PROPOSED CORRECTION TO THE TOVS FAST TRANSMITTANCE MODEL TO ACCOUNT FOR VARIABLE CO_2

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1. INTRODUCTION

Many schemes for retrieving temperature profiles from TOVS radiances, such as the one used by the National Environmental Satellite, Data and Information Service (NESDIS) (*Fleming et. al.*, 1986), use a physical or physical statistical approach. This type of approach depends on comparison between observed radiances and values computed from assumed atmosphere temperature and moisture profiles. Generally, operational retrieval schemes such as that of NESDIS apply empirical corrections for non-identifiable systematic errors in the radiative transfer equations. However, it is desirable to remove identifiable systematic errors physically rather than empirically where possible.

A previous simulation of TOVS brightness temperatures in the $15 \mu m$ CO_2 band (*Turner*, 1992) indicates that the 30 ppmv change in CO_2 mixing ratios observed over the past 17 years may be associated with brightness temperature changes as large as one degree. This result indicates that ignoring long term changes in CO_2 amounts may lead to systematic errors in TOVS radiative transfer models large enough to be of concern in applications such as the assimilation of TOVS radiances into NWP models.

A simple correction term is proposed that could be applied to the HIRS transmittance calculated by the NESDIS fast transmittance model (*Weinreb et. al.*, 1981) to correct for this systematic error.

2.1 THEORETICAL BASIS FOR THE CORRECTION TERM

For a given temperature-moisture profile the top of the atmosphere (TOA) radiance observed by a satellite is simulated by:

$$R^{TOA}(p_s) = R(p_s) \cdot \overline{\tau}(p_s:0) - \int_0^{P_s} B(p') \cdot \frac{d\overline{\tau}(p')}{dp} \cdot dp' . \tag{1}$$

The TOA radiance is the sum of the attenuated radiation from a surface at p_s, plus the attenuated emissions from the atmosphere. Equation (1) assumes that spectrally averaged (mean) transmittances can be used to derive the mean radiance. As this study is focused on sensitivity to CO_2, the $\overline{\tau}$'s used in this study consist only of the uniformly mixed gases component (CO_2, N_2O, CO, CH_4, O_2 and N_2); the O_3 and the H_2O components are excluded.

A possible form for a CO_2 correction term can be determined by writing the monochromatic transmission function as;

$$\tau(p,q) = e^{-(q \cdot H + A)}$$

where $H = \dfrac{\sec(\theta)}{g_0} \cdot \int_0^p k(\tilde{v}, p', T(p)) \cdot dp'$ & $A = \dfrac{\sec(\theta)}{g_0} \cdot \sum \int_0^p q_i \cdot k_i(\tilde{v}, p', T(p)) \cdot dp'$

$q \cdot H$ is the optical depth contribution due to CO_2, A is the contribution due to the other uniformly mixed gases, q is the mass mixing ratio of CO_2, k is the

absorption coefficient of CO_2, q_i is the mass mixing ratio of the ith absorber, excluding CO_2, k_i is the absorption coefficient of the ith absorber, T is the temperature (a function of p), θ is the satellite zenith angle, and g_0 the gravitational acceleration.

If one assumes that the CO_2 mixing ratio varies, but that it is still constant with pressure, a form of a correction term that may be applied to a reference transmittance, $\tau_0(p,q)$, can be found, ie;

$$\tau(p,q) = \tau_0(p,q_0)^{1+\beta \cdot (q-q_0)}, \qquad \beta = \frac{H}{q_0 \cdot H + A} \qquad (2)$$

The term $\beta \cdot (q-q_0)$ represents an adjustment to τ_0, the transmittance at q_0, in order to approximate the transmittance at q. It should be noted that β is not a simple number, but a intricate function of pressure, temperature and the absorption coefficients of all the absorbers involved, including CO_2. Although β is not constant, it can be shown that for practical purposes it can be approximated by a single constant for each channel.

It would be desirable that β be temperature independent, but this not obvious due to the implicit temperature dependencies of H and A. However intuitively one would expect the dependency on temperature to be weak since presumably the ratio of H and $q_0 \cdot H + A$ will cancel out most of the effect of any temperature dependencies. Furthermore, β is a second order correction term that only accounts for the change in the temperature dependence of the transmittance with q since τ_0, already includes the effect of T(p) and θ at q_0. It is expected that β should have no dependency on zenith angle since the sec(θ) factor in H and $q_0 \cdot H + A$ cancel out.

