NO. 5

REPORT TO THE UNIVERSITY OF WISCONSIN

LAKE INVESTIGATIONS COMMITTEE

ON CERTAIN OSCILLATORY MOTIONS OF LAKES

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OF CERTAIN OSCILLATORY MOTIONS OF LAKES

In lakes and bays a periodic rise and fall of the water level along a particular longitudinal axis has been called a "Seiche". This oscillation is regarded as a standing wave with free oscillations of a period depending upon the horizontal and vertical cross sectional dimensions of the lake (or bay) and upon the number of nodes in the standing wave. Thus in a uninodal system the wave length will correspond to the length of the lake. Since the length of the lake is generally much greater in absolute magnitude than the depth, the waves will be of the nature of long waves.

Any influence which can produce a temporary elevation or depression at one place on the lake will give rise to a seiche. Sharp wind changes, rapid and large scale changes in barometric pressure, subsurface earth tremors, heavy runoff at one end of a lake are all possibilities in the initiation of seiche movement.

Our work in the Four Lakes region has had a dual purpose in establishing the physical significance of the periodic oscillations of the lakes, namely, the application of the classical Defant method[1] of period and node calculation together with the establishment of the general agreement of such theoretical calculations with actual observations; and secondly, the application of seiche observations in a study of stress, momentum and energy relations.

In considering a free oscillation, antinodes of the wave in a closed lake are necessarily located at the ends of the lake, while the nodal points will be located along the principal axis. The vertical displacement is zero at each shore as a necessary boundary condition. Defant's method of computing the period, magnitude of vertical displacement and position of nodal lines is in essence a numerican integration of the hydrodynamic equation freed from the time factor. Lamb's textbook on "Hydrodynamics"[2] provides the background equations of much of Defant's work. Practically, the Defant method embodies the choice of a principal longitudinal axis of oscillation followed by a rough calculation of the period of oscillation by "Merrian's Formula," where the period of oscillation, T, is given by

$$T = (2l)/\sqrt{gh}$$ [7]

where l length, g acceleration due to gravity, and h mean depth of lake. After these preliminary steps the horizontal displacement and vertical displacement may be calculated.

To establish agreement between theory and observation, a series of recording stations were set up in strategic locations on the four lakes. A Stevens Water Level Recorder employing a counter-balance float and clock driven rotating drum was employed for this purpose with a stilling well of 55 gallon capacity sunk into the bottom some six inches and rigidly fastened between the posts of the recorder housing. Figure 1 is an actual record from the Lake Lab Station of the seiche resulting from the severe storm of 8 July 1951.

Figure 2 illustrates the results of the Defant method of period and vertical displacement computation with a north-south longitudinal axis and a uni-nodal wave for Lake Kegonsa on 3 September 1951. Eleven sections were chosen with the intersectional distance approximately 1/4 mile. The resulting period of 12.3 min from calculations compared excellently with the observed period of 12.10 minutes.

On a graph with ordinate as vertical amplitude and abscissa as horizontal extent of the principal axis, the oscillation forms of Lake Mendota for a generally east-west and north-south axis are shown in Figure 3. The agreement between theoretical period and observed period was again within 12 seconds, the observed period for the east-west axis being 25.8 minutes as against 25.5 minutes by computation, and for the north-south axis and 25.6 minutes as against 25.3 minutes by computation. The location of the nodal point in each case is subsequently employed in a computation to define a complex surface of deformation. The longitudinal axis in each case was chosen from a consideration of the wind direction initiating the oscillation and lies along this axis. The table summarizes theory and observational agreement on three of the four lakes.

AXIS	Computed Period (minutes)	Observed Period (minutes)
N-S - Kegonsa	12.30	12.10
E-W - Mendota	25.50	25.80
N-S - Mendota	25.30	25.60
E-W - Monona	-	-

Table 1.

Insofar as the lakes as a whole have been considered, we find no deviation of the theoretical period calculations from the observed periods when considering a uninodal seiche along a single axis. However when Defant's method of period computation, modified to compute a period for a bay of a lake was carried out, large discrepancies between theory and observation resulted. Defant has assumed that there must always be a nodal line across the mouth of a bay. And he further assumes that the uninodal period of this bay would be the same as that of two such bays joined at their bay mouths. Following the manner of his computations the period for University Bay of Lake Mendota was found to be 4.5 minutes. However a permanent Seiche Station deep in the bay regularly gave periods of 25.6 minutes while one on the east side of Picnic Point gave a period of 15.1 minutes indicating a complexity in a bay oscillation.

