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PRELIMINARY REPORT ON INTERNAL WAVES IN LAKES

by

Reid A. Bryson

and

Robert A. Ragotzkie

Department of Meteorology

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R. A. Ragotzkie

I

Introduction

A remote reading thermometer with little time lag suspended in Lake Mendota will rarely indicate a steady value. This fact, in itself, would not be surprising if the fluctuations were nearly random and there were some evidence of turbulent motion, but even with a completely smooth surface and sufficient stratification to strongly inhibit turbulence, the temperature unrest continues. The strongly periodic nature of the record obtained on many occasions is also very striking. Whether near the surface, within the epilimnion, at the thermocline or below, these temperature oscillations of a few minutes duration and up to 2°C. amplitude are more characteristic than a state of quietude.

That the temperature unrest is associated with motions of the water is hardly to be questioned. No other possible cause comes to mind. Furthermore, periodic motions of the same general frequency have been observed: both oscillation of the horizontal current direction and of the relative vorticity.

The periodic nature of the phenomenon, its occurrence at all levels in the lake, and its apparent lack of relationship to the external surface state suggest that <u>internal waves</u> were being observed.

II

The Nature of Internal Waves

Internal waves are an inherent characteristic of any disturbed stratified medium, i.e. a medium in which buoyant forces are present to restore a disturbed parcel to its original position. They should be found, therefore, at the thermocline, in the hypolimnion, in the epilimnion when it is stratified, and in the atmosphere as well[1]. Indeed, surface waves are a special case where the medium as well as the density changes in the vertical and the density of the upper medium is negligible compared to that of the lower.

The theory of internal waves was first treated by Stokes[2]. An excellent discussion of the theory is to be found in Sverdrup[3].

The essential features of internal waves are diagrammed in Figure 1.

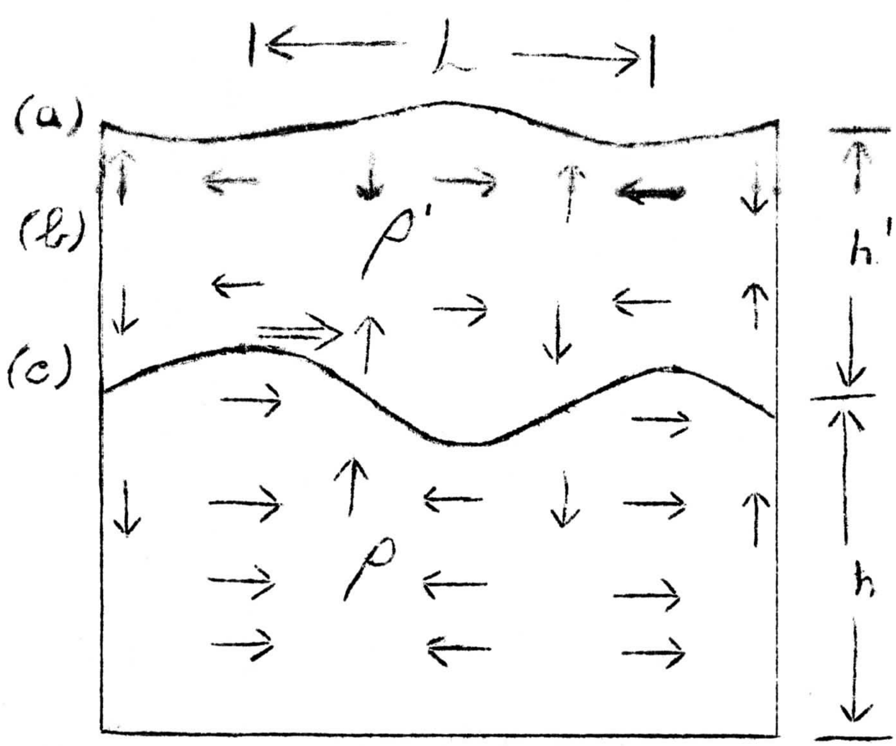


Figure 1

⟹ - direction of progress of internal wave at phase velocity c.

⟶ = horizontal and vertical components of orbital motion of water.

(a) = free surface, vertical displacement h

(b) = nodal surface, i.e. no vertical displacement

(c) = surface of discontinuity between upper layer of density ρ' and lower layer of density ρ. Vertical displacement h_{max}.

For this arrangement a quadratic expression for the phase velocity may be derived with the roots

$$c_1^2 = \frac{gL}{2\pi}; \quad c_2^2 = \frac{gL}{2\pi} \frac{\rho - \rho'}{\rho \coth kh' + \rho'} \qquad (1)$$

where g = gravity
L = wavelength
$k = \frac{2\pi}{L}$

This expression applies approximately for short waves. A further approximation for the case where h is large compared to L gives

$$c_1^2 = \frac{gL}{2\pi}; \quad c_2^2 = gh' \frac{\rho - \rho'}{\rho} \qquad (2)$$

For long internal waves the corresponding roots are

$$c_1^2 = g(h + h'); \quad c_2^2 = \frac{ghh'}{h + h'} \frac{\rho - \rho'}{\rho} \qquad (3)$$

In both cases c_1 represents the phase velocity of ordinary short and long surface waves. The phase velocity of the internal wave in (3) reduces to that in (2) if h is great compared to h'.

The maximum amplitude is at the boundary surface. At the free surface the amplitude is reduced to

$$\eta_0 = \frac{-\eta_{max}(\rho - \rho')}{\rho} \qquad (4)$$

the negative sign indicating a reversal of phase. There is then a level between these two with no vertical displacement, i.e. a nodal surface.

Long internal waves in a lake are represented primarily by the standing internal waves or "temperature seiche" well known to limnologists[4]. The author is not aware of any literature dealing with short internal waves in lakes. Indeed there is very little observational material on short interval waves at sea.

It is important to note at this point that the energy of an internal wave is given by

$$E = \tfrac{1}{2}\,\frac{\rho - \rho'}{\rho}\,g\,\eta^2 \qquad (5)$$

while that of a surface wave is given by

$$E = \tfrac{1}{2}\,\rho' g\,\eta^2 \qquad (6)$$

This means that waves on a weak thermocline may be of very large amplitude and still not represent as much energy as rather small surface waves. (See Table 1)

Table I

Heights of internal waves at the thermocline with the same energy as surface waves of given heights.

Surface wave height cm.		Approximate internal wave height				
	Epilimnion T°C.	6	10	15	20	25
	Hypolimnion T°C.	4	5	7	10	10
1		18 m.	62 cm.	35 cm.	26 cm.	19 cm.
5		90 m.	3 m.	2 m.	1.3 m.	95 cm.
10		180 m.	6 m.	3.5 m.	2.5 m.	2 m.
20		!	12 m.	7 m.	5 m.	4 m.

III

Further Theoretical Developments

In the previous paragraphs certain theoretical conclusions with regard to a surface of discontinuity were described. Fjeldstad (5) discussed the case for continuous variation in density. His conclusion was that an infinite number of internal waves could be simultaneously present each with a different number of nodes between surface and bottom, and each with its own phase angle.

Most oceanic verification of the importance of internal waves has been with reference to long waves, especially of tidal period (6, 7, 8, 9, 10, 11). Considerable material on internal waves at sea, especially with regard to their stability, was summarized by Defant(12). (See appendix)

On the other hand observations of short interval waves at sea are rare (13, 14, 15) and non-existent for lakes.

In a lake, the thermocline sometimes resembles a true surface of discontinuity, sometimes the continuous density variation of Fjeldstad (op.cit.) but more often is a finite layer of density transition between rather uniform epilimnion and hypolimnion. This case has been treated recently by Groen(16), whose results are summarized here:

"The problem of internal waves is dealt with theoretically for certain continuous density distributions of the general type shown in his fig. The relative variation of density is supposed to be small. The fluid is supposed to be incompressible and to be at rest in the non-perturbed state; the internal waves are treated as small perturbations.

"If we describe the simple harmonic, basic waves by means of a stream-function

$$\varphi(x,z,t) = \varphi(z)\exp i(\kappa x - vt),$$

it appears that $\varphi(z)$ may be found with, in general, sufficient accuracy as a solution of the equation

$$\frac{d^2}{dz^2}\left(\frac{\varphi}{\sqrt{S}}\right)+\left(\frac{g\,dS/dz}{c^2 S_0}-\kappa^2\right)\frac{\varphi}{\sqrt{S}}=0,$$

where $S(z)$ = specific volume in the equilibrium state, S_0 = mean specific volume, $c = v/\kappa$ = velocity of propagation.

"Together with the boundary conditions, this equation gives an eigen-value problem, solution of which gives relations between wave-length and period.

"When using, as an analytical representation of the density distribution, the function

$$S(z) = S_0 + \tfrac{1}{2}\Delta S \tanh(2z/b),$$

where b is a measure of the thickness of the transition layer (see fig. 1) and ΔS is the total variation of the specific volume, we may solve the above differential equation analytically by means of hypergeometric series. When the fluid is sufficiently deep on both sides of the transition layer, the relation between the wave length $L = 2\pi\lambda$ and the period $T = 2\pi\tau$ is given by

$$\frac{g \Delta S}{b S_0} \tau^2 = n(n+1)\left(\frac{2\lambda}{b}\right)^2 + (2n+1)\left(\frac{2\lambda}{b}\right) + 1,$$

where λ is positive; n has one of the values 0, 1, 2, 3, etc. (any integer) and represents the order of the mode of oscillation, which is equal to the number of zeros of the corresponding solution $\varphi_n(z)$.

"When $L \to 0$, the period approaches a minimum value, which is independent of n, viz:

$$T_{min} = 2\pi\sqrt{\frac{b S_0}{g \Delta S}} = \frac{2\pi}{\sqrt{g\,(S^{-1} dS/dz)_{max}}}.$$

"The existence of this lower limit of the period of internal waves appears to be a general feature, not restricted to the special type of density distribution assumed here.