Equation (2) applies to monochromatic transmittances. Nevertheless the use of average transmittances in place of the monochromatic transmittances will not change the form of (2), just the value of β. A dependency on the zenith angle is introduced due to the wavenumber averaging, however it is expected to be weak.

2.2   Calculation of $\overline{\tau}$

The GENLN2 line-by-line (LBL) radiative transfer model (*Edwards*, 1992) is used to evaluate the required $\overline{\tau}(p,q)$'s. The LBL model utilizes the HITRAN86 spectral database (*Rothman et. al.*, 1987). $\tau(p,q)$ is calculated at 0.002 (cm^{-1}) resolution assuming a heterogenous path defined by a string of homogenous cells whose boundaries are defined by the standard NESDIS pressure levels. The cell absorber amounts are determined by the U.S. standard atmosphere and the mixing ratios of the uniformly mixed absorbers. The $\overline{\tau}(p,q)$'s are formed by integrating over the NOAA-10 HIRS response functions from *Planet* (1988).

The values of $\overline{\tau}(p,q)$ for values of CO_2 mixing ratio from 320 ppmv to 380 ppmv in 10 ppmv increments were calculated using GENLN2. The reference value q_0 is set as 330 ppmv and the satellite zenith angle is set to 0°. Unless otherwise noted, the temperature and water vapour profiles used were from the U.S. standard atmosphere.

2.3   Determination of β

Rearrangement of (2) leads to an expression linear in $(q-q_0)$, ie;

$$\alpha'(p,q) = \left(\frac{\log(\overline{\tau}(p,q))}{\log(\overline{\tau}_0(p,q))} - 1 \right) = \beta \cdot (q-q_0) \tag{3}$$

The character of $\alpha'(p,q)$ may be examined by plotting it against pressure as in figure 1.

Figure 1 illustrates the values of $\alpha'(p,q)$ for HIRS 6[1] as a function of pressure and CO_2 mixing ratio. The weighting function is superimposed. As one would expect α' becomes larger and less constant with pressure as the amount of CO_2 deviates away from 330 ppmv. The values of α' over that region of the atmosphere where the greatest contribution of upwelling radiation originates (ie weighting function values greater than half the peak value) are relatively constant with pressure for smaller deviations from q_0. This implies that it may be possible to assign a single value of α for each $\alpha'(p)$ curve.

An average value for each $\alpha'(p)$ curve, denoted as α, is determined by weighting $\alpha'(p)$ with the channel's weighting function, ie:

$$\alpha(q) = \frac{\int \left| \frac{d\overline{\tau}(p,q)}{d\log(p)} \right| \cdot \alpha'(p,q) \cdot d\log(p)}{\int \left| \frac{d\overline{\tau}(p,q)}{d\log(p)} \right| \cdot d\log(p)} \tag{4}$$

The vertical lines in figure 1 represent the values of α for each q as evaluated by equation (4). If the values of $\alpha(q)$ are plotted against $(q-q_0)$ (figure 2), it can be seen that an excellent linear relationship exists. A value for β is determined by a least squares fit of the line $\alpha = \beta \cdot (q-q_0)$ to these data. This procedure was applied to all the HIRS channels and the resulting values for β are tabulated in table I.

Henceforth transmittances which have been calculated by applying equation (2) to transmittances at q_0=330 ppmv are referred to as 'β-approximation' transmittances.

3.   RESULTS

In order to gain confidence as to how well the correction term simulates the actual transmittances, the following differences for HIRS 6, as an example,

$$\overline{\tau}^{LBL}(p,q) - \overline{\tau}^{LBL}(p,q_0), \quad \overline{\tau}^{\beta}(p,q) - \overline{\tau}^{\beta}(p,q_0), \quad \overline{\tau}(p,q) - \overline{\tau}(p,q_0)^{1+\beta(q-q_0)} \tag{5}$$

are plotted in figures 3a and 3b.

