A Hybrid Iterative Method for ATOVS Temperature Profile Retrieval

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ABSTRACT

A hybrid iterative method is proposed for temperature profile retrieval. The method combines the advantages of the Smith (1970) iterative method and the Chahine (1970) relaxation method. Using ATOVS channels, the validity of the method is illustrated through determination of atmospheric temperature profiles and surface temperatures of five distinct model atmospheres, ranging from tropical to subarctic winter. The results indicate that the method is relatively insensitive to the quality of a first guess profile and can be used effectively to infer the diverse temperature structures in different regions of the earth's atmosphere.

1. INTRODUCTION

The interpretation of radiance measurements in terms of meteorological parameters requires the inverse solution of the radiative transfer equation (RTE). The RTE possesses nonunique inverse solutions for a given set of observed radiances, the degree of non-uniqueness dependent upon spectral resolution, the number of spectral channels, and instrument noise. Many retrieval methods have been developed in the past to obtain meteorologically meaningful solutions to the RTE or its linearized version which is a Fredholm integral equation of the first kind.

A large class of retrieval methods involve a matrix inverse operation for the solution to a discrete linearized RTE. This is mathematically ill-posed. The matrix inverse method for such a noise-sensitive system needs some constraints or statistical functions to stablize the solution. *Constrained* matrix inverse methods may provide good results if a regularization parameter is properly determined by trial-and-error or by some objective parameter-choice criteria, e.g. GCV (O'sullivan and Wahba, 1985). *Statistical* matrix inverse methods are advantageous for obtaining the meteorological profiles in regions where current or historical empirical observations are available to generate representative statistical functions (Smith, 1970).

Iterative methods belong to another class of retrieval methods, that do not involve any matrix inverse computation. Landweber (1951) first proposed a general iteration formula for Fredholm integral equations of the first kind. Fleming (1977) discussed the virtual duality between iterative and inverse matrix methods. Two well-known iterative methods applied to atmospheric remote sounding are the Chahine (1970) relaxation method and the Smith (1970) iterative method. Chahine's method calculates temperature values at prespecified pressure levels where the weighting functions peak, and then uses interpolation to recover the whole atmospheric temperature profile. The sufficient conditions for this method to converge were given by Barcilon (1975). Smith's method needs no interpolation for recovering temperature profiles and the solution is an extremely stable one because it is obtained by an averaging process weighted with differential transmittances. The weighted average is the key to insure that the inferred profile, starting from an arbitrary first guess, can approximate the true profile after several iterations. The disadvantage of Smith's method is that the method converges more slowly than the Chahine relaxation solution, assuming that the latter does not diverge. The greater stability but slower convergence of Smith's iterative relation is a result of its *addition* operation, whereas Chahine's iterative relation is a *multiplication* operation.

In this paper an iterative method is presented which is a hybrid of the Chahine and the Smith methods. Like Chahine's method the iterative relation in this method is achieved by a multiplication operation. Like Smith's method no interpolation is necessary for recovering the complete temperature profile stability is achieved by employing a weighted average. The performance of the method is demonstrated by using ATOVS channels to retrieve temperature profiles and surface temperatures of five distinct model atmospheres (McClatchey et al. 1972), ranging from tropical to subarctic winter.

2. MATHEMATICAL DEVELOPMENT

For a nonscattering stratified atmosphere in local thermodynamic equilibrium, the upwelling spectral radiance observed by the satellite at a channel centered at wavenumber v is described by the radiative transfer equation (RTE):

$$R_v = \varepsilon_{vs}\,\tau_{vs}\,B_v(T_s) + \int_{p_s}^{0} B_v[T(p)]\,\frac{\partial \tau_v(p)}{\partial p}\,dp + r_{vs}\,\tau_{vs}\int_{0}^{p_s} B_v[T(p)]\,\frac{\partial \tau_v^*(p)}{\partial p}\,dp, \quad (1)$$

where B_v is the Planck function:

$$B_v[T(p)] = \frac{c_1\,v^3}{\exp[c_2\,v / T(p)] - 1}, \quad (2)$$

c_1 and c_2 are known constants, T is temperature, p is pressure, ε_{vs} is the surface emissivity, r_{vs} is the surface reflectivity, $\tau_v(p)$ is the channel transmittance between the pressure p and the satellite, and $\tau_v^*(p)$ is the channel transmittance between the pressure p and the surface. The subscript s denotes the earth's surface. The first term of the RTE represents the surface radiance to the satellite, the second term the upwelling radiance from the atmospheric layer of temperature T and pressure p to the satellite, and the third term the downwelling radiance being reflected by the surface upward to the satellite.