"The theory is extended so as to include the earth's rotation. In this case the same relation as exists between τ and λ in the previous (non-rotating) case, now exists between τ and $\lambda\sqrt{1-(2w_z t)^2}$, w_z being the vertical component of the angular velocity of rotation."

Groen's treatment differs from that of earlier authors in that he considers short internal waves in a thermocline-like density distribution. One further result of interest at this point is Groen's graph (Fig. 2) of τ versus η for various wave orders (essentially the order gives the number of wave maxima in a vertical section). Here $\tau = g\delta T^2/\ell$, $\eta = 2\lambda/\ell$, where the meanings of the symbols are as given above and $\delta = \Delta S/S_0$. Only the physical properties of the thermocline are needed to use this graph.

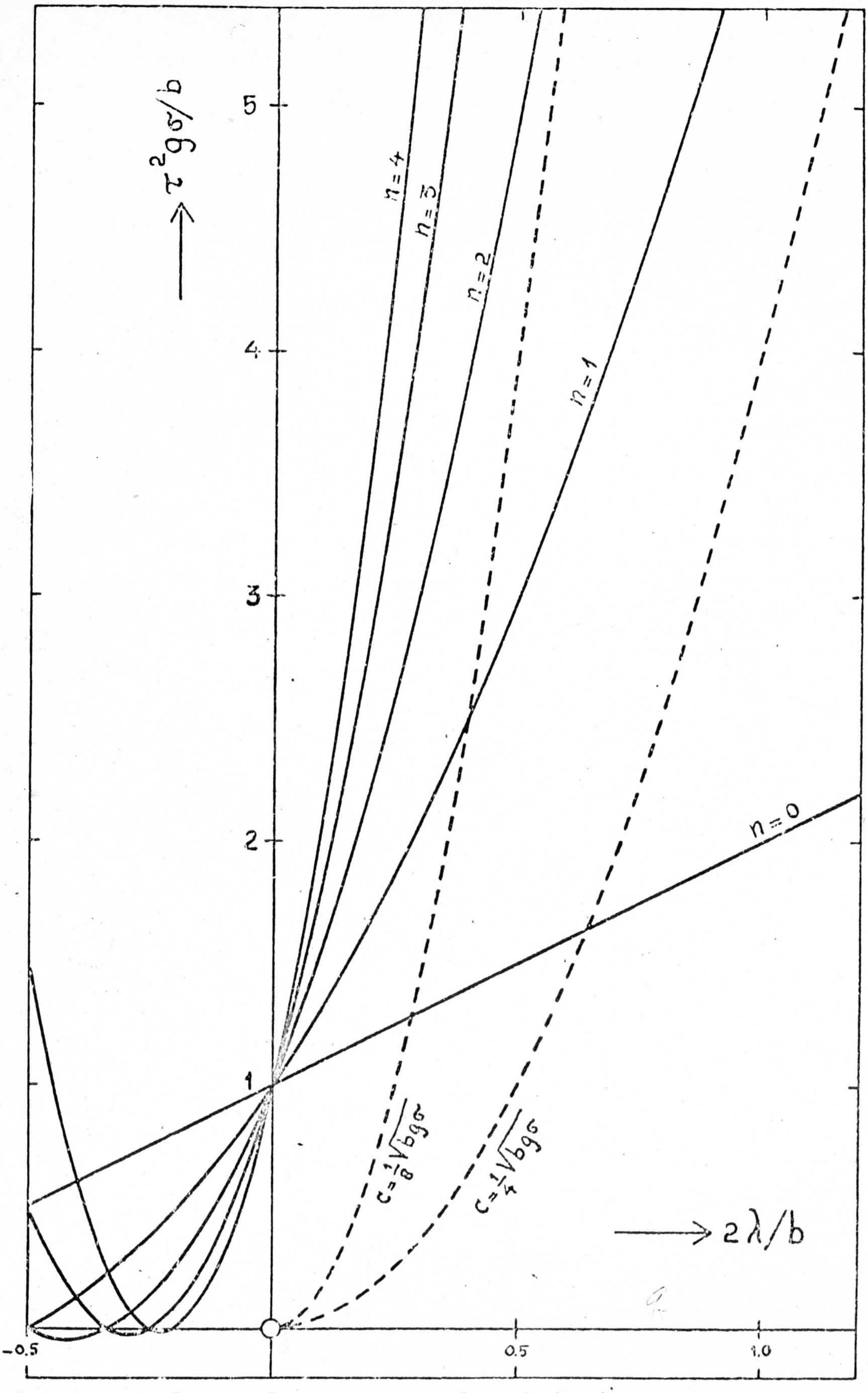


Fig. 2. Relations between wave-length ($2\pi\lambda$) and period ($2\pi\tau$) of internal waves.

Laboratory Studies

Observations of internal waves in lakes or in the sea are by necessity of an indirect nature. However, it is possible to make direct visual observations of internal waves artificially produced in a laboratory tank model. Mortimer (17) made extensive use of a model to study the water movements in a stratified lake. He was primarily concerned with the horizontal currents associated with the temperature seiche. Our use of a tank model has been chiefly to demonstrate and measure the period of an internal standing wave.

A glass aquarium 75 centimeters by 25 centimeters and 25 centimeters deep was filled with water. The system was thermally stratified by heating from above with two 100-watt light bulbs and cooled by cold water flowing through a copper tube running around the periphery 10 centimeters below the surface. Immediately above the cooling tube maximum thermal stability occurred. This layer will be termed the thermocline. Methylene blue dye was carefully injected into the warm epilimnion so that it "floated" on the cool hypolimnion, thereby sharply delineating the thermocline. Stress was applied to the water surface by means of an electric fan. A tilt of the thermocline resulted representing the storage of potential energy in the system. When the fan was suddenly stopped, this potential energy expressed itself in the form of an internal wave at the thermocline which was readily observable by means of the blue dye.

If it is assumed that the oscillations of the thermocline are internal standing waves, then the expression developed by Defant (18) for the period,

$$T = 2l\sqrt{\left(\frac{\rho}{\rho-\rho'}\right)\left(\frac{h+h'}{ghh'}\right)}$$

may be applied. Table II shows the periods of internal waves produced under two sets of temperature conditions.

TABLE II

Temperature of upper layer, h'	Temperature of lower layer, h	Observed period (sec.)	Theoretical period (sec.)
18°C.	13°C	82	70
16°C.	13°C	94	95

h' = 10 centimeters

h = 15 centimeters

The observed periods are in reasonable agreement with those calculated by use of Defant's formula.

V

<u>Field Studies</u>

Evidence for short internal waves in Lake Mendota is of several sorts. The first indications were obtained by serial observations at a fixed depth and fixed location.

Figure 3 shows this variation of temperature at 7.2 m depth on July 12, 1949. The observations were made just off Picnic Point. In the space of five minutes the temperature varied more than 1.5°C. A rough periodicity of about 3 minutes is rather clear, as is the fact that the record is not a single sinusoidal variation. Only occasionally in the many such observations made during 1949-1953 have simple patterns been observed. An example is given in Figure 4. Complex patterns such as those in Figure 3 and Figure 5 are far more common.

Late in 1952 and during the summer of 1953 a tripartite method for measuring internal waves was adopted (14). By this method the temperature fluctuations at three fixed points in a horizontal plane are recorded. Fluctuations associated with progressive internal waves will appear at each of the three points at different times. By analyzing the phase differences of the fluctuations, it is possible to determine not only the periods of the waves but also their direction of progress, their speed, and hence their length.

The rig used to obtain most of the measurements reported here is shown diagrammatically in Figure 6. The temperature elements were 10-junction thermopiles. One set of couples was exposed directly to the water while the reference junctions were imbedded in beeswax to increase their time constant beyond the expected period of the waves. Thus the temperature anomaly due to the internal wave was measured as

