270 degrees. This goal could be difficult to reach in the carbon dioxide absorption band at 15 μm, hence the supposedly need of apodization. Apodization is a tool to smooth spectra and it is relevant to spectra obtained by Fourier Transform Spectrometers or Interferometers. With reference to the case of a one-sided interferogram signal, which is the quantity measured by a FTS, we have that the relation between the interferogram signal and the spectrum is expressed by a cosine Fourier Transform:

$$R(\sigma) = 2\Delta \left(I(0) + 2 \sum_{k=1}^{N-1} w_k I(k) \cos(2\pi\sigma k\Delta) \right) \quad (1)$$

where

$I(k)$: denotes the sampled interferogram signal

Δ : is the sampling interval in the interferogram domain;

$R(\sigma)$: denotes the radiance at the wavenumber σ;

N : indicates the number of interferogram samples;

w_k : denotes the lag window function.

In the Fourier transform above the interferogram samples are multiplied by weights (the w_k's) decreasing to zero at the end of the record. This practice is commonly referred to as windowing and has the effect of suppressing part of the noise affecting the data points. Common windows are the *rectangular* window:

$$w_k = \begin{cases} 1 & 0 \leq k \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

which is equivalent to no smoothing, and the Triangular, Fejer or Bartlett window:

$$w_k = \begin{cases} 1 - \frac{k}{N} & 0 \leq k \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In the spectral domain the windowing operation is equivalent to convolve the true spectrum, $R_t(\sigma)$, with the Fourier Transform of the window function, $W(\sigma)$:

$$R(\sigma) = \int_{-\infty}^{\infty} R_t(\sigma_o) W(\sigma - \sigma_o) d\sigma_o \tag{4}$$

However, in practice, an additive noise term, $\epsilon(\sigma)$, has to be considered which comes from the noise affecting the interferogram:

$$R(\sigma) = \int_{-\infty}^{\infty} R_t(\sigma_o) W(\sigma - \sigma_o) d\sigma_o + \epsilon(\sigma) \tag{5}$$

Since the Fourier transform is a linear operation, we have that the noise term transforms as the interferogram function does, i.e.:

$$\epsilon(\sigma) = 2\Delta \left(\eta(0) + 2 \sum_{k=1}^{N-1} w_k \eta(k) \cos(2\pi\sigma k \Delta) \right) \tag{6}$$

where η denotes the measurument error affecting the interferogram signal. Following the common assumption of uncorrelated, Gaussian, measurement errors:

$$E\left[\eta(i)\eta(j)\right] = \begin{cases} \sigma_\eta^2 & i = j \\ 0 & i \neq j \end{cases} \tag{7}$$

($E[\cdot]$ denoting expectation value), the variance affecting the spectral ordinates can be easily computed

$$E\left[\epsilon^2(\sigma)\right] = 4\Delta^2\sigma_\eta^2\left(1+4\sum_{k=1}^{N-1} w_k^2\cos^2(2\pi\sigma k\Delta)\right) \quad (8)$$

In the same way we can compute the covariance matrix V_{ij} of the spectral estimates

$$V_{ij} = E\left[\epsilon(\sigma_i)\epsilon(\sigma_j)\right] = 4\Delta^2\sigma_\eta^2\left(1+4\sum_{k=1}^{N-1} w_k^2\cos(2\pi\sigma_i k\Delta)\cos(2\pi\sigma_j k\Delta)\right) \quad (9)$$

In general, we have that, compared with the rectangular window, other windows reduce the variance, that is the diagonal terms of the covariance matrix, while increasing the off-diagonal values. That is, windowing introduces linear dependence in the data points. Furthermore, classic windowing is empirical in principle. With any window we cannot optimize the transfer of the noise power from the diagonal terms to the off-diagonal ones, which is not good, in general.

2.1 The ASE smoothing of spectra.

For this reason we have developed an optimal procedure (ASE; *Amato and Serio* 1991; *V. Cuomo et al* 1993) which yields adaptive window functions. The analytical form of such window functions is:

$$w_k = \begin{cases} \left(1+(2\pi k)^4\lambda N\right)^{-1}; & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where N is always the number of interferogram samples and λ is trade-off parameter whose optimal values is found by using the General Cross Validation (GCV) criterion (*Wahba* 1977; *Amato and Serio* 1991; *V. Cuomo et al* 1993)).