With the approximations of $r_{vs} \approx 1 - \varepsilon_{vs}$ and $\tau_v^*(p) \approx \tau_{vs}/\tau_v(p)$, Eq. (1) becomes

$$R_v = \varepsilon_{vs}\,\tau_{vs}\,B_v(T_s) + \int_{0}^{p_s} K_v(p)\,B_v[T(p)]\,dp, \quad (3)$$

where

$$K_\nu(p) = -\left\{1+(1-\varepsilon_{\nu s})\left(\frac{\tau_{\nu s}}{\tau_\nu(p)}\right)^2\right\}\frac{\partial \tau_\nu(p)}{\partial p} \tag{4}$$

is the weighting function enhanced by surface reflection. Writing Eq. (3) in an iterative form:

$$R_\nu^n = \varepsilon_{\nu s}\,\tau_{\nu s}^n\,B_\nu(T_s^n) + \int_0^{p_s} K_\nu^n(p)\,B_\nu\left[T^n(p)\right]dp, \tag{5}$$

where the superscript n on K_ν^n suggests recomputing the weighting function with each new temperature profile. If the value of R_ν^n in Eq. (5) is not equal to that of R_ν in Eq. (3), we rewrite Eq. (5) as

$$R_\nu = \varepsilon_{\nu s}\tau_{\nu s}^n\,\frac{R_\nu}{R_\nu^n}B_\nu(T_s^n) + \int_0^{p_s} K_\nu^n(p)\frac{R_\nu}{R_\nu^n}B_\nu\left[T^n(p)\right]dp \tag{6}$$

and define

$$\begin{cases} B_\nu\left[T_\nu^{n+1}(p)\right] = \dfrac{R_\nu}{R_\nu^n}B_\nu\left[T^n(p)\right] \\[2ex] B_\nu\left(T_{\nu s}^{n+1}\right) = \dfrac{R_\nu}{R_\nu^n}B_\nu\left(T_s^n\right) \end{cases} \tag{7}$$

Eq. (7) is the same iteration relation obtained by Chahine (1970) except that the pressures are now taken as continuous values instead of the discrete pressure levels where weighting functions peak. The channel dependent estimate of the temperatures $T_\nu^{\,n+1}(p)$ and $T_{\nu s}^{n+1}$ in Eq. (5) can be obtained from the inverse Planck function:

$$\begin{cases} T_\nu^{n+1}(p) = c_2\,\nu \Bigg/ \ln\left[1+\dfrac{c_1\,\nu^3}{\dfrac{R_\nu}{R_\nu^n}B_\nu\left[T^n(p)\right]}\right], \\[4ex] T_{\nu s}^{n+1} = c_2\,\nu \Bigg/ \ln\left[1+\dfrac{c_1\,\nu^3}{\dfrac{R_\nu}{R_\nu^n}B_\nu\left(T_s^n\right)}\right] \end{cases} \tag{8}$$

where c_1 and c_2 are the same constants used in Eq. (2). The channel independent temperature profile at the next iteration is given by a *Smith*-type weighted average of channel dependent temperature profiles:

$$\begin{cases} T^{n+1}(p) = \dfrac{\sum_{v} W_v^n(p)\, T_v^{n+1}(p)}{\sum_{v} W_v^n(p)}, & where \quad W_v^n(p) = K_v^n(p) \\ T_s^{n+1} = \dfrac{\sum_{v} W_{vs}^n\, T_{sv}^{n+1}}{\sum_{v} W_{vs}^n}, & where \quad W_{vs}^n = \varepsilon_{vs}\, \tau_{vs} \end{cases} \tag{9}$$

The solution is valid for all regions except where all $W_v^n(p) = 0$ or $W_{vs}^n(p) = 0$.

3. COMPUTATIONAL PROCEDURES

Thus, given radiance observation in a spectral region where the absorbing gas distributions and surface type are known, the temperature profile can be calculated through the following steps:

(i) Guess at the temperature profile $T^n(p)$ and T_s^n, $n = 0$.

(ii) Compute $\tau_v^n(p)$ using a transmittance model, $K_v^n(p)$ using Eq. (4), $B_v\left[T^n(p)\right]$ using Eq. (2) and subsequently R_v^n using Eq. (5).