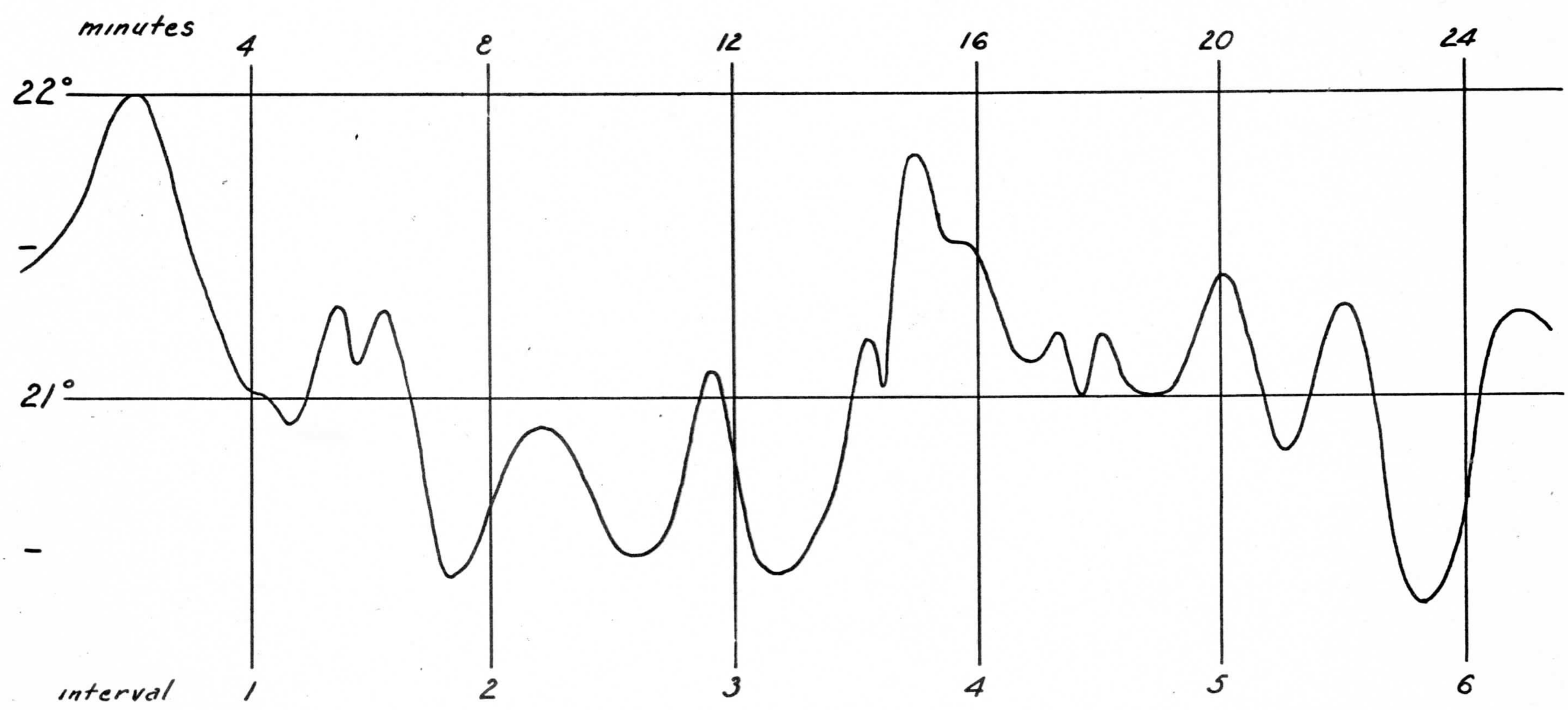


Figure 3. Temperatures observed at 7.23 m depth off Picnic Point, 12 July 1949. Temperature was observed every 15 seconds with a Whitney resistance thermometer.

Figure 4. Temperature variation record illustrating the simple periodic nature occasionally observed.

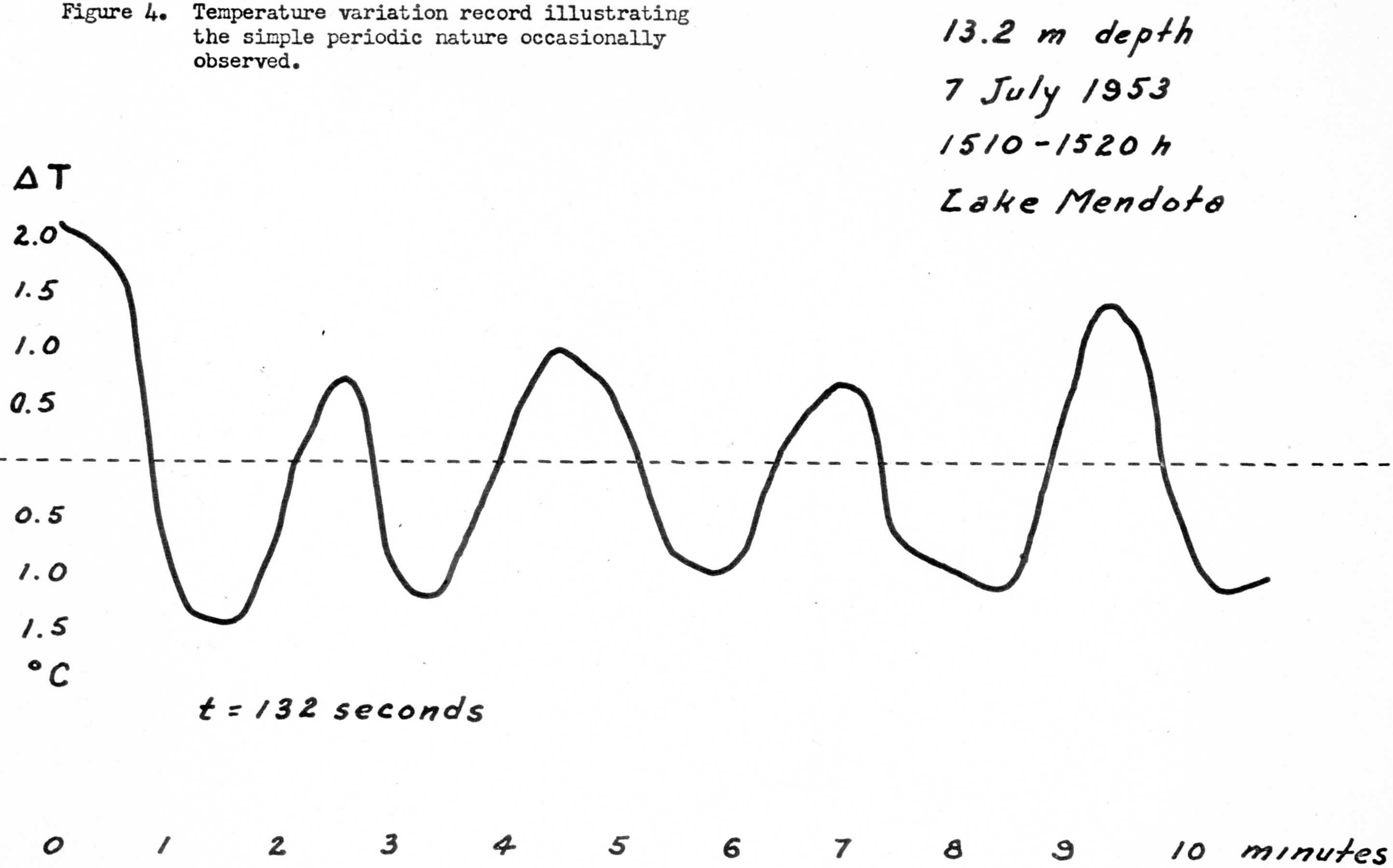
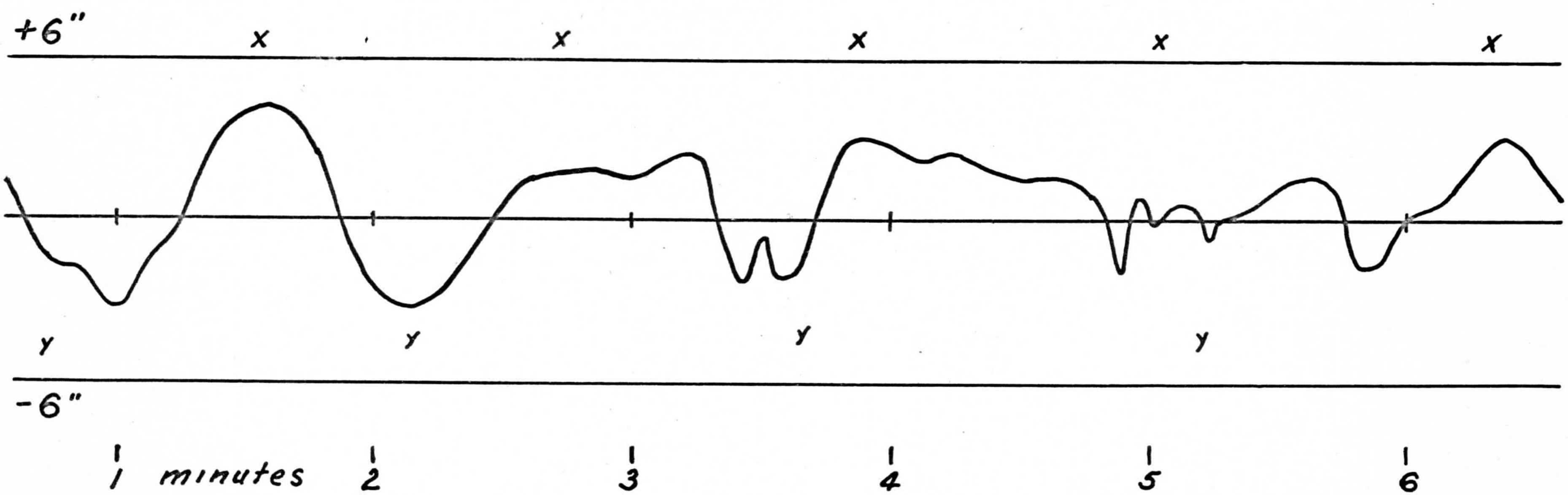


Figure 5. Record of temperature variation at one point (Mouth of University Bay) 15 July 1952. It is evident that the variation is roughly periodic, but consisting of several components. Depth 8 meters.