Following the discrepancy in bay period calculations it was assumed that Defant had perhaps oversimplified the surface of vertical deformation of a lake. The oscillations on two axes were, consequently, combined and a chart (Figure 4) of isolines of Relative Vertical Amplitude was prepared. The chart was constructed by superimposing lines of relative vertical amplitude at right angles to the principal longitudinal east-west and north-south axis. Isolines giving contours of equal relative vertical displacement were then drawn in to complete the final chart. It was found that the resulting nodal lines and nodal point near the northwest tip of Picnic Point coincided with previous observations.

With a view to checking the standing wave nature of the seiche, the arrival time of high water at each of the stations was compared. If we assume, as Defant did, that the seiche is a standing wave, then high water would arrive at each recording station at precisely the same time instant, a phase difference for maximum vertical amplitude was apparent. The location of six seiche recording stations around Lake Mendota together with their time phase angle is shown in Figure 5. If we assume the Lake Lab Station as our base station with zero phase angle we observe that the time of maximum vertical amplitude is observed at successively later times as one moves in a clockwise direction around the perimeter of the lake.

Considering the circumference of the lake as 25.8 km and the period as 25.6 minutes we arrive at a velocity of 16.8 m/sec for a progressive wave of this nature. This value is in agreement with the theoretical computation of the velocity of progress of a long wave for Lake Mendota of 12 m/sec. If the perimeters of the smaller bays are disregarded and we assume a perimeter of 20 km we obtain a velocity of 13.3 m/sec for the progressive wave which is in close agreement with the theoretical value of 12 m/sec.

H. U. Sverdrup in his work on the "Dynamics of Tides on the North Siberian Shelf"[3] has concluded that it is not sufficient to assume that a free tidal wave proceeds under the influence of gravitational forces only. He takes into account the effect of Coriolis force as well as the effect of internal resistance. He shows that the rotary character of the tidal waves can be explained as a result of the deflecting force of the earth's rotation. His findings as to the rotational direction of the free tidal wave oscillations which can be likened as he states to a (seiche) oscillation agree with results on Lake Mendota in that the theoretically progressive "seiche", which is a free oscillation, moves in a clockwise direction about the lake. Further work is being carried out on the progressive nature of the seiche and on the discrepancy between the observed velocity with which the wave proceeds and that computed by the simplified equation, $C = \sqrt{gh}$.

The second phase of the study of the oscillations of lake surfaces in the four lakes region deals with the application of seiche observations to sediment transport and boundary process problems.

Obtaining sufficient data to carry through a standard type study of bottom stress is, to say the least, a difficult problem in instrumentation and requires an elaborate multi-recording current meter with the customary tripping mechanism to obtain a quantitative measure of the velocity distribution with height in the turbulent layer. However, it is possible to arrive at the bottom stress by measuring the rate of seiche amplitude damping.

The steps employed in determining bottom stress, τ_B , require the following basic assumptions and definitions:

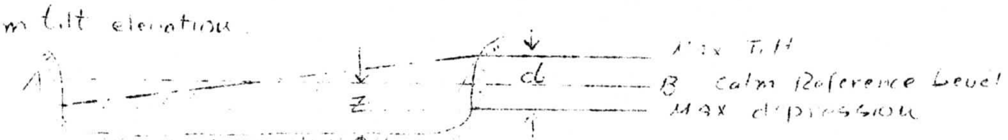
(1) Stress is defined as the rate of transfer of horizontal momentum in the vertical.

(2) Rate of momentum transport $\frac{d(mv)}{dt}\cdot\frac{1}{A}$ across the surfaces normal to the velocity gradient has the dimensions of stress $M/L/T^2$ and is, after reversing the sign identical with τ .

(3) The maximum potential energy of a lake undergoing periodic free oscillations at the point of maximum tilt is equal to the maximum kinetic energy at the point in the oscillation at which the surface is horizontal. Potential energy is represented by .