As the amount of CO_2 increases the transmittance decreases. The greatest sensitivity of the transmission function occurs in the region where the weighting function is greatest. Generally the β-approximation tends to underestimate the transmittance below the region where the weighting function peaks and overestimate in the regions above. This pattern is repeated for both the

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[1]figures corresponding to figures 1 to 4 for other HIRS channels may be found in *Turner*, 1993.

longwave and shortwave temperature sounding channels.   Overall the β-approximation works very well.   The maximum deviation is of order ± 0.004 (HIRS 1 at 6 mb) at a mixing ratio difference of 50 ppmv.

In a physical retrieval system, information about the atmosphere is inferred by comparing observations to TOA calculations from an assumed atmospheric state. Consequently, the best method of determining how well the β-approximation would perform in such a scheme is by comparing the β and LBL radiances.

For the purposes of this comparison the surface (clear sky) or cloud (100% coverage) is considered to be black.   Only the differences between LBL and β-approximation simulations at varying mixing ratios are considered.   The effect of a non-black surface (ie; non-unity emissivity, reflected downwelling and solar terms) is ignored since it is small compared to the surface emission and the atmospheric term. It is reasonable to assume that the error due to neglecting this term is approximately equal in the two simulations.

The LBL and β-approximation forms of the TOA (equation 1) radiance are:

$$R^{LBL}(p_s, q) = B(p_s) \cdot \overline{\tau}(p_s, q) - \int_0^{p_s} B(p') \cdot \frac{d\overline{\tau}(p', q)}{dp} \cdot dp'$$

$$R^{\beta}(p_s, q) = B(p_s) \cdot \overline{\tau}(p_s, q_0)^{1+\beta \cdot (q-q_0)} - \int_0^{p_s} B(p') \cdot \frac{d\overline{\tau}(p', q_0)^{1+\beta \cdot (q-q_0)}}{dp} \cdot dp' \tag{6}$$

Table I lists the percentage differences between LBL clear-sky radiances at 330 ppmv and 360 ppmv of CO_2, "A", and the differences between the LBL and β-approximation radiances at 360 ppmv, "B".

$$A = 100 \cdot \frac{R_{360}^{LBL} - R_{330}^{LBL}}{R_{360}^{LBL}}, \qquad B = 100 \cdot \frac{R_{360}^{LBL} - R_{360}^{\beta}}{R_{360}^{LBL}} \tag{7}$$

Examination of column "A" shows that as expected the effect of CO_2 mixing ratio changes in window (HIRS 8, 18 and 19), water vapour (HIRS 10, 11 and 12), and ozone channels is negligible.   The effect of CO_2 changes in the lower shortwave temperature sounding channels (13 and 14) is also small due to the dominance of N_2O absorption over CO_2 absorption in these channels.

The fifth column in the table is a measure of the performance of the β-approximation in accounting for CO_2 changes.   The overall performance of the β-approximation is better than 90% for the longwave stratospheric temperature sounding channels and better than 97% for the longwave tropospheric channels. Channel 3 which straddles the tropopause performs poorly but the sensitivity to changing CO_2 is negligible, thus it is not important.   In the shortwave temperature sounding channels the performance is better than 82%.

Channel 15 is an oddity.   This channel is the second most sensitive to CO_2 changes.   The β-approximation performs at 88%, but the difference between the LBL and the β simulations is an order greater than any other channel.   This might be due to a transition from N_2O to CO_2 dominance in the mid-troposphere.   The final column tabulates the absolute difference in brightness temperature (a more

commonly used unit) between 330 ppmv and 360 ppmv. These values represent systematic errors in current models for TOA brightness temperature that assume a constant CO_2 mixing ratio of 330 ppmv. Although the errors are generally small, those which are greater than approximately .25 K may have an impact on data assimilation routines (C. Chouinard, 1992).

Table I illustrates the performance of the β-approximation for TOA radiances in clear sky conditions, however the effects of varying CO_2 on TOA radiances from clouds must also be considered. Figure 4 plots a comparison between the LBL and the β-approximation HIRS 6 TOA radiances for various mixing ratios of CO_2 as a function of the cloud top pressure.

As with surface results (table I), examination of similar graphs of the TOA radiance from clouds for the other HIRS channels show that the temperature sounding channels are the channels with significant sensitivity to CO_2 changes, with the same exceptions (HIRS 3, 13, 14).

The sensitivity to CO_2 changes varies with height in all channels. The regions with the greatest sensitivity coincide with the regions that contribute the most to the TOA signal (compare with the weighting functions). With the exception of channels 1, 2, 3 and 17 all the channels show a decrease in their sensitivity to CO_2 changes as the cloud top pressure decreases to about 200 mb.