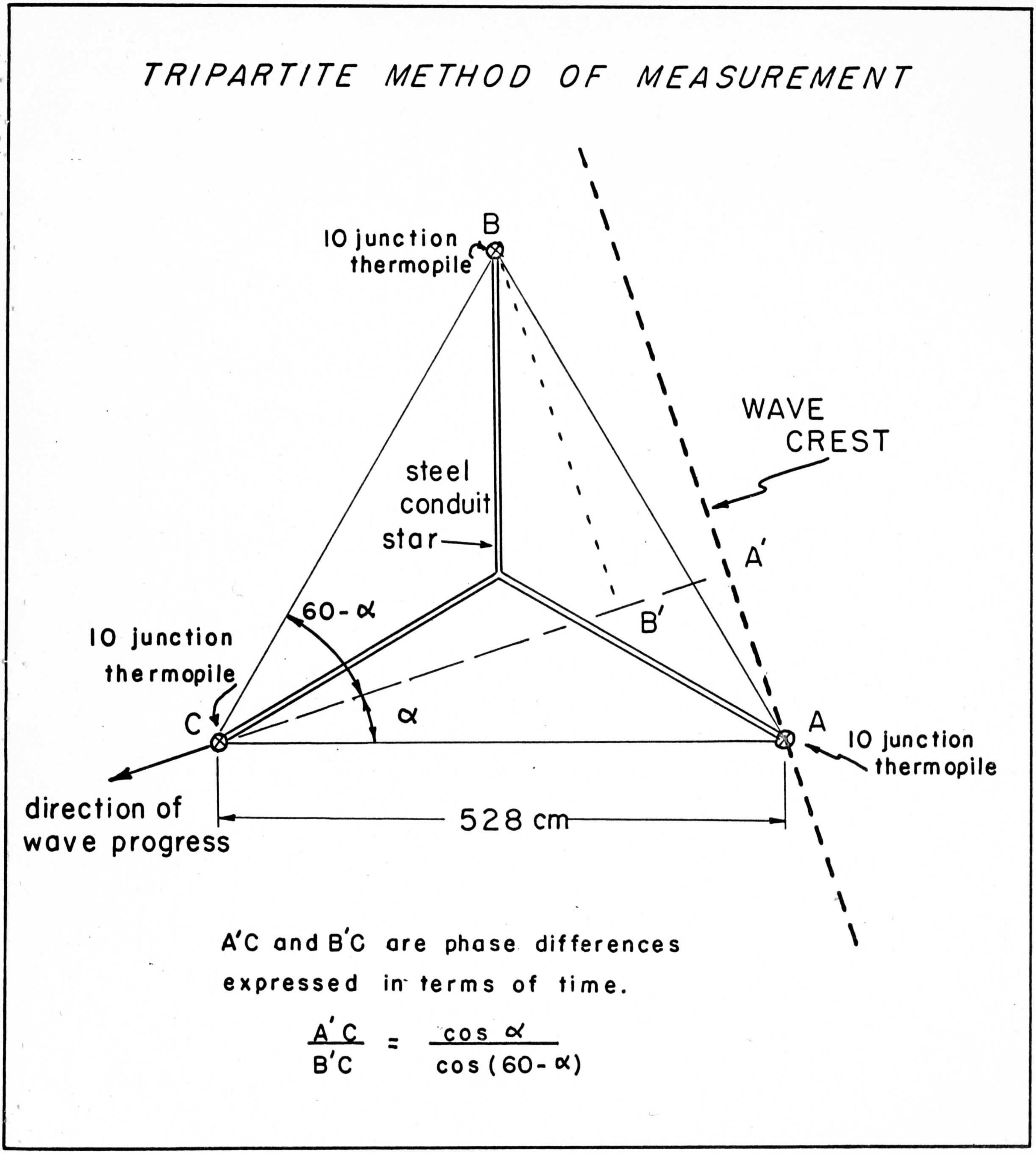


Figure 6. Layout of the thermopile rig used to determine the direction and velocity of progressive internal waves. From the known orientation of the rig and the computed angle of the direction of progress was derived. The velocity could then be determined from the time difference A'C.

a departure from the mean temperature at the depth of measurement. All thermopiles used were carefully checked to be sure the time constant was the same in each case. Thermopiles were mounted on the vertices of a triangle with 528 cm. sides (See Figure 6). The entire rig was suspended from submerged buoys so that it could not tilt, rotate, or move up and down.

The thermocouple emf's were amplified by a portable chopper-type amplifier (designed by Prof. V. E. Suomi and built by Mr. C. R. Stearns) and recorded on an Esterline-Angus 0-1 ma portable recorder.

The input was switched from one thermopile to the next each four to five seconds. For purposes of analysis the records so obtained were re-plotted on a much compressed time scale and an expanded temperature scale. All three thermopile records were plotted on the same time scale (Figure 8). The phase differences in terms of time among the three curves were determined by tracing one of the curves on a transparent overlay and shifting it along the time scale until it best matched each of the other two curves. This was repeated by tracing all three curves and comparing the results. Where poor agreement was found, the record was discarded.

As the speed of the waves may be regarded as essentially constant, the phase differences in terms of time may be used to calculate the wave direction, speed and hence the wavelength. This is done geometrically by using the diagram in Figure 6. The angle α is found by graphical solution of the equation

$$\frac{A'C}{B'C} = \frac{\cos\alpha}{\cos(60-\alpha)}.$$

Then, since the orientation of the frame is known, the wave direction can be determined. The length of A'C divided by the time interval of A'C gives the speed of the wave, C, and the wavelength, L, is obtained by the relationship CT = L, where T is the wave period.

Lake Mendota

In June 1953 a station for measuring progressive internal waves by the tripartite method was established 2400 feet north of the Lake Laboratory in 55 feet of water. This will henceforth be referred to as the "55 ft. station". A total of eleven tripartite records and six single thermopile records were obtained at this station. Of these, seven of the tripartite records could be analyzed for period, direction of travel, speed, and length, and six of the remaining records of both types were analyzed for period only. One other tripartite measurement was made on 9 October 1952 by use of three Whitney Electric Thermometers which were suspended from two securely anchored boats. Temperatures were read and recorded at five-second intervals. This measurement was made at a point about one-half mile south-southwest of Governor's Island.

Typical tripartite records are shown in Figure 7 and 8. It will be noticed that the amplitudes are most conveniently expressed in terms of °C.; however, this has been approximately converted to linear units from the vertical temperature distribution measured sometime during the

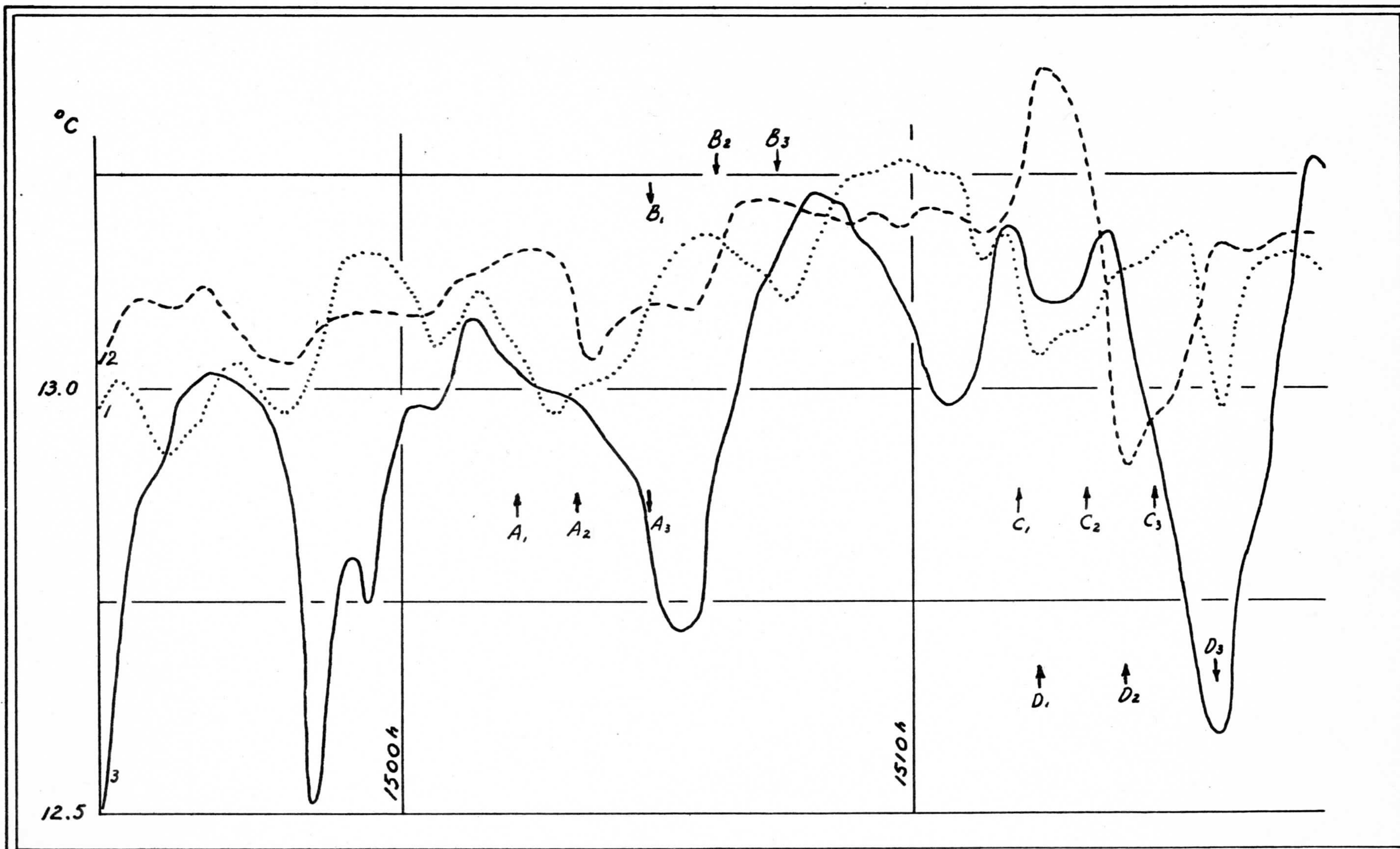
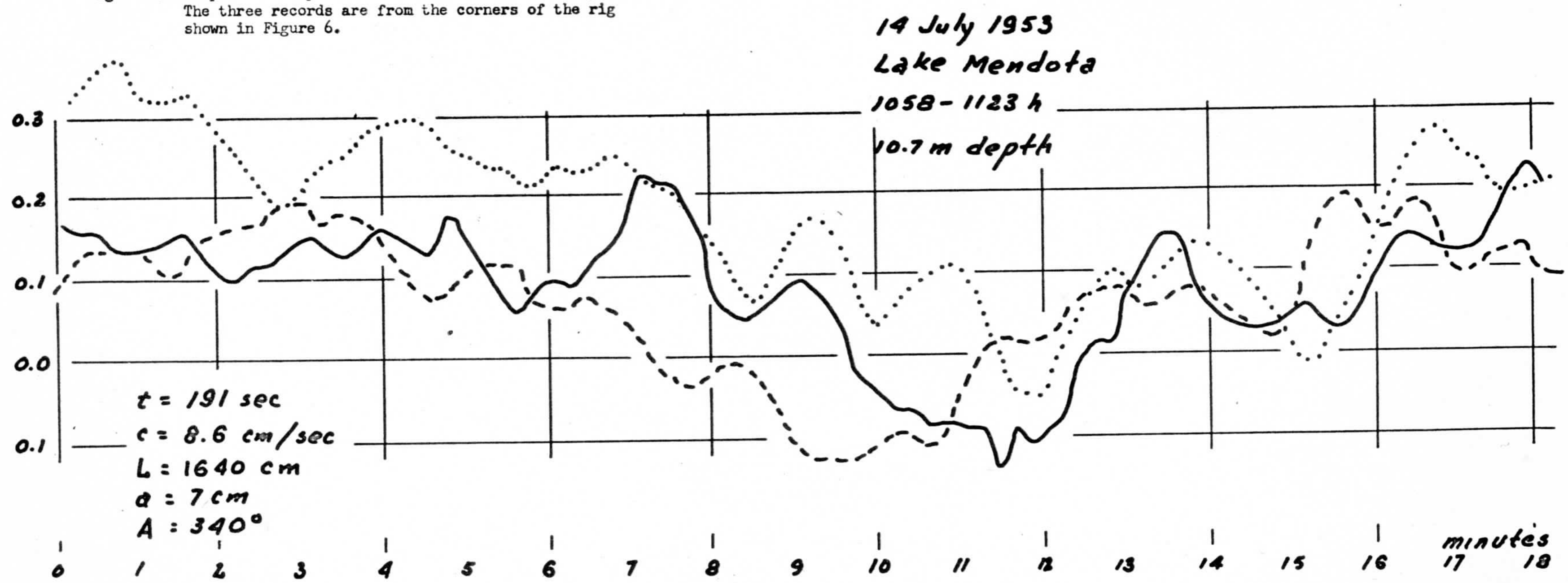