3. THE INVERSION ALGORITHM

To evaluate the impact of apodization over the inversion products, we have developed a straightforward but robust inversion scheme. A Newton-Raphson scheme has been used where the inverse problem is linearized by Taylor expanding the signal, $R(\sigma; T(z))$, with respect to the temperature profile:

$$R(\sigma;T(z)) = R(\sigma;T_o(z)) + \int_0^\infty \left.\frac{\partial R(\sigma;T(z))}{\partial T(z)}\right|_{T(z)=T(z)} (T(z)-T_o(z))\,dz \quad (11)$$

Here $T_o(z)$ is a suitable first guess profile, and $R(\sigma; T(z))$ is the radiance at the wavenumber σ, the notation emphasizes that the radiance R is a function of the temperature profile.

For simplicity only variations in the temperature profile have been considered. Furthermore, for computation the intregral (11) has been separated in 17 layers, 1 km thick up to ten kilometers, and with varying thicknesses up to 100 km. Once the inverse problem has been linearized it can be put in the usual form

$$\mathbf{y} = A\mathbf{x} \tag{12}$$

where $\mathbf{x}$ is the unknown profile (M-dimensional vector), $\mathbf{y}$ is a vector of quantities (radiances) to be measured (N-dimensional vector) and A is an $N \cdot M$ matrix. We consider N greater than M and therefore we get a solution, $\hat{\mathbf{x}}$, by Least Square:

$$\hat{\mathbf{x}} = \left(A^T V^{-1} A\right)^{-1} A^T V{-}1\mathbf{y} \tag{13}$$

where V denotes the input matrix covariance and $V(\hat{\mathbf{x}})$ the output covariance matrix; the suffix T denotes transpose and -1 inverse matrix. Furthermore, the output covariance matrix, $V(\hat{\mathbf{x}})$, reads:

$$V(\hat{\mathbf{x}}) = \sigma_\eta^2 \left(A^T V^{-1} A\right)^{-1} \tag{14}$$

4. ASSESSING THE IMPACT OF APODIZATION

To assess the impact of apodization on the retrievals the following exercise has been carried out. Using the FASCOD2 code (which was used to do all the line-by-line calculations), the interferogram corresponding to the spectral region 600-800 cm^{-1} was obtained for the U.S. Standard Atmosphere. The resolution was 0.25 cm^{-1} which corresponds to the design unapodized resolution of the IASI interferometer. In order to have a suitable statistics to compute the r.m.s. error of the retrievals, the inteferogram signal was corrupted with one thousand different sets of noise term so that 1000 noisy inteferogram signals were obtained. Each corrupted interferogram was FFT transformed, and in this operation three different windows were considered: the rectangular window, the Bartlett one, and the ASE window. Of course, for each window the radiative transfer equation was convolved accordingly. Fig. 1 show the Brightness Temperature (BT) spectrum corresponding to the U.S. standard atmosphere (no external noise added) which was used in our simulations. Although different signal-to-noise ratios were simulated, for the sake of brevity, only the case corresponding to a signal-to-noise ratio of 5000:1 at zero delay in the interferogram domain will be shown. Fig. 2 shows how the error propagates to the BT's in the spectral domain (unapodized case). This case is very interesting, since, on average, the r.m.s error of the spectral estimates is about 0.47 K. At this level of the error, apodization is considered to be important. Convolving the noisy interferograms with the different windows we had the following results.