(iii) If the residual norm, $\left\|R_v^n - R_v\right\|$, is not larger than the norm of instrumental noise, $\left\|\sigma_v\right\|$, and the convergence $\left| R_v^n - R_v^{n-1} \right| / R_v^{n-1}$ is less than a preset criterion (say, less than 0.00001) for each channel, then $T^n(p)$ and T_s^n are the solutions.

(iv) Otherwise, compute $B_v\left[T_v^{n+1}(p)\right]$ and $B_v\left(T_s^{n+1}\right)$ using Eq. (7) and subsequently compute $T_v^{n+1}(p)$ and T_{vs}^{n+1}, using Eq. (8).

(v) Compute new estimates of $T^{n+1}(p)$ and T_s^{n+1}, using Eq. (9). Go to step (ii).

4. RETRIEVAL RESULTS

To assess the applicability of the current method to the forthcoming Advanced TIROS Operational Vertical Sounder (ATOVS), we performed atmospheric and surface temperature retrievals by using 39 ATOVS channels (19 HIRS channels plus 20 AMSU channels). Figure 1 shows the HIRS and AMSU weighting functions for the 1976 U.S. Standard atmosphere.

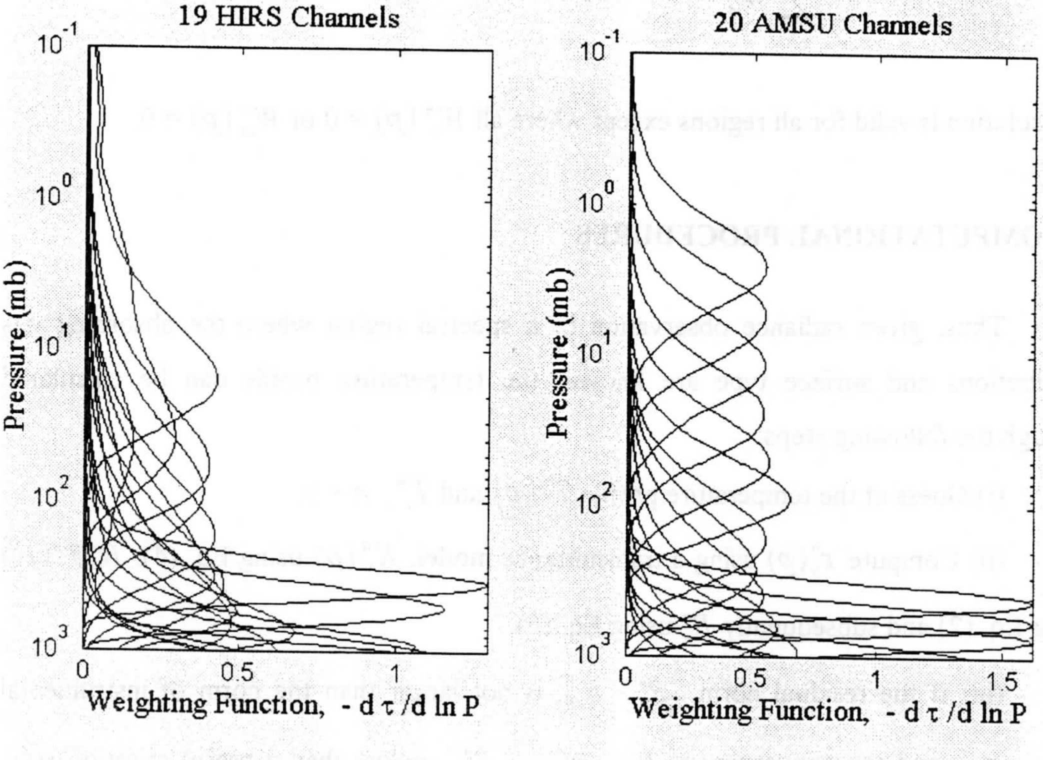


Figure 1. HIRS and AMSU weighting functions for the 1976 U.S. Standard atmosphere.

To explore the performance of this method for retrieving diverse atmospheric temperature profiles, we applied it to five distinct model atmospheres (McClatchey et al. 1972), ranging from tropical to subarctic winter. Furthermore, a poor isothermal first guess profile of 250 °K was used. Figure 2 shows the first guess profile and the five model atmospheres.