Figure 7. Tripartite temperature variation record using Whitney direct reading resistance thermometers. Near Governors Island, Lake Mendota, 9 October 1952. Depth 16 meters. Observations made every five seconds.

Figure 8. Tripartite temperature variation record 14 July 1953. The three records are from the corners of the rig shown in Figure 6.



record interval. The results of all analyzed records are shown in Table III, Most of the measurements are for the 10.7 meter level which lay within the thermocline during the period July 9-16, None of the observed values for the period, T, are less than the theoretical minimum period, $T_{min.}$, calculated by the theory of Groen.

Comparison of the directions of the wind and internal waves (Table IV) shows that, in general, the internal waves in the region of the thermocline move to the right of the wind. There seems to be no pronounced tendency for the waves to move upwind. Therefore the origin of these waves is not at the downwind end of the lake. This is surprising since the concentration of kinetic energy in the form of surface waves and turbulence is probably highest at the downwind end of the lake. Where and how progressive internal waves originate remains unknown at this time.

Trout Lake

Trout Lake, which is in Vilas County, is composed of two basins connected by a channel. This channel is approximately 425 ft. wide and 720 ft. long and has maximum depth of slightly more than 15 meters. Several measurements of internal waves were made at the south end of the channel. Because of the configuration of the channel, it seemed reasonable to assume that if internal waves were present, they would probably move through the channel parallel to it. Therefore a bipartite system with two thermopiles one meter apart horizontally and oriented parallel to the channel was used. A typical record obtained is shown in Figure 11.

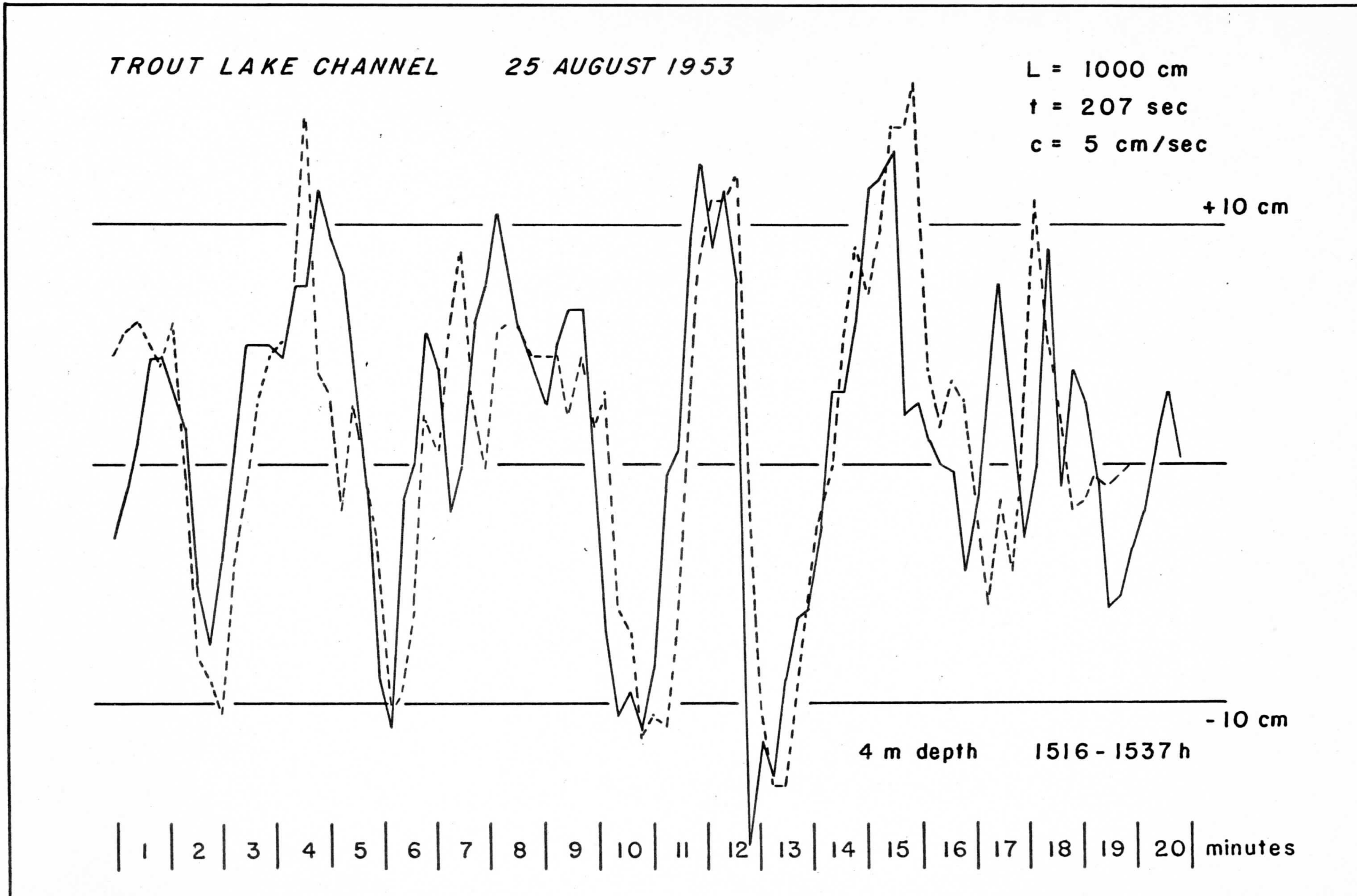


Figure 11. Variation of temperature at two points one meter apart horizontally. Solid line southerly point, dashed line northerly point.

Date	Time	Location	Depth(m)	Observed: Major Period, T, (sec)	Length, L (cm)	Speed, C (cm/sec)	Amplitude, a (cm)	Direction of progress	Theoretical minimum period, $T_{min.}$ (sec.)
521009	1500-1520	Gov. Is.	16	330	1000	3	~100	20°	162
530629	1400	55' Sta.	8	187	1619	8.6	~ 10	44°	103
530707	1500-1530	"	13.1	132			10		57
530707	1558-1608	"	12	90			~ 1		
530709	1400-1411	"	10.7	233	744	2.42	< 1	287°	72
530709	1537-1550	"	10.7	270			~ 1		
530709	1552-1601	"	10.7	180			0.5		56
530710	1527-1547	"	10.7	319	1746	5.4	5-10	205°	82
530711	1124-1147	"	10.7	200	1540	7.7	8	190°	98
530714	1058-1123	"	10.7	150	1105	6.7	7	303°	87
530715	0955-1010	"	10.7	163	388	2.38	6	119°	88
530715	1016-1044	"	10.7	474	3010	6.35	6	325°	101
530716	0846-0902	"	10.7	109			~ 10		83
530716	0902-0914	"	10.7	109			~ 15		84

Table III. Summary of Lake Mendota Internal Wave Measurements.

Table IV.. Wind Conditions and Internal Wave Directions and Amplitudes

	Wind		Internal Waves		
Date	Dir.	Aver. Vel.	Dir. + 180°*	Amplitude(cm)	Angle with wind positive to right
521009	45°	9.7	200°	100	+ 155°
530629	202°	6.5	222°	10	+ 20°
530709	315°	6.1	107°	< 1	+ 152°
530710	338°	5.6	25°	7	+ 47°
530711	248°	7.1	10°	8	+ 122°
530714	112°	5.3	123°	7	+ 11°
530715	68°	6.2	299°	6	- 129°**
			145°	6	+ 77°

*180° added to wave direction to conform to convention of describing wind direction by direction from which it comes

**short period waves

On August 25, 1953 three separate wave records were obtained at this station. A summary of the results is given below:

Time of record	Depth (m.)	Period, T, (sec)	Speed, c (cm./sec)	Wavelength, L, (cm)	Direction of progress of waves	Theoretical minimum period by Groen's theory (sec.)
1516-1537	4	207	5.00	1035	45°	259
1558-1619	4	141	3.85	544	45°	185
*1637-1651	4.5) 5.5)	143				172

*Thermopiles oriented vertically to measure "lean" of waves.