Channels 1, 2 and 17 have near vertical structure in the troposphere due to the dominance of the atmospheric emission term. These are the stratospheric temperature sounding channels. Consequently the ground and all 'realistic' clouds are invisible to the satellite due to the opaqueness of the atmosphere at these wavelengths.

In general the β-approximation curves parallel the LBL simulations quite well. Deviations are generally less than 0.1 to 0.2% and at worse less than half a percent. Although the differences from the LBL model are negligible, the β-approach for the shortwave channels that have a fair amount of N_2O contamination (HIRS 13, 14 and 15 (figure 5)) does not parallel the LBL simulation as well. The β-simulation improves with height lending some credence to the idea that N_2O dominates over CO_2 in the lower troposphere.

Without exception all the channels that can sense the surface or clouds show a decrease in CO_2 change sensitivity with height illustrating that a correction to changing CO_2 cannot be applied equally to clear and cloudy atmospheres. Frequently, correction terms (or biases) for retrievals are statistically derived from clear sky (surface) measurements and are then applied equally to observations from clouds. This practise is not a good one. In the case of the aforementioned CO_2 corrections, it is certainly not the case since the errors in ignoring CO_2 clearly change with cloud top pressure. They decrease with height in regions where 'real' clouds exist.

Finally, simulations were run in order to determine if β has a significant dependence on the temperature profile and zenith angle, details of which can be found in Turner (1993). LBL simulations were carried out using a mid-latitude summer temperature profile and a sub-arctic winter temperature profile. The LBL

TOA radiance results were compared to the reference LBL radiances corrected for q using equation (2) and the table I values of β, and it was found that these values of β were applicable with only a slight loss of accuracy. A negligible impact on the accuracy was found after a similar comparison was done assuming an extreme satellite zenith angle of 52°.

4.   CONCLUSIONS

A simple method has been developed to correct for systematic errors in calculated HIRS radiances generated by changes in CO_2 mixing ratios. The method consists of multiplying the mean transmittance corrected for temperature and zenith angle by an exponent which is a linear function of the deviation of the CO_2 mixing ratio from a reference value, ie:

$$\overline{\tau} = \overline{\tau}_0^{\,1+\beta\cdot(q-q_0)} \qquad (9)$$

where $\overline{\tau}$ is the transmittance evaluated for an arbitrary temperature profile and zenith angle assuming 330 ppmv of CO_2. One value of β can be used for each channel regardless of temperature or zenith angle. The values may differ slightly from satellite to satellite.

The term $1+\beta\cdot(q-q_0)$ should not be confused with a current method of correcting transmittances, which is known as the γ-correction. The γ-correction is an empirical coefficient derived from satellite-radiosonde collocation statistics that is applied to the transmittances in an analogous manner, ie $\overline{\tau} = \overline{\tau}^{\gamma}$. The term $\beta\cdot(q-q_0)$ corrects for an identified systematic error, whereas the γ-correction is a statistical correction for unidentified systematic and random errors.

It should be noted that there are two caveats regarding this method. First the errors in the application of $\beta\cdot(q-q_0)$ increase with increasing $(q-q_0)$, thus it would be ideal to propagate q_0 with time. Currently, the reference CO_2 mixing ratio is 330 ppmv, a value that was applicable in the mid-seventies. Although the β-approximation does not produce large errors at the current mixing ratio of 360 ppmv, it will not be long before the errors are unacceptable. It would be prudent to recalculate the basic NESDIS transmittance model coefficients at a new reference mixing ratio, say 380 ppmv.

The values of β defined by this paper were calculated using more recent spectral line data than the original coefficients were. In using a different set of spectral data the mean transmissions calculated in this study will not necessarily equate to the NESDIS transmittances, thus the values of β implicitly assumes that a constant systematic error accounting for the differences in the spectral line data will occur that will be removed by the usual calculation of NESDIS correction biases. This problem will vanish upon recalculating all the NESDIS transmittance coefficients.