Assume $KE = \frac{1}{2}mv^2 = \frac{1}{2}\rho z A v^2$; where z is the average depth of lake and "v", current velocity. From a consideration of the drawing below:

d = maximum tilt elevation



we see that PE = KE when the oscillation swing is in position AB, the horizontal.

$$\therefore \frac{1}{2}\rho g h^2 A = \frac{1}{2}\rho z A v^2$$

Thus $v = h\sqrt{g/z}$, where "h" is the maximum amplitude change from the Seiche Record.

Assume:

$$\tau_B = \frac{d(mv)}{dt}\cdot\frac{1}{A} \qquad \text{(Basic Assumption)}$$

Substituting for v and m,

$$\tau_B = \frac{d(\rho A z h\sqrt{g/z})}{dt}\cdot\frac{1}{A}$$

$$\tau_B = \frac{d(\rho h\sqrt{gz})}{dt} = gm\ cm^{-1}\ sec^{-2}$$

The following table compares calculations of stress on the bottom of Lake Mendota by the method just outlined with stress calculations made by R. H. Lesser[4] with a current meter under similar conditions over a mud-sand bottom.

Date	Initiating Wind	τ_B (As above) gm/cm/sec^2	τ_B (Lesser) gm/cm/sec^2
6/28/51	SW 29	.10	.13
7/8/51	W 62	.48	.49
8/14/51	NW 18	.20	.21
8/15/51	E 15	.17	.21
8/28/51	NW 25	.23	

Table 2

Referring to the drawing on Page 5, we may express the potential energy equation per unit area due to the tilt of the lake by $P.E. = \frac{1}{2}\rho g d^2$ where d is the maximum height increase of the lake due to the tilt since the total Potential Energy is given by, $P.E. = \frac{1}{2}\rho g d A d$

where ρ is the density of water,

g is the acceleration due to gravity, and

A is the surface area of the lake.

In considering the energy dissipated by a lake during the frictional damping out of a seiche, the record from the Stevens Recorder is again employed. From this record we obtain the average height increase of the lake, d, from the tilt of its surface above the calm weather reference level. To compute the energy added to the lake per unit area by the surface tilt or water pileup we employ the energy equation: $P.E. = \frac{1}{2}\rho g d^2$

The rate of dissipation as the oscillation dies out we obtain from the envelope of the oscillation. Thus the rate of dissipation of energy per unit area is given by

$$\frac{d(P.E.)}{dt} = \frac{\frac{1}{2}\rho g d^2}{dt} = \text{ergs cm}^{-2}\text{ sec}^{-1}$$

From data obtained from the seiche record in July and August, a table was prepared showing the close agreement between the Potential Energy per unit area and the rate of dissipating of Potential Energy.

Date	Pot. E./ Area	Rate of Dissip. of Pot. E.	τ_B
7/8/51	4.9 x 10 ergs	.20 ergs per sec.	.490
7/20/51	5.65 10 ergs	.22 ergs per sec.	.112
8/14/51	4.91 10 ergs	.19 ergs per sec.	.210
Mean	5.15 10 ergs	.20 ergs per sec.	.271

Table 3

Returning to our basic assumption that at some time in the free oscillation of the lake such as position "AB" in the drawing on Page 5, we may say that the Potential Energy per unit volume is equal to the Potential Energy per unit area divided by the mean depth of the lake,

$$\frac{P.E.}{Vol} = \frac{P.E./\text{unit area}}{\text{Mean depth}} = P.E.$$

From the mean potential energy per unit area given in the table above we have $K.E. = (5.15 \times 10^3)/(1.2 \times 10^3) = 4.3$ ergs