The wind was from the south at about 15 miles per hour. In the records where the wave direction could be measured the waves were travelling downwind. In all three cases the observed periods were less than the theoretical minimum periods calculated according to Groen's theory. These discrepancies should not be taken as a refutation of the theory without first considering all possible sources of observational error.

In order to calculate the theoretical minimum period, $T_{min.}$, it is necessary to know the vertical distribution of density. In freshwater lakes this is obtained directly from the temperature distribution. When the temperature gradient is not steep, a small error in temperature measure- can result in fairly large errors in the theoretical $T_{min.}$ Similarly an error in the measurement of the depth at which the temperatures were taken can leas to errors in temperature and hence $T_{min.}$ The errors in temperature or depth measurement which would explain the discrepancy between the observed and theoretical $T_{min.}$ were computed and are given below.

Table V Analysis of Errors

Time of Record	Depths used for theoretical $T_{min.}$ (m.)	Error Temperature (°C.)	Error Depth (cm.)
1516-1536	3.0	-0.07	-10
	4.0	0.07	10
1558-1619	3.5	-0.13	-12
	4.5	0.35	12
1637-1651	4.0	-0.13	-8.5
	5.0	0.29	8.5

It is possible that the discrepancies were due to a combination of errors. As the scale of the Whitney Thermistor-type electric thermometer is graduated in 0.1° C., an error of ±0.07°C. is not excessive. Nor is an error of ±10 cm. at the 4-meter level unreasonable when the temperature sounding is being taken from a small boat in a 15-mile-per-hour wind.

It should be noted also that although the theoretical $T_{min.}$ is calculated from a single temperature sounding taken at about the midpoint of the period of record of the internal waves, the temperature distribution is constantly changing. By calculating $T_{min.}$ for all the soundings and plotting $T_{min.}$ as a function of time, the theoretical $T_{min.}$ can be clearly compared with the observed $t_{min.}$ (Figure 12). The graph shows that the theoretical values are fluctuating over a range of 172 to 259 seconds while the observed values varied from 141 to 207 seconds. However, the general trend of theoretical and observed values is similar with the observed values being consistently lower. It is not clear why the errors should have always been in the direction of lower than

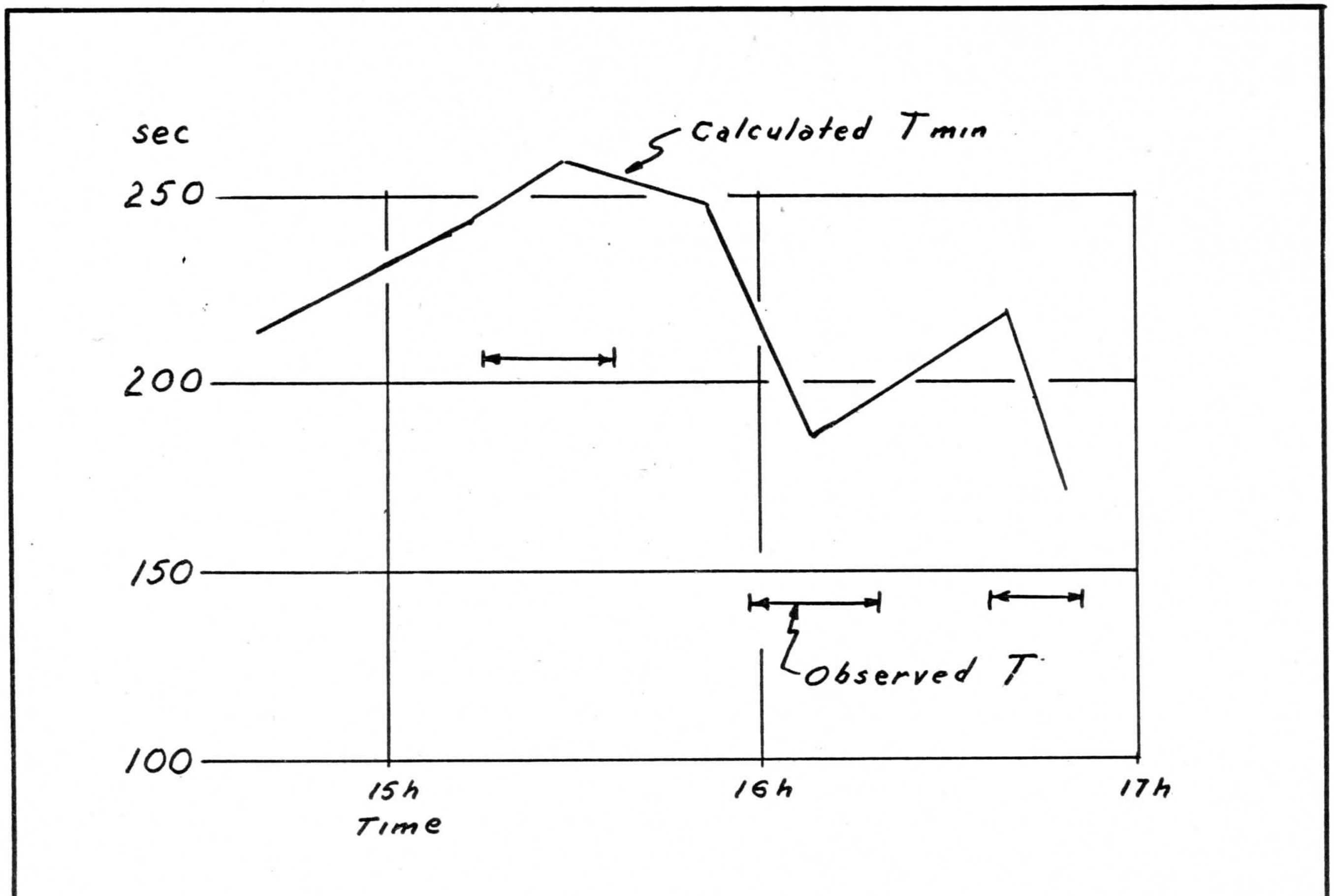


Figure 12. Comparison of minimum period calculated by the formula of Groen with the actual period observed in the Trout Lake Channel. The horizontal arrows indicate the period during which internal waves were observed. 25 August 1953. Depth 4 m.

theoretical $t_{min.}$ values. Nevertheless the analysis of errors illustrates the care required to obtain precise measurements of progressive internal waves.

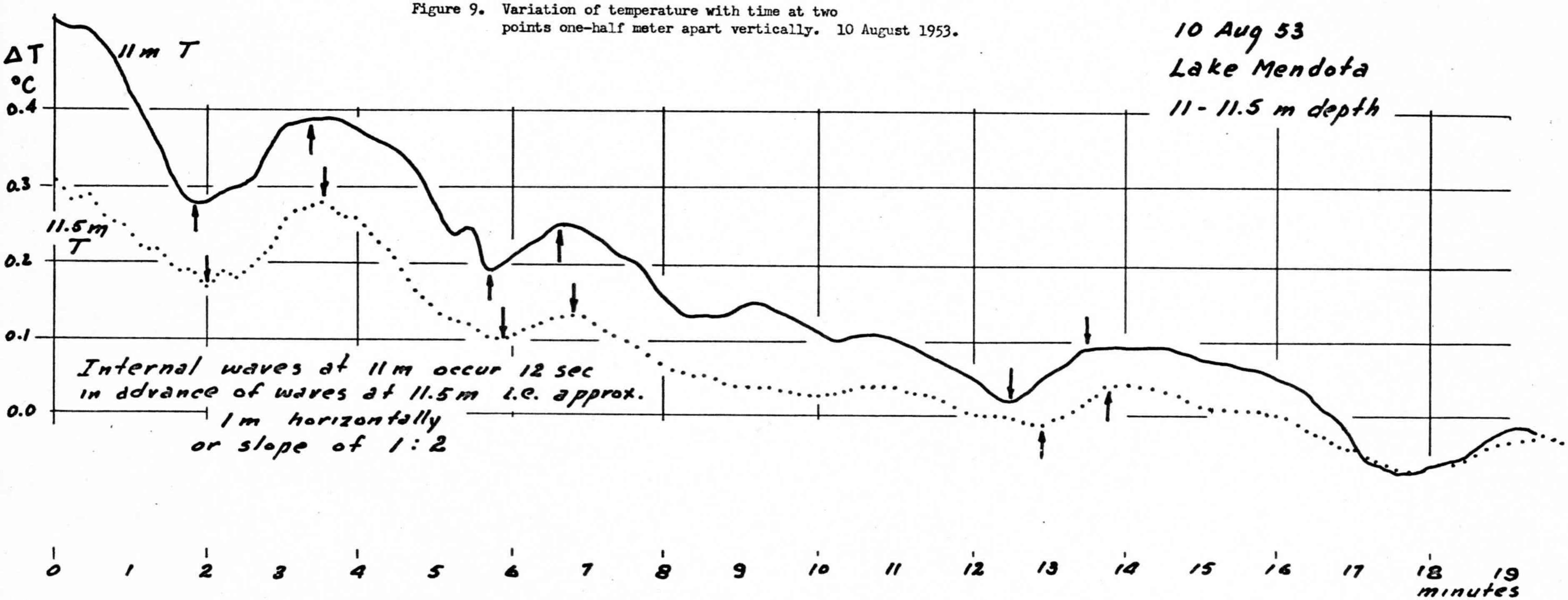
VI.