5.   ACKNOWLEDGEMENTS

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HIRS	β ppmv^{-1}	A	B	$1-|B/A|$	C
1	1.243×10^{-3}	.648	$-$ .071	.890	.362
2	1.705×10^{-3}	.476	$-$ .033	.931	.240
3	1.571×10^{-3}	$-$ .056	.063	.125	$-$ .027
4	1.630×10^{-3}	-1.516	$-<.001$	$>$ .999	$-$ .785
5	1.628×10^{-3}	-1.909	$-$ .057	.970	-1.080
6	1.870×10^{-3}	-1.877	$-$ .036	.981	-1.146
7	1.818×10^{-3}	-1.023	$-$ .003	.997	$-$ .686
8	3.010×10^{-3}	$-$ .003	$<.001$	$>$ .999	$-$ .002
9	2.863×10^{-3}	$-$ .031	$<.001$	$>$ .999	$-$ .017
10	3.238×10^{-5}	$-$ .006	$<.001$	.981	$-$ .003
11	6.175×10^{-5}	$-$ .026	$<.001$	.992	$-$ .010
12	1.105×10^{-7}	$-<.001$	$<.001$	.999	$-<.001$
13	1.202×10^{-5}	$-$ .025	$-$ .004	.840	$-$ .006
14	3.247×10^{-5}	$-$ .105	$-$ .019	.819	$-$ .023
15	7.240×10^{-4}	-2.566	.301	.883	$-$ .479
16	1.730×10^{-3}	-3.459	$-$ .064	.981	$-$ .540
17	1.584×10^{-3}	2.520	.099	.961	.429
18	1.960×10^{-4}	$-$ .023	.002	.897	$-$ .007
19	2.108×10^{-4}	$-$ .008	$<.001$	.956	$-$ .002

TABLE I: The table lists the values of β for each channel and the following relative TOA clear sky radiance differences from a black surface located at 1000 mb.

$$A = 100 \cdot \frac{R_{360}^{LBL} - R_{330}^{LBL}}{R_{360}^{LBL}}, \quad B = 100 \cdot \frac{R_{360}^{LBL} - R_{360}^{\beta}}{R_{360}^{LBL}}$$

An U.S. standard atmosphere, a surface temperature of 288 K and a zenith angle of 0° is assumed. The fifth column is a measure of the performance of the β-approximation in accounting for CO_2 changes. "C" is the TOA brightness temperature difference in K between 330 and 360 ppmv. The long and shortwave temperature sounding channels are highlighted in bold type.

FIGURES

Fig 1: Plotted are the values α' as a function of pressure for HIRS 6 assuming CO_2 mixing ratios ranging from 320 ppmv to 380 ppmv. The mixing ratio values can be identified using the legend in figure 3a. The vertical lines represent the weighted mean of the α' curves. A vertical path through an U.S. standard atmosphere is assumed. The thick solid line is the HIRS 6 weighting function.

Fig 2: The weighted mean of each α' for each value of q used in figure 1 (solid circles) is plotted against its corresponding value of $q-q_0$. The line represents the curve $\alpha(q)=\beta(q-q_0)$, where β (applicable to HIRS 6) is determined by a least squares fit to these data.

Fig 3: Figure (a) depicts the HIRS 6 mean transmittance differences between,

$$A = \overline{\tau}^{LBL}(p,q) - \overline{\tau}^{LBL}(p,q_0), \quad \text{and} \quad B = \overline{\tau}^{\beta}(p,q) - \overline{\tau}^{\beta}(p,q_0)$$

The solid lines, A, are paired with the legend defined lines, B, one pair for each CO_2 mixing ratio. Figure (b) shows the difference between the curves in figure (a), ie;

$$A = \overline{\tau}^{LBL}(p,q) - \overline{\tau}^{\beta}(p,q)$$

The legend in figure (3a) applies.

Fig 4: Figure (a) illustrates two sets of differences defined as;

$$A = 100 \cdot \frac{R^{\beta}(p_s,q) - R^{\beta}(p_s,q_0)}{R^{\beta}(p_s,q)}, \quad B = 100 \cdot \frac{R^{LBL}(p_s,q) - R^{LBL}(p_s,q_0)}{R^{LBL}(p_s,q)}$$

as a function of cloud top pressure for HIRS 6. Clouds are assumed to be black. Curves A, defined by the legend in figure (5b), are paired with solid lines defined by B. Figure (b) depicts the difference between the curves in (a), ie;

$$100 \cdot \frac{R^{LBL}(p_s,q) - R^{\beta}(p_s,q)}{R^{LBL}(p_s,q)}$$

The U.S. standard atmosphere and a zenith angle of 0° is assumed.