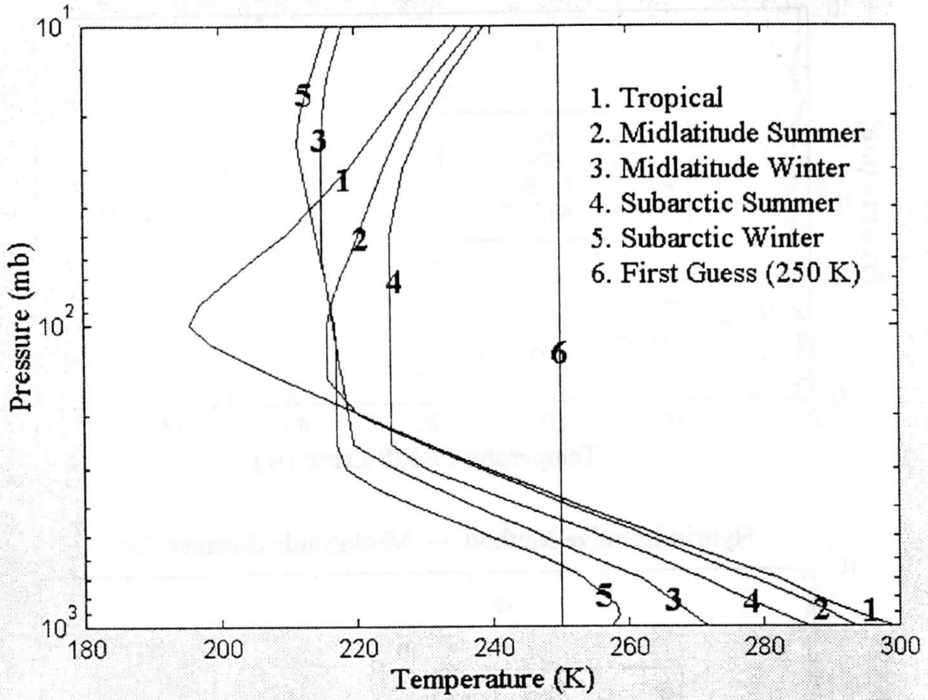
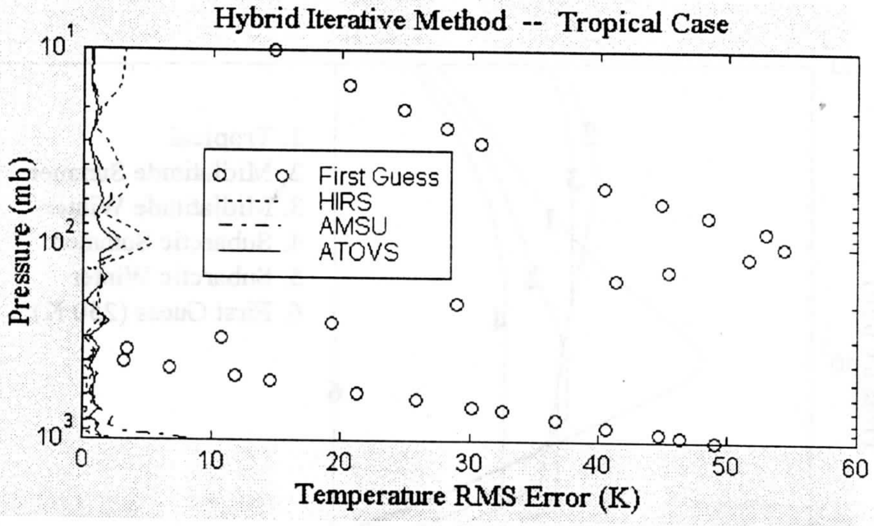
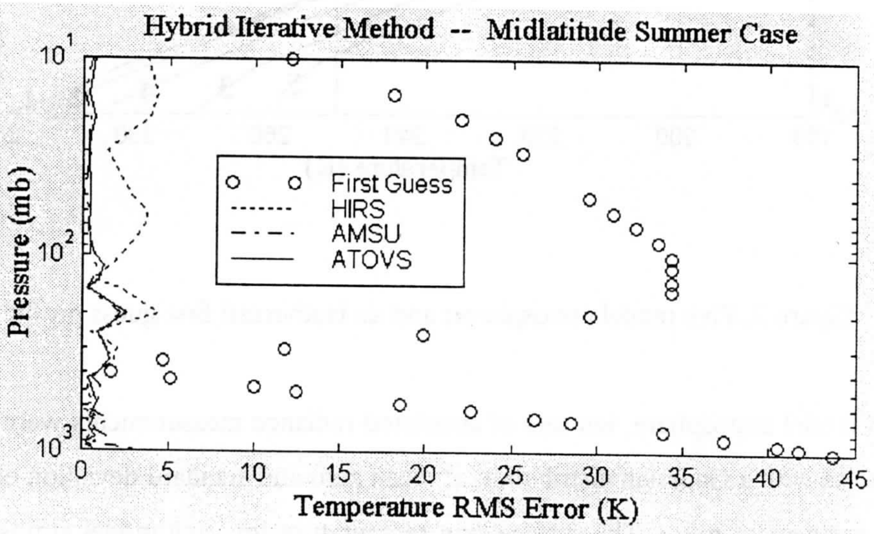
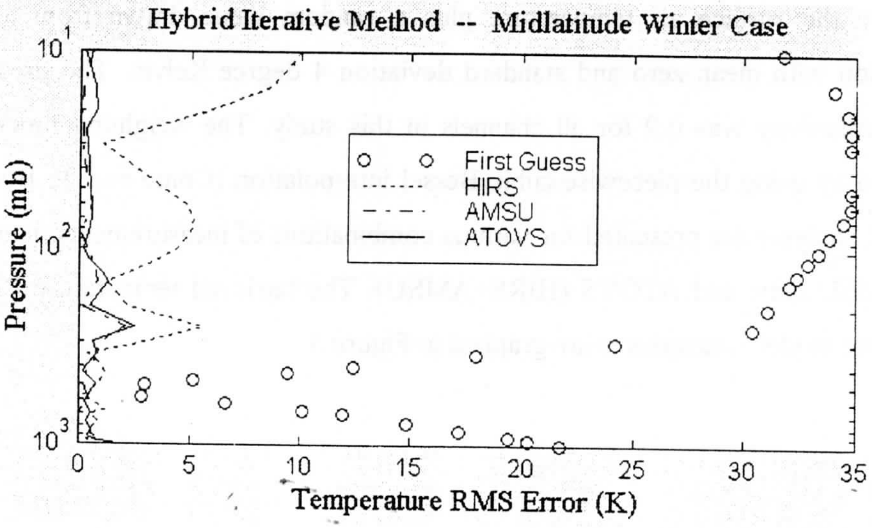