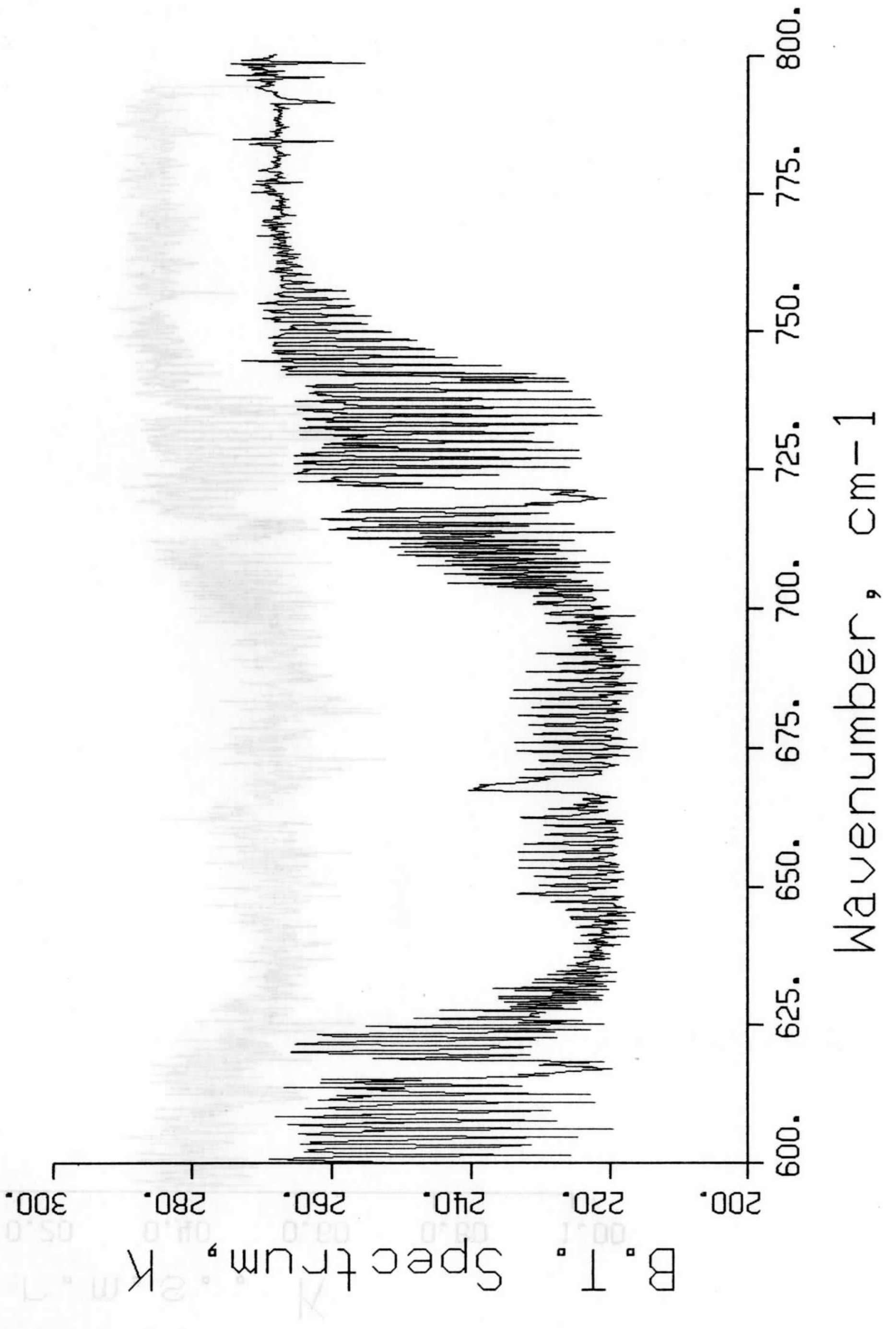

Fig. 1 Brightness Temperature spectrum for the U.S. standard atmosphere in the spectral region 600-800 cm^{-1}. The resolution is 0.25 cm^{-1}.