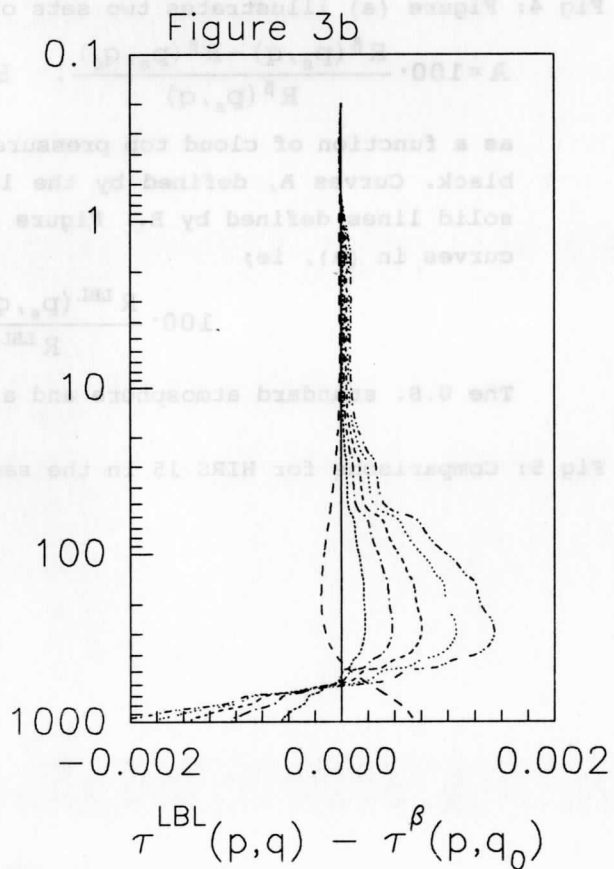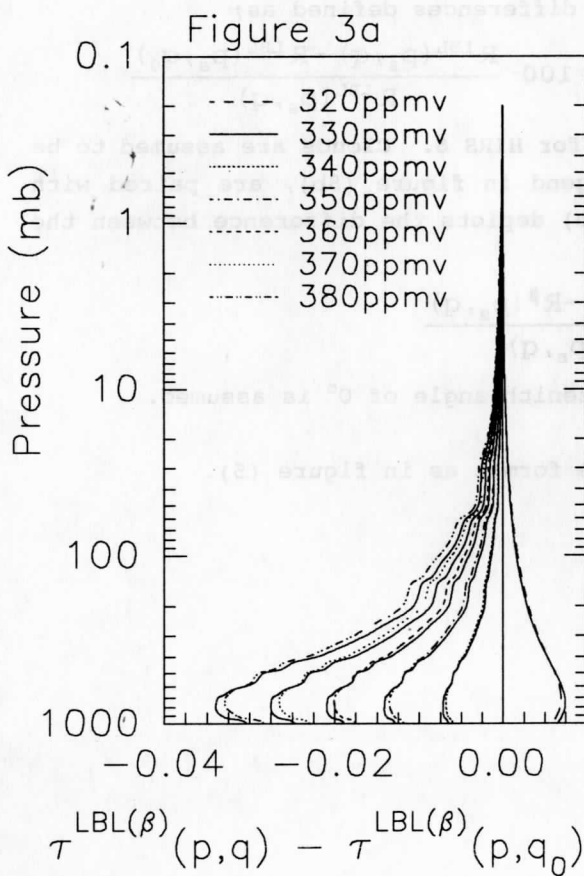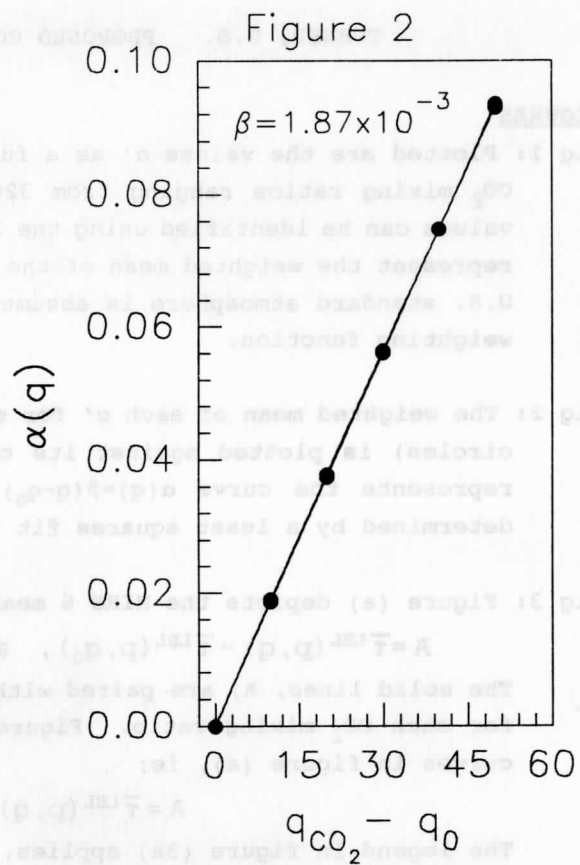