Figure 2. Five model atmospheres and an isothermal first guess profile.

For each model atmosphere, ten sets of simulated radiance measurements were generated adding noise with a Gaussian distribution of mean zero and standard deviation equal to the ATOVS instrument noise. The surface skin temperature for each model atmosphere was set to be the surface air temperature plus a random value drawn from a Gaussian distribution with mean zero and standard deviation 4 degree Kelvin. The predetermined surface emissivity was 0.9 for all channels in this study. The weighting functions were computed by using the piecewise-cubic Bessel interpolation (Conte and de Boor, 1980). Retrieval analyses are presented for various combinations of measurements, namely HIRS only, AMSU only, and ATOVS (HIRS+AMSU). The retrieved temperature RMS errors for the five model atmospheres are graphed in Figure 3


Hybrid Iterative Method -- Tropical Case
Pressure (mb)
10^1
10^2
10^3
First Guess
HIRS
AMSU
ATOVS
0
10
20
30
40
50
60
Temperature RMS Error (K)


Hybrid Iterative Method -- Midlatitude Summer Case
Pressure (mb)
10^1
10^2
10^3
First Guess
HIRS
AMSU
ATOVS
0
5
10
15
20
25
30
35
40
45
Temperature RMS Error (K)


Hybrid Iterative Method -- Midlatitude Winter Case
Pressure (mb)
10^1
10^2
10^3
First Guess
HIRS
AMSU
ATOVS
0
5
10
15
20
25
30
35
Temperature RMS Error (K)