Breaking of internal waves

The thermocline represents the region of maximum stability and therefore the region where mixing through normal turbulence should be suppressed. Wherever density stratification exists, internal waves may occur. Indeed, our measurements have never failed to demonstrate the presence of internal waves when such stratification was present. This suggests that the internal waves may provide the mechanism for the transfer of heat and other properties through the thermocline.

It is generally assumed that one of the symptoms of breaking in waves is increasing asymmetry. While we were not able to observe <u>increasing</u> asymmetry, examination of the examples presented in the previous paragraphs shows that truly symmetrical waves are rare. If there is a phase shift in the vertical, there must be either asymmetry of the wave or of the associated stability field. One method for measuring this phase shift uses a bipartite thermopile rig in a vertical position. The phase difference between the two levels then appears directly as a time phase shift in the two records. An example from Lake Mendota is shown in Figure 9. The upper curve led the lower curve by about 12 seconds. As the approximate speed for waves of the period observed is 8.5 cm./sec., the waves were leaning forward with a slope of 1:2. Another record from the channel at Trout Lake showed a lean of 1:4.5. In this

Figure 9. Variation of temperature with time at two points one-half meter apart vertically. 10 August 1953.



case the phase difference of two temperature curves from depths of 3.5 and 4.5 meters was approximately 100 seconds, and the speed of the waves from measurements at 4 meters was about 4.5 cm./sec.

A short term temperature inversion (at temperatures above 4°C.) in the presence of internal waves demonstrates the breaking of the wave. Two instances of this phenomenon were observed. The first was observed by a series of bathythermograph casts at one location (Figure 10). The presence of internal waves is clearly indicated by the periodic fluctuations of the isotherms, especially the 63, 62.5, and 62° F. isotherms. One case showed that the 63°F. isotherm occurred three times in the same water column with colder water above warmer water. To be sure the temperature gradient was small where the inversion occurred, but the wave was in the process of breaking and the temperature gradient was gradually being destroyed by mixing.

The second case of a temperature inversion in the presence of internal waves was observed in the course of making direct measurements of vertical temperature gradients. A five-couple thermopile was connected to measure the temperature gradient through a 10 cm. stratum. The emf. developed was recorded by the same system described for the tripartite thermopile rig. Below is a record obtained at the 3.95 to 4.05 meter level in the channel at Trout Lake.

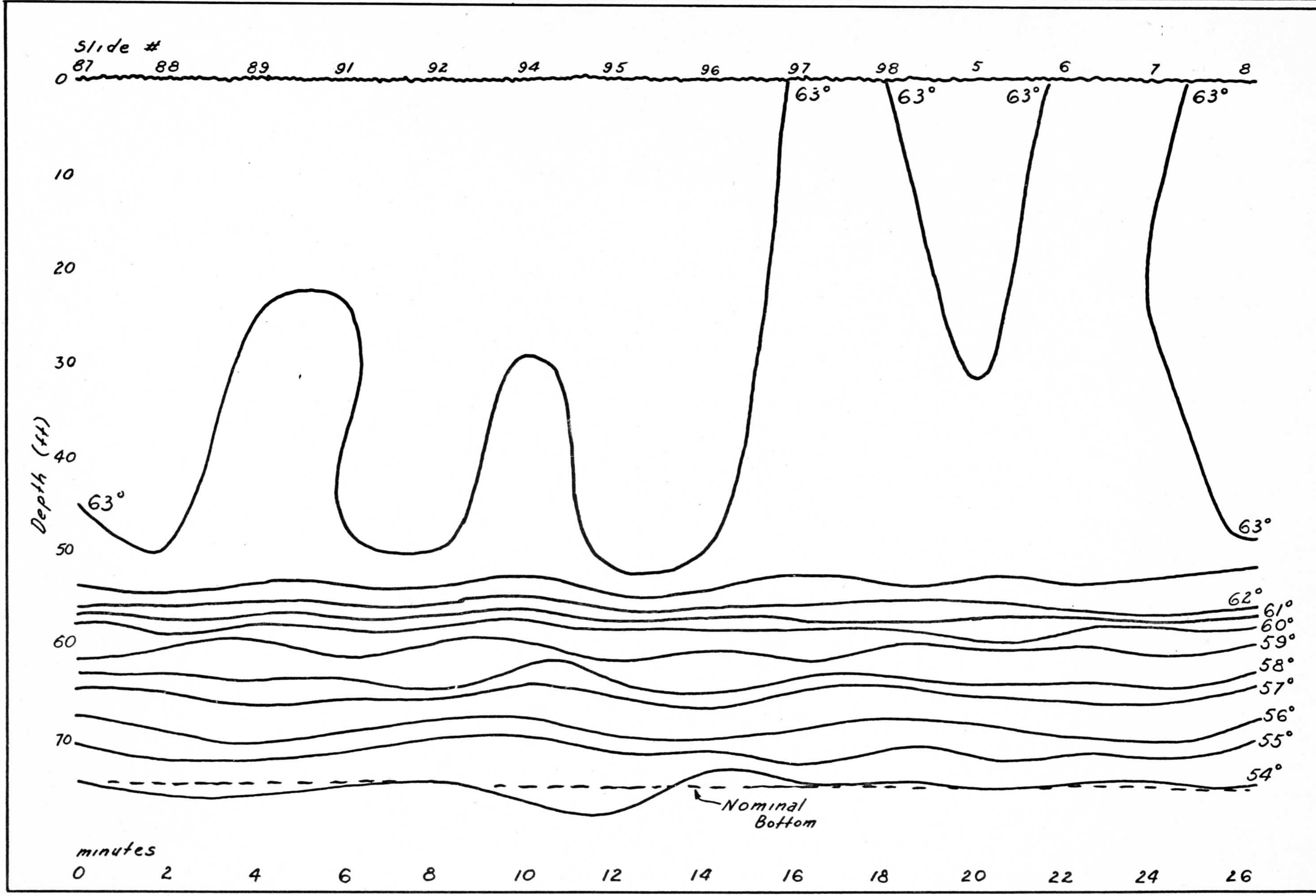
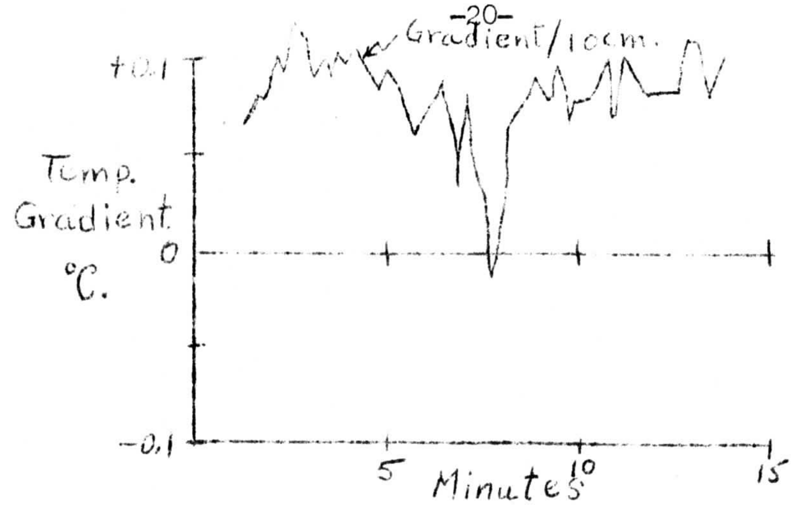


Figure 10. Variation of temperature with time and depth in Lake Mendota on 7 October 1950. The actual depth of the isotherms was read from the bathythermograph slides.



Measurements immediately before and after this record clearly demonstrated internal waves at 3.5 to 4.5 meters (Trout Lake section). The temperature gradient per 10 cm. fluctuated around 0.1°C. except for a short period when the gradient dropped to zero and actually reversed for an instant. The record was continuous so that there could be no doubt of the gradient reversal. The lean of the waves at this level was found to be 1:45 less than one hour later. It is very probable that the negative gradient represented a breaking internal wave with the associated mixing and destruction of thermal gradient.

VII

Standing Internal Waves

Commonly referred to as the temperature seiche, standing internal waves are frequently present in Lake Mendota. Measurements of these waves were obtained by two methods: (1) repeated temperature soundings at one location over a period of several days, and (2) measurement of the surface reflection of internal oscillations by the water level recorder used to study the surface seiche.

In one attempt to measure the standing internal waves in Lake Mendota temperature soundings were obtained at 1-1/2-hour intervals for nearly four days with the exception of the hours from midnight to 8 a.m. Even with these lapses in the record it was possible to plot the fluctuations of the isotherms and to detect some periodicity in their movements. The thermocline did not rise and fall as a unit; rather, each isotherm behaved slightly different from all the rest indicating the complexity of the temperature seiche. The most pronounced fluctuations occurred in the isotherms nearest the bottom and the surface. There was an indication of a nodal surface near the middle of the water column. It has been shown by Proudman (19) that in a stratified medium which also possesses velocity shear there should be both a vertical and horizontal wavelength, i.e. nodes such as the one described above.

A standing internal wave will be reflected by a small oscillation of the surface of the lake. These surface reflections are discernible on the seiche records obtained with the water level recorder. (Lakes Investigations Report No. 5, Department of Meteorology, University of Wisconsin) The complexity of the internal seiche is masked by this method since only the net changes in density distribution appear as a surface reflection. From these records can be obtained the period (T) and the amplitude of the surface reflection (η_0). Defant (18) has shown that the period of a standing internal wave is as follows:

$$T = 2l\sqrt{\left(\frac{\rho}{\rho-\rho'}\right)\left(\frac{h+h'}{g h h'}\right)}$$

Calculation of the period (T) is somewhat unprecise because the values of ρ and ρ' for the layers h and h' must be approximated from one or two temperature soundings, which are subject to changes due to the standing internal wave. However, typical summer conditions in Lake Mendota give values for T which vary between 10 and 12 hours. The observed values for the period of the standing internal wave are given in Table VI. The amplitude of the surface reflection was shown by Sverdrup (3) to be as follows:

$$\eta_0 = \frac{-z(\rho - \rho')}{\rho}$$

where η_0 is the amplitude of the surface reflection and $-z$ is the amplitude of the standing internal wave. Table VI also shows the observed values of ρ and the calculated values of $-z$.