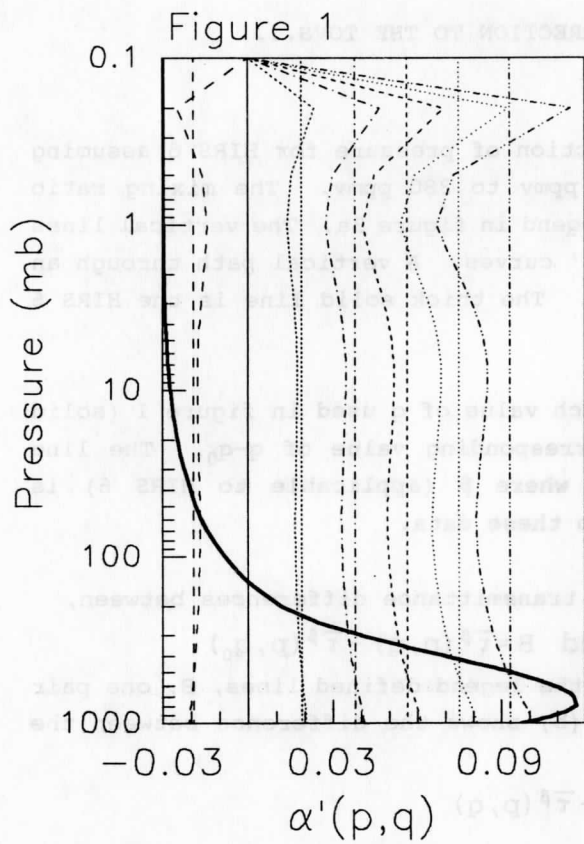Fig 5: Comparisons for HIRS 15 in the same format as in figure (5).