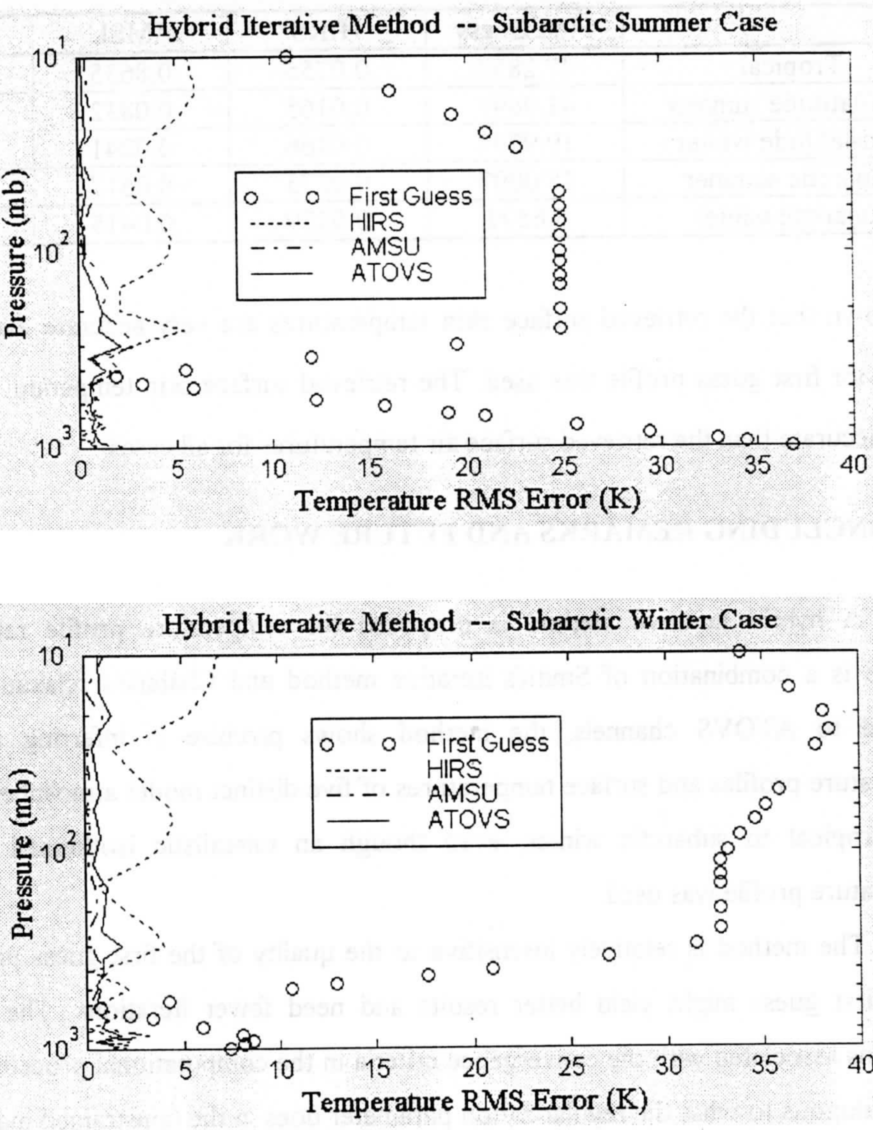


Figure 3. Retrieved temperature RMS errors for the five model atmospheres.

Figure 3 indicates that the retrieval algorithm performs quite well even though a poor first guess profile was used. Finally, the surface skin temperature RMS errors are summarized in Table 1.

Table 1. Surface Skin Temperature RMS Errors (in degrees Kelvin)

	First Guess	HIRS	AMSU	ATOVS
Tropical	47.2897	0.0755	0.8635	0.0684
Mid-latitude summer	41.9697	0.0165	0.0832	0.0299
Mid-latitude winter	19.9797	0.0366	0.0241	0.0227
Subarctic summer	35.0097	0.0275	0.0617	0.0289
Subarctic winter	5.6597	0.0329	0.0415	0.0229

As shown that the retrieved surface skin temperatures are very accurate even though a very poor first guess profile was used. The retrieved surface skin temperatures are much more accurate than the retrieved surface air temperatures for all cases.

5. CONCLUDING REMARKS AND FUTURE WORK

A hybrid iterative method is proposed for temperature profile retrieval. The method is a combination of Smith's iterative method and Chahine's relaxation method. Applied to ATOVS channels, the method shows promise in inferring atmospheric temperature profiles and surface temperatures of five distinct model atmospheres, ranging from tropical to subarctic winter, even though an unrealistic isothermal first guess temperature profile was used.

The method is relatively insensitive to the quality of the first guess profile, but a good first guess might yield better results and need fewer iterations. The number of iterations associated with the convergence criteria in the computational procedures plays a role analogous to what the regularization parameter does in the constrained matrix inverse methods. We found that the single criterion of fitting the residual norm to the norm of instrumental noise is usually not enough to yield the best solution. In the near feature we will better define optimal iteration termination criteria. The current version of this method requires independent estimates of the absorbing gas profiles. Recently, we have developed two new iterative methods for simultaneous retrieval of the profiles of temperature and absorbing gases. Numerical implementation of these new methods is in progress.

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