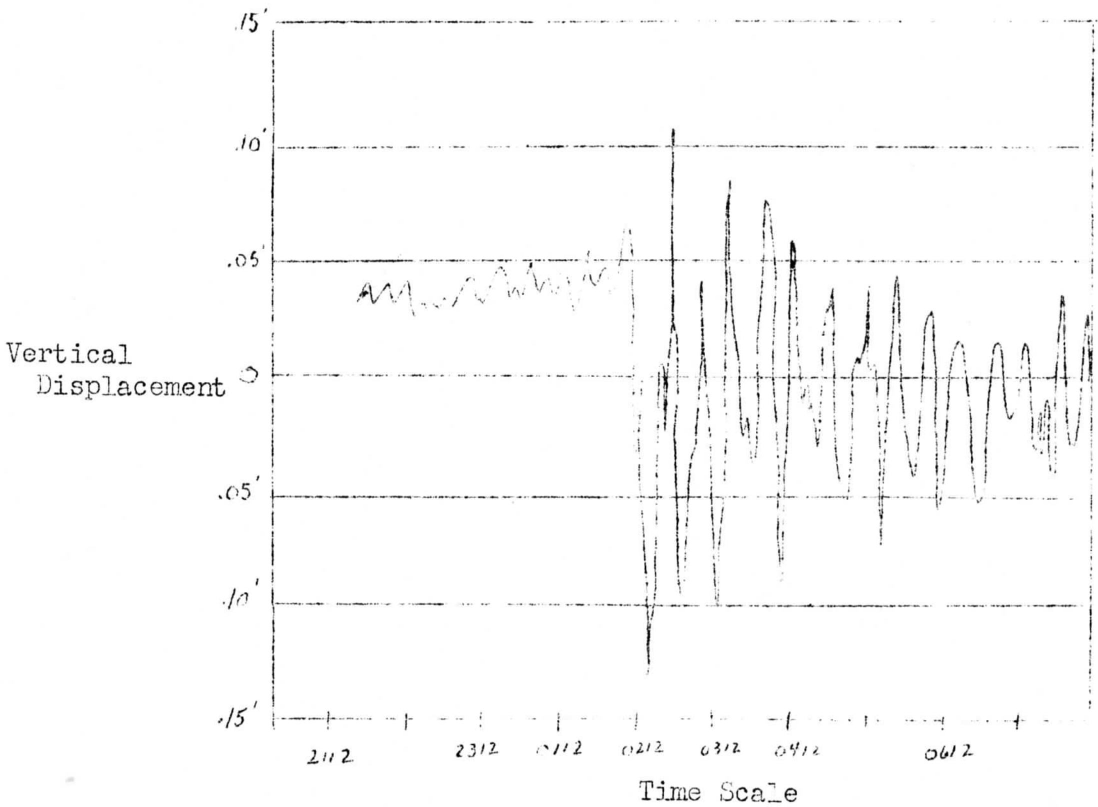
And from the relation, $K.E. = \frac{1}{2}\rho v^2$, we obtain a mean current velocity of approximately 3 cm per second at depth throughout Lake Mendota which is in accord with a number of drag measurements of currents at depth. This is of great importance in its relation to the movement of algal producing nutrients and water renewal or water shifting along the bottom. Defant's method of seiche period computation also leads to this value of 3 cm/second from a consideration of currents due to the seiche resulting from vertical displacements. Here, then, we have a direct application of the seiche in dealing with nutrient movement and the currents necessary to shift bottom deposits.

SEICHE RECORD

STATION #1

LAKE LAB - UNIV. OF WISCONSIN

7 and 8 July 1951



Remarks:
(1) At 0012 hours wind from S-W at 15 mph. (7-8-51)
(2) At 0223 hours wind from West at 78 mph.
Heavy thunderstorm.
(3) At 0312 hours wind from North at 32 mph.
(4) At 0412 hours wind from North at 25 mph.