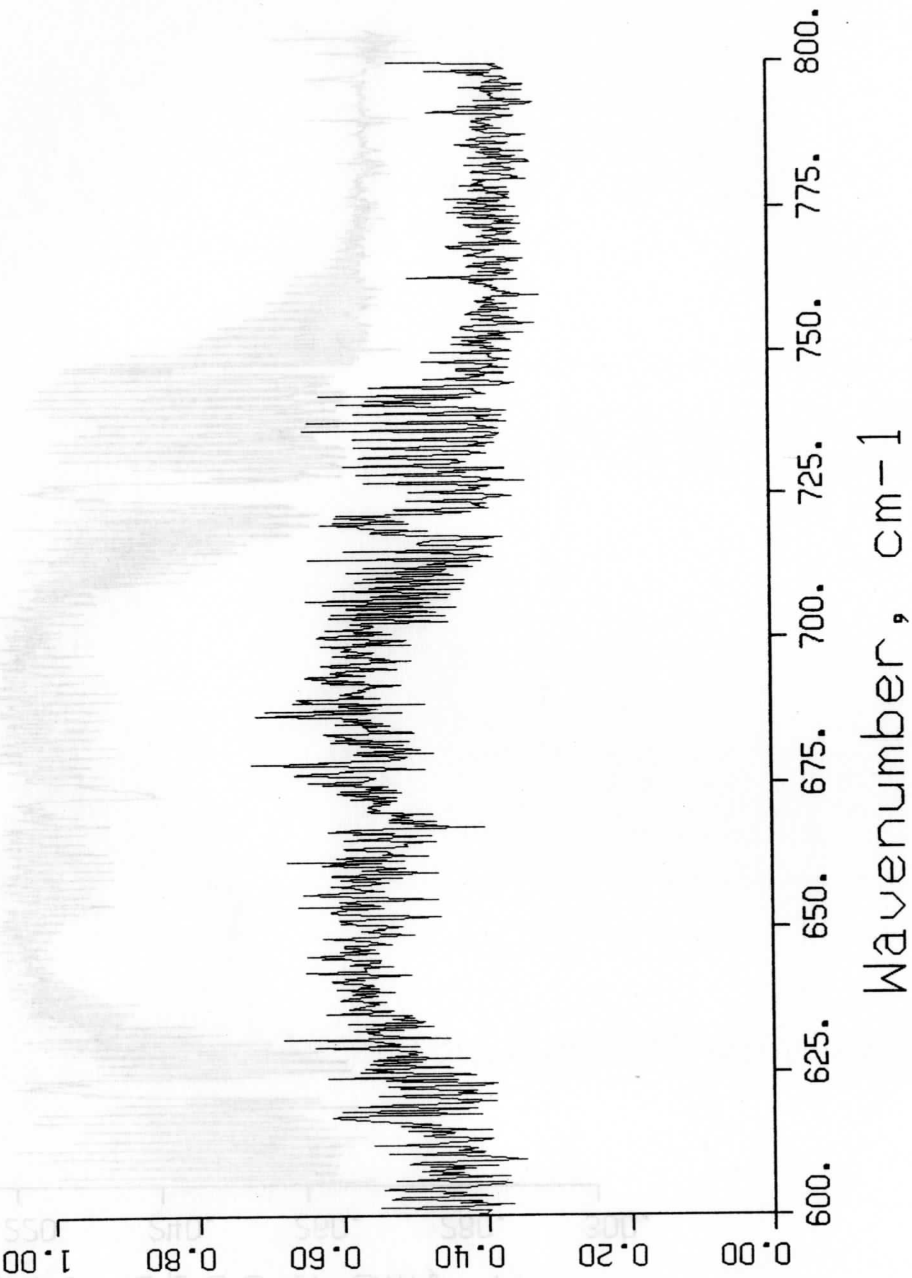

Fig. 2 Root mean square error affecting the spectral ordinates (unapodized case) assuming a constant error (signal-to-noise ratio equal to 5000:1 at zero delay) affecting the interferogram signal.