Figure 1

Pressure (mb)

0.1
1
10
100
1000

-0.03   0.03   0.09

$\alpha'(p,q)$

Figure 2

$\beta=1.87\times10^{-3}$

$\alpha(q)$

0.10
0.08
0.06
0.04
0.02
0.00

0   15   30   45   60

$q_{CO_2} - q_0$

Figure 3a

Pressure (mb)

0.1
1
10
100
1000

320ppmv
330ppmv
340ppmv
350ppmv
360ppmv
370ppmv
380ppmv

-0.04   -0.02   0.00

$\tau^{LBL(\beta)}(p,q) - \tau^{LBL(\beta)}(p,q_0)$

Figure 3b

0.1
1
10
100
1000

-0.002   0.000   0.002

$\tau^{LBL}(p,q) - \tau^{\beta}(p,q_0)$

Figure 4a

HIRS 6

Cloud Top Pressure (mb)

% difference (A,B)

Figure 4b

% difference (C)

Figure 5a

HIRS 15

Cloud Top Pressure (mb)

% difference (A,B)

Figure 5b

- - - - 320ppmv
— — — 330ppmv
———— 340ppmv
— · — · 350ppmv
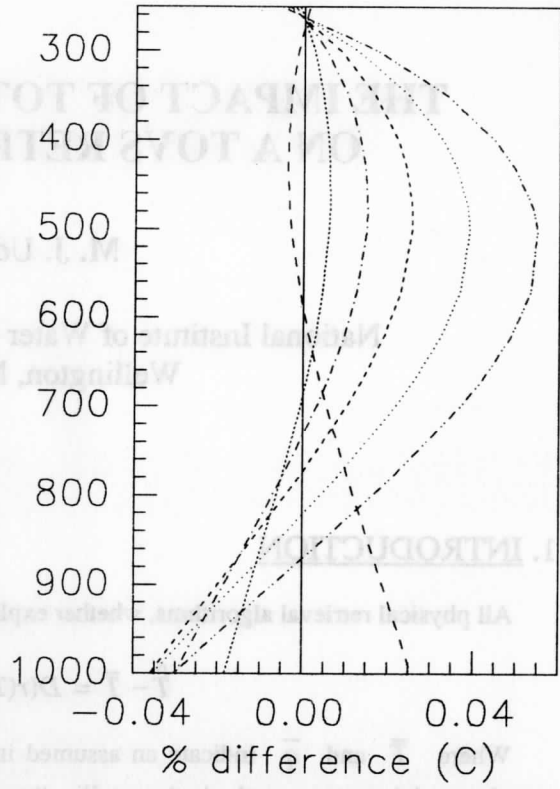· · · · · · · 360ppmv
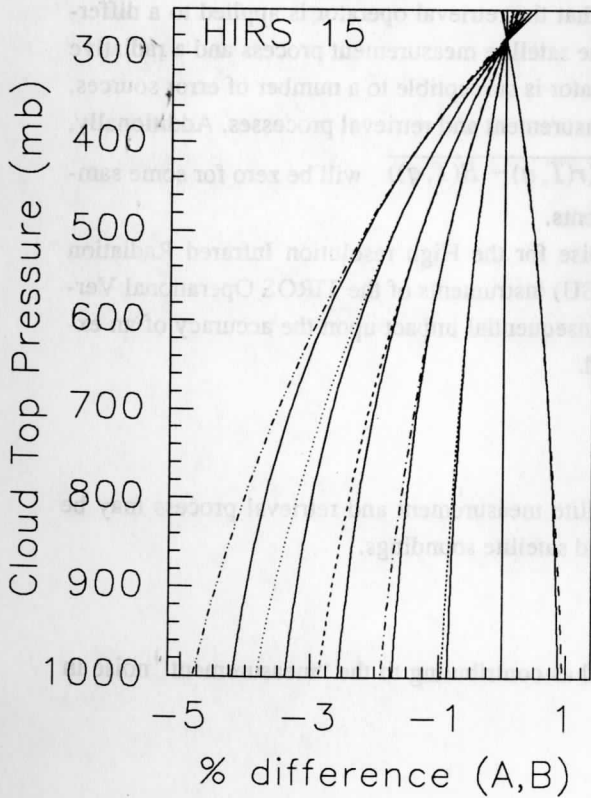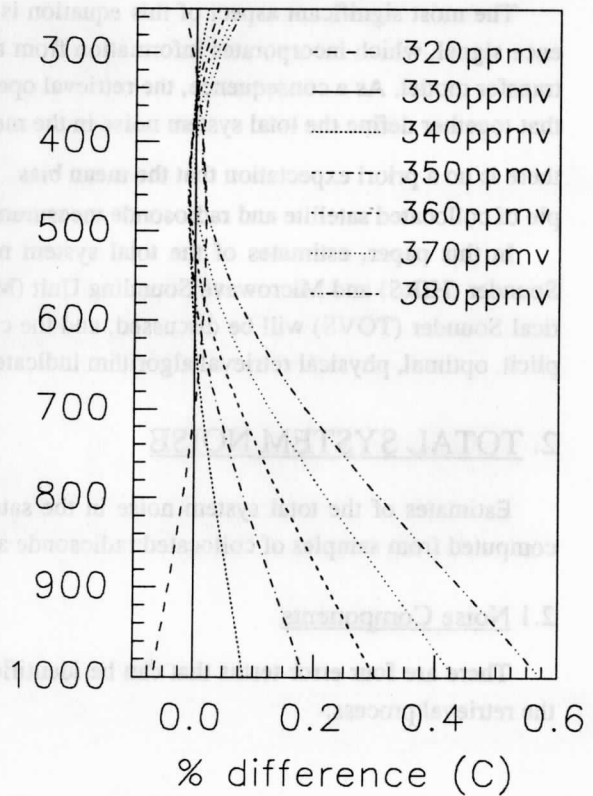· · · · · · 370ppmv
— · · — 380ppmv

% difference (C)

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