Figure 1

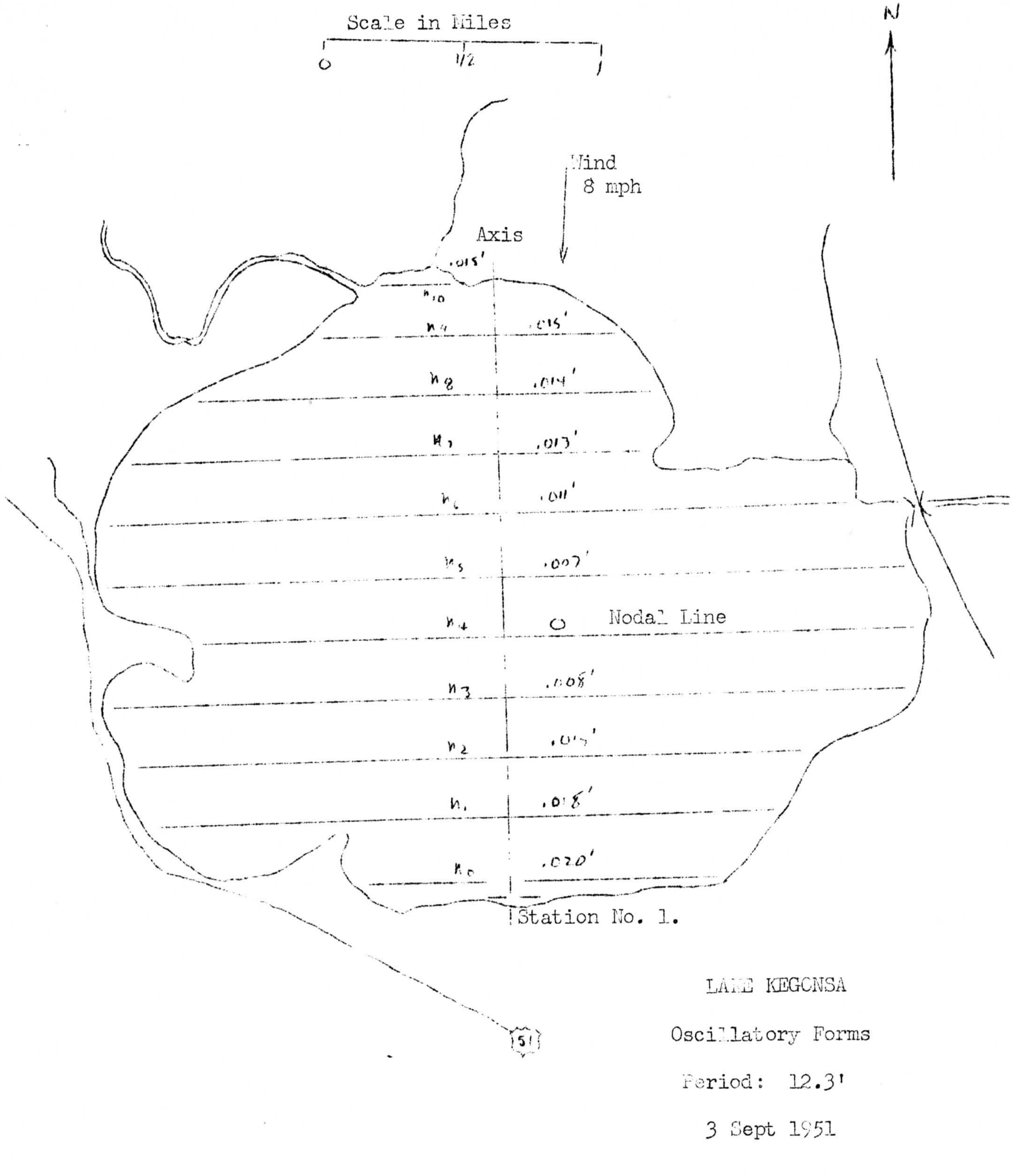

Scale in Miles
0
1/2
1
N
Wind
8 mph
Axis
.015'
h_{10}
h_9
.015'
h_8
.014'
h_7
.013'
h_6
.011'
h_5
.007'
h_4
0
Nodal Line
h_3
.008'
h_2
.015'
h_1
.018'
h_0
.020'
Station No. 1.
51
LAKE KEGONSA
Oscillatory Forms
Period: 12.3'
3 Sept 1951