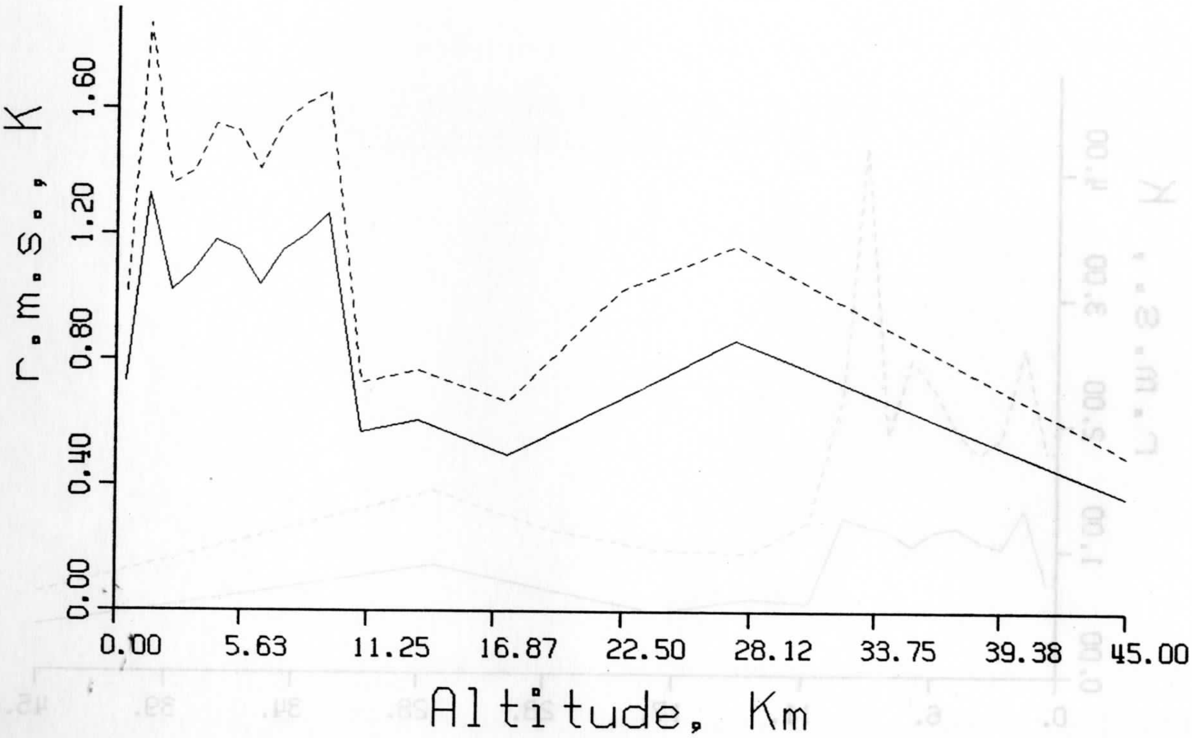

Fig. 3 Root mean square error of the temperature profile obtained by smoothing the spectral estimates before applying the inversion algorithm. Dashed line: Bartlett window; solid line: ASE window; dotted line: rectangular window (unapodized case). Note that ASE and rectangular smoothing give the same result.

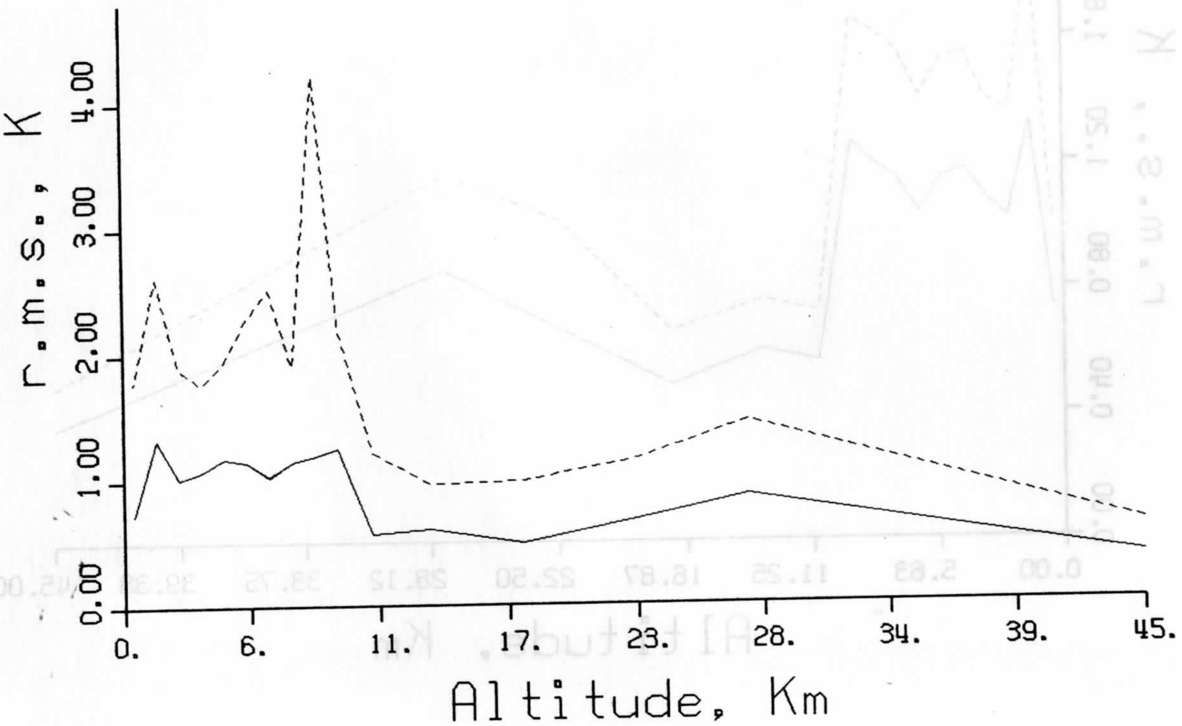

Fig. 4 As figure 3, but using the correct input covariance matrix for each window. Note that ASE and rectangular smoothing produce the same result.