Figure 2

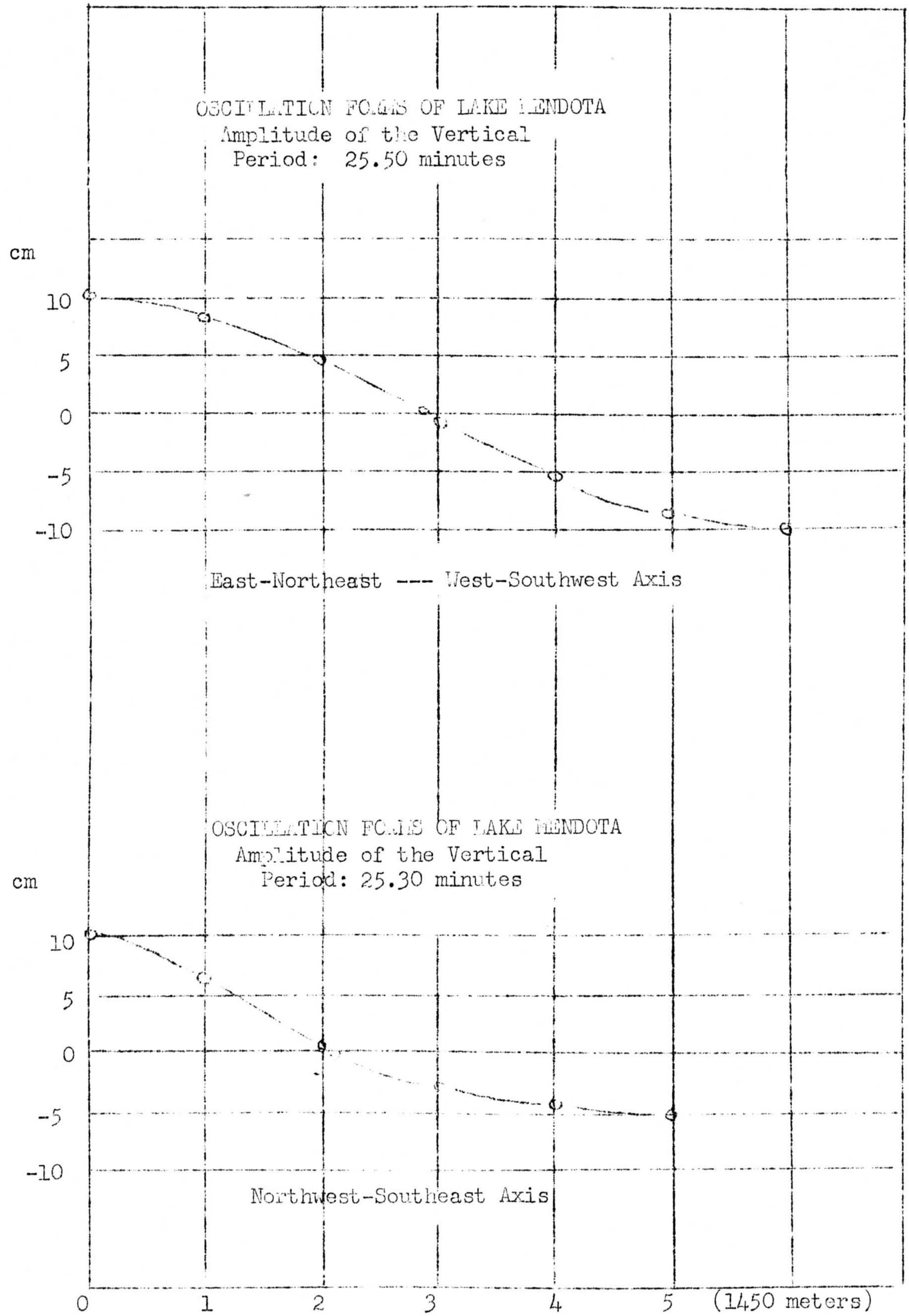

OSCILLATION FORMS OF LAKE MENDOTA
Amplitude of the Vertical
Period: 25.50 minutes
cm
10
5
0
-5
-10
East-Northeast --- West-Southwest Axis
OSCILLATION FORMS OF LAKE MENDOTA
Amplitude of the Vertical
Period: 25.30 minutes
cm
10
5
0
-5
-10
Northwest-Southeast Axis
0
1
2
3
4
5
(1450 meters)

Figure 3

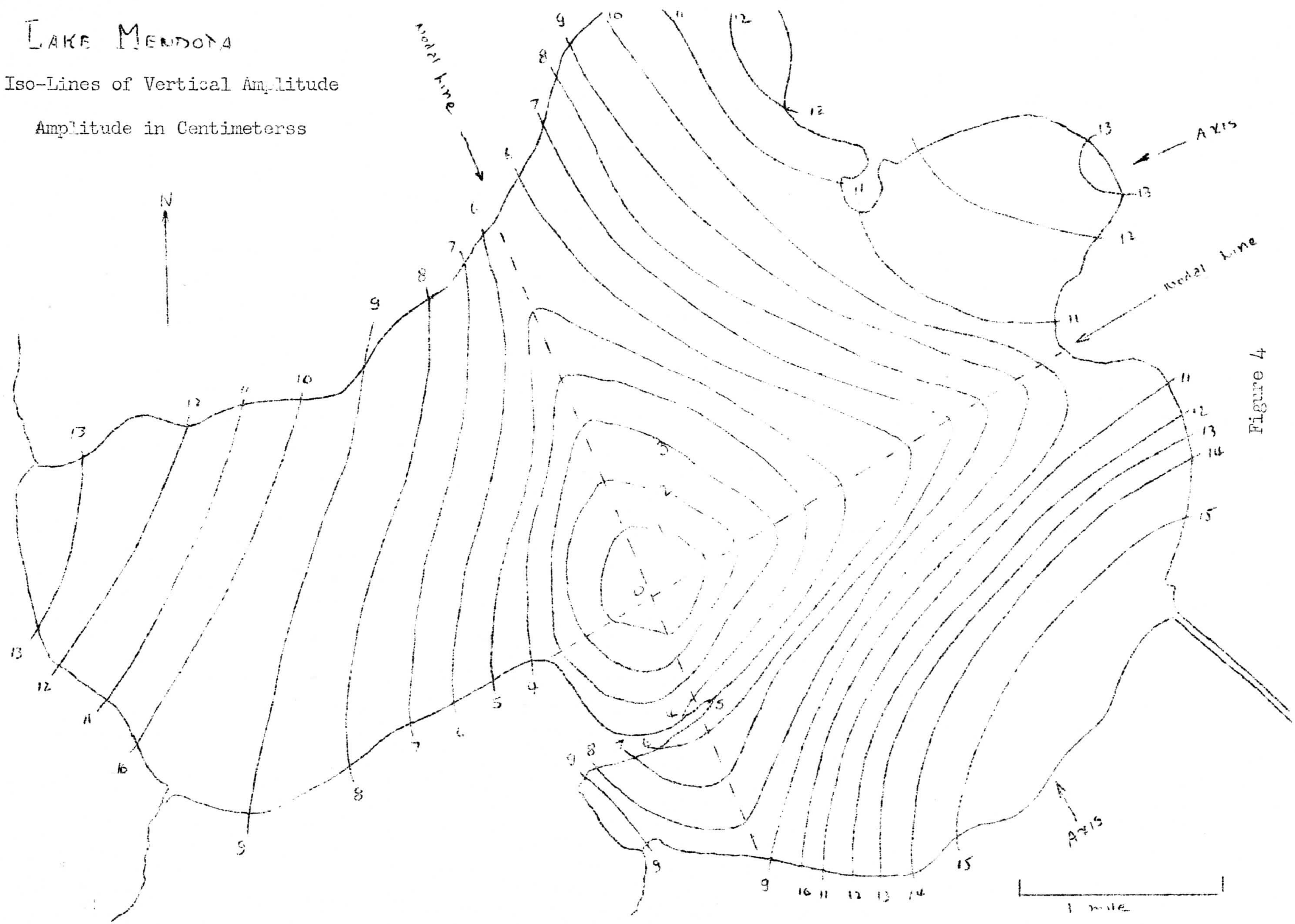

LAKE MENDOTA
Iso-Lines of Vertical Amplitude
Amplitude in Centimeterss
N
Nodal Line
Nodal Line
AXIS
AXIS
1 mile