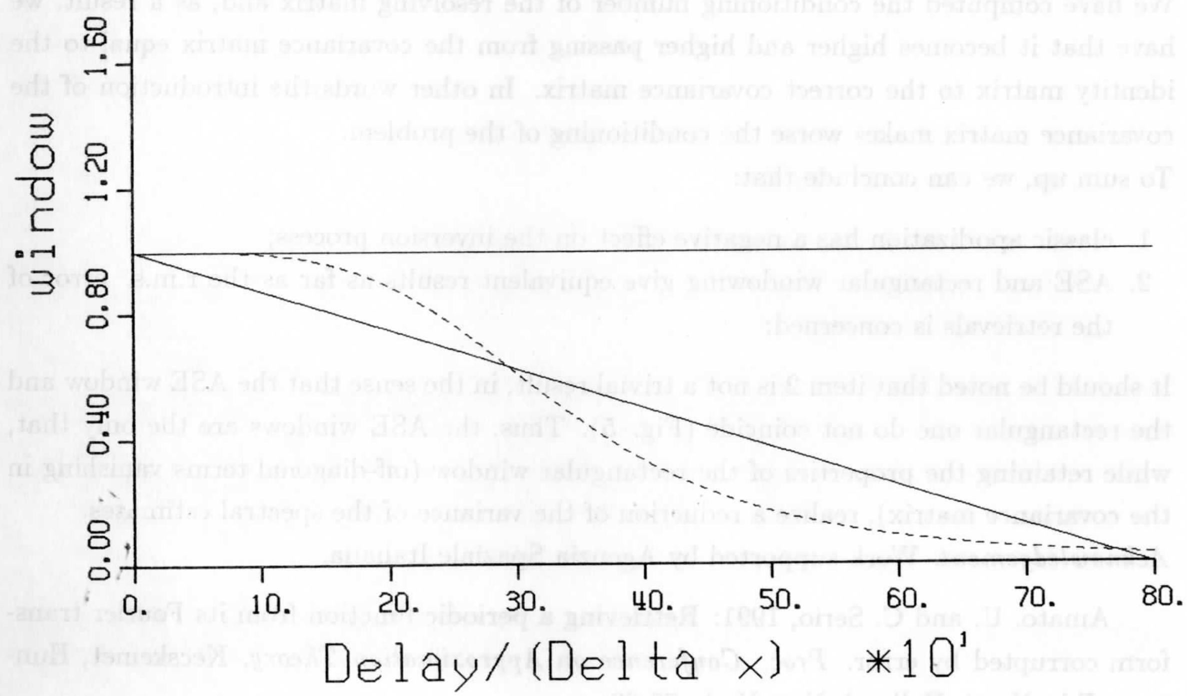

Fig. 5 Comparison between the ASE window (dashed line) the Bartlett window and the rectangular one.

The Bartlett window realized the best reduction of the variance of the spectral estimates, since, on average, the r.m.s. error affecting the spectral ordinates was 0.27 K against 0.47 K (unapodized case). The equivalent figure for ASE was 0.40 K. Yet, coming to the result of the inversion process we see (Fig. 3) that the Bartlett window gives the worst r.m.s error of the retrievals (i.e. temperature profile). Furthermore, the ASE smoothing gives the same results as the ones obtained by using the rectangular window. For the case illustrated in Fig. 3, the input covariance matrix was set equal to the identity matrix, therefore it could be argued that better results might be obtained by using the correct input covariance matrix (9). However, using the correct covariance matrix for each window, the Bartlett window gets worse and worse (Fig. 4). This behaviour deserves a bit of further analysis. We have computed the conditioning number of the resolving matrix and, as a result, we have that it becomes higher and higher passing from the covariance matrix equal to the identity matrix to the correct covariance matrix. In other words the introduction of the covariance matrix makes worse the conditioning of the problem.
To sum up, we can conclude that:

1. classic apodization has a negative effect on the inversion process;
2. ASE and rectangular windowing give equivalent results as far as the r.m.s. error of the retrievals is concerned;

It should be noted that item 2 is not a trivial result, in the sense that the ASE window and the rectangular one do not coincide (Fig. 5). Thus, the ASE windows are the only that, while retaining the properties of the rectangular window (off-diagonal terms vanishing in the covariance matrix), realize a reduction of the variance of the spectral estimates.
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Amato, U. and C. Serio, 1991: Retrieving a periodic function from its Fourier transform corrupted by error. *Proc. Conference on Approximation Theory,* Kecskemet, Hungary, Ed., North-Holland, New York, 79-92

U. Amato, V. Cuomo and C. Serio (1992): An Optimal Advanced Spectral Estimation Algorithm in Fourier Spectroscopy with Application to Remote Sensing of Atmosphere. *J. Appl. Meteor.* (in press)

Wahba, G., 1977: Practical approximate solutions to linear operators equations when the data are noisy. *SIAM J. Numer.Anal.,* **14**, 820-831.

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