Figure 4

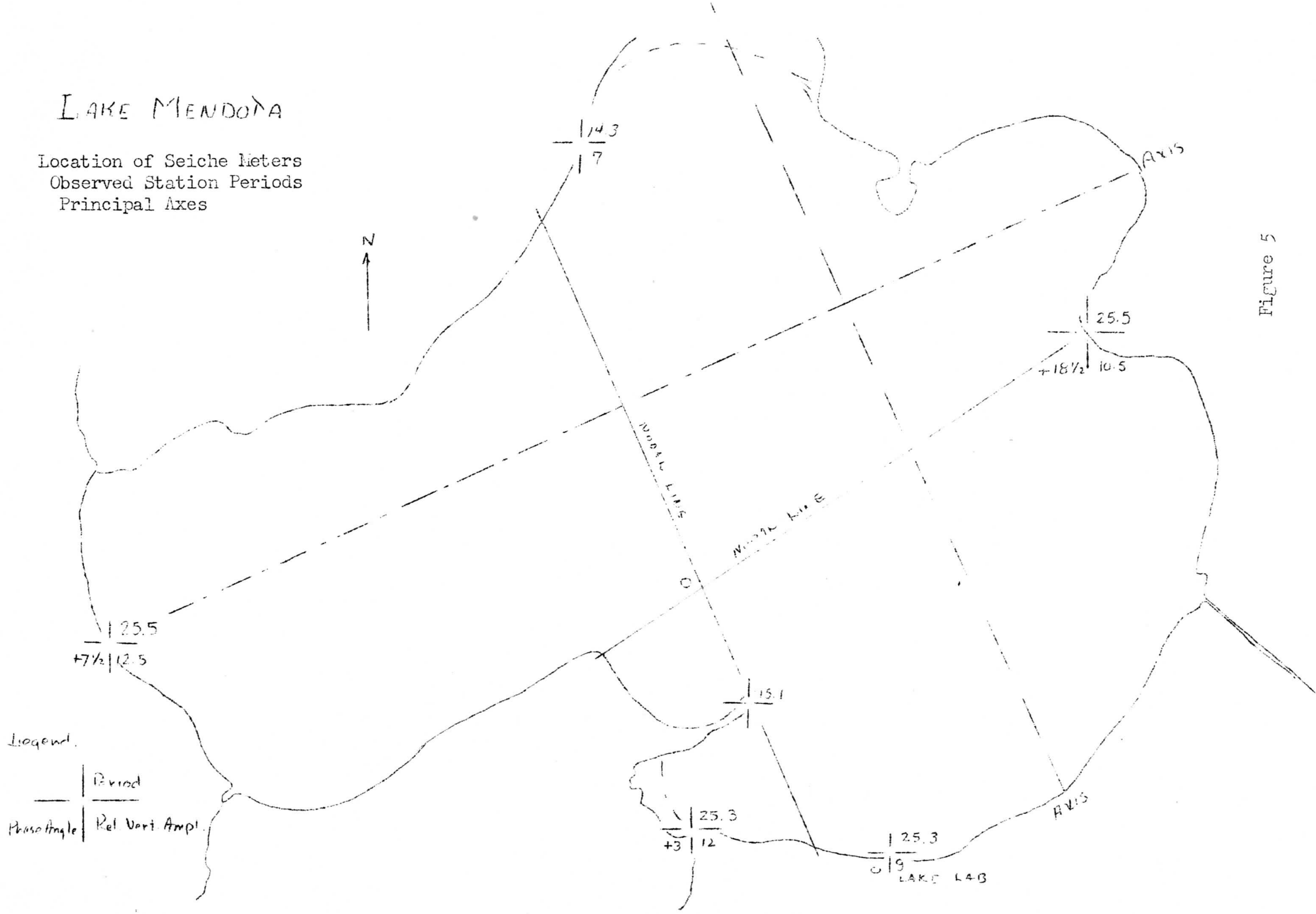

LAKE MENDOTA
Location of Seiche Meters
Observed Station Periods
Principal Axes
N
14.3
7
25.5
+18½
10.5
AXIS
25.5
+7½
12.5
15.1
25.3
+3
12
25.3
0
9
LAKE LAB
AXIS
Legend.
Period
Phase Angle
Rel. Vert. Ampl.

Figure 5

REFERENCES

1. Defant, A. 1929. "Standing Waves", _Dynamische Ozeanographie_.

2. Lamb, Sir Horace, 1932. "Hydrodynamics".

3. Sverdrup, H. U., 1926. "Dynamics of Tides on the North Siberian Shelf", _Geofysiske Publikationer_.

4. Lesser, R. M., 1951. "Some Observations of the Velocity Profile near the sea floor." _Transactions American Geophysical Union_., v 32, No. 2.