Table VI

Summary of Standing Internal Wave Data Obtained by the Surface Reflection Records

Date	Observed T (hours)	Observed η_0 (cm.)	Calculated z (cm.)
510720		1.5	1000
510720	10.8	.15	100
530628	10.8	.05	45
530629	10.0	.15	135
530630	10.5	.10	90
530703	10.2	.15	83
530704	10.5	.15	83

Under the influence of a prolonged strong wind the lake surface and the thermocline are forcibly displaced. The first case in Table VI occurred under these conditions. All the other amplitudes were observed when the lake surface and thermocline were oscillating freely. The ordinary amplitude appears to be about one meter with occasional very large forced fluctuations.

VIII

The Role of Internal Waves

Planktonic debris in its slow settling from the upper epilimnion to the bottom of the lake slows up at the thermocline. Oxygen consumption should, therefore, increase at the thermocline, especially in the warmer period. By analogy with the sea there should be an oxygen minimum at that depth. The fact that there is not suggests that the rate of replenishment from above is sufficient to prevent the formation of such a minimum. Is this downward diffusion due to turbulence in the ordinary sense or is it due to breaking internal waves? Our evidence suggests that it is the latter.

Similarly, as the summer progresses the portion of the lake occupied by the epilimnion increases at the expense of the hypolimnion. This means a downward flux of heat through the thermocline. This could be accomplished either by the breaking of internal waves or by exchange with the portion of the bottom which is alternately in contact with the epilimnion and the hypolimnion as a result of the internal seiche. This question deserves further study.

The same alternating exposure of the bottom allows replenishment of oxygen for the bottom organisms such as the chironomid larvae. This alternation affects about 10 per cent of the total Lake Mendota bottom in mid-summer and may be important in the prevention of nutrient release from bottom muds by maintaining higher oxidation reduction potentials than would occur in the completely anaerobic bottom (20). The solubility of iron is especially sensitive to the redox potential.

Associated with both the surface and internal seiche, and the progressive internal waves there are alternating or periodic currents. These must play a role in the winnowing of sediment and production of microturbulence about the fixed aquatic plants, and other benthos, which in turn is necessary to free access to a larger supply of carbon dioxide, oxygen, etc. than could be reached by molecular diffusion.

IX

Conclusions

1. Both standing and progressive internal waves exist in the lakes which were studied.

2. In general, progressive internal waves have periods exceeding Green's calculated minimum.

3. Both progressive and standing internal waves are generally complex, variable in amplitude, direction, and period, and frequently polynodal both vertically and horizontally.

4. Internal waves occur with sufficient frequency and amplitude that the ecological implications should not be ignored.

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APPENDIX

CONCERNING INTERNAL TIDAL WAVES AND THEIR STABILITY CRITERIA by A. Defant

translated by Norman F. Islitzer

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That internal boundary waves with periods of the most varied lengths are present in the open oceans is well established. They were first determined by Fridtjof Nansen (1) on the North Pole expedition. Similar observations were made later by him and Helland Hansen (2) in the Norwegian Sea, and were thoroughly discussed in the well-known work "The Norwegian Sea". Otto Petterssen (3) had shown, especially in the Danish and Swedish fiords, large vertical displacements of the internal boundary surfaces, and first called attention to the fact that the tidal force stands in close connection with its effects; that is, that they are of the tidal period nature. Observations were also made on the Michael-Sars Expedition in the North Atlantic Ocean, and on the occasion of the long series of simultaneous observations in the Faroer-Shetland-Ridge, internal waves with tidal characteristics could be observed for a long period (4).

The most comprehensive and thorough observations concerning internal waves on layers of density in the free ocean has probably been collected on the Anchor stations of the "Meteor" Expedition of 1925-27 (5). Further measurements were collected on the "Snellius" Expedition (6), and by L. Lek (6). A partial examination was also carried out with the exhaustive observations which W. Ekman and B. Helland Hansen (7) obtained with their current measurements in the North Atlantic Ocean. Finally, with the longest previous anchor station, that of the investigation ship "Altair" in the current of the Gulf Stream north of the Azores on the occasion of the international Gulf Stream Investigation, 1938 (9), it has become clear without doubt that internal tidal waves are always coupled with the course of the tidal phenomena of free oceans, and must be regarded as continuously occurring phenomena accompanying periodic water movements produced by the flood causes.

In the following sections, after short discussions concerning the character of internal waves (particularly those with tidal character), the points which concern the stability of such internal tidal waves will be investigated, and with several selected examples it will be shown how internal tidal surf is possible under certain conditions of oceanic structure and current characteristics. This internal surf phenomenon is of interest in the question of the mixing of water masses, especially upon the continental shelf or coastlines, and deserves to be followed closer.

On the boundary surface between a homogeneous lower layer of dense water and a homogeneous upper layer of less dense water, internal waves are possible. The current paths and orbits of water particles in the case of such progressive waves are given in illustration 1. The propagation velocity, c, for waves of definite wave length, λ, along the boundary surface, is decreased by the presence of the upper layer relative to the speed on the free surface, c_0, in the relationship $\sqrt{\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}$ that is, $c = c_0 \sqrt{\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}$ in which case ρ_1, the density of the lower layer and ρ_2, is the density of the upper layer. The decrease is considerable. For example, for the cases which occur in the ocean, it is for density of $\rho_2 = 1.027$ $\rho_1 = 1.028$ (density difference is 10^{-3} and c is equal to $\frac{c_0}{45.3}$; that is, the wave velocity of the internal waves is approximately 45 times smaller than the waves of the same wave length on the free surface. With these internal waves the vertical displacement of the water masses at the boundary surface is maximum. Upwards and toward the bottom the amplitude of this displacement rapidly decreases. With a not particularly great thickness of the upper layer the amplitude of the internal waves on the free surface is so small that wave motion is not seen. The effect of the internal waves is limited, to the extent that it is a question of pure waves on a surface of discontinuity, to the direct environment of the boundary surface; a fact which is very important to the thermal and haline stratification in the boundary water masses. We can also observe internal waves in a trough filled with liquids of different densities. By means of a slow displacement of a sector shaped body on the bottom of the trough, we can produce internal waves, and we can see that although the upper layer is not very thick relative to the lower layer, the free surface of the upper liquid has small waves even if the wave motion on the boundary surface of the two liquids assumes considerable size.[1]

1. We find photographs of such experiments in A. deFant: Theoretical Considerations and Experimental Investigations of the Structure of High Cyclones and Anticyclones. Ber. d. Akad. d. Wiss. Wien. II. A. 1913.

In the ocean and in the lakes with internal boundary surfaces, internal waves can be formed which travel along the boundary surface with speeds which depend upon the difference in density of the two types of water and upon their vertical thickness, which, however, are often smaller than the wave speeds of waves of similar wave lengths on the free surface. In actuality, sharp physical discontinuity surfaces very seldom occur in nature. On the other hand in the perturbated layers there is a larger or smaller change of density with depth. J. E. Fjeldstad (10) has developed a theory of internal waves for such vertical density structure. He finds that in this case at each depth small waves of infinitely many periods are possible, all of which have different speeds. However, the velocities are relat to one another in an harmonic fashion. The longest wave is most of all not always the most important. Fjeldstad has also indicated a method which permits the calculation of the various possible internal waves of tidal periods from the given vertical density structure, and also the calculation of their respective periodical current velocities. Comparison can then be carried out with the results of current measurements.

The reason why relatively few fully satisfactory observations of internal waves in the free oceans exist lies in the external conditions under which such observations can be obtained. From measurements at one station alone no data concerning the direction of propagation of these waves, with such observations at a station, nothing can be said. Only from the theoretical speed of propagation which we can determine approximately from the vertical decrease of density at the observation point, since the period of the wave is given, do we obtain a picture of the length of the internal waves. Such calculations have produced, for the case of a tidal period of about 12 hours, that such internal waves have long wave character, that is, waves whose wave length often is larger than the depth of the water which occurs in the oceans. They have therefore the character of flood waves, which is important for the meaning of many observational facts. The daily series of measurements at the anchor stations in the free oceans and simultaneous current measurement confirm this conclusion. The internal displacement of water masses has tidal characteristics. It appears that they are periodically coupled with the periods of the upper surface and with the tidal currents of these fluctuations. Apparently this coupling is not always the same and differs from place to place.

A visual example of internal waves might explain their behavior. In the straits of Gibraltar we always find two different types of water lying over one another; the upper layer the lighter Atlantic water flows eastward, while underneath the heavier Mediterranean water flow westward. The transition from one to the other furnishes a well developed layer of density contrast, especially in the middle part of the Straits (1)

2. See here particularly G. Schott Die Wasserbewegungen im Gebiete der Gibraltarstrasze. Journ. du Conseil, III/2. Kopenhagen 1928.

which in the Straits themselves rises from west to east. Illustration 2 gives a cross section of the density through the entire straits (to be sure, average conditions which are present in the transition from spring to summer). At this time the Atlantic Current achieves greatest intensity in the upper layer, the Mediterranean Current in deep water becomes on the other hand weaker. The layer of density contrast as seen in the vertical distribution of isopykes is in the mean depth of 200 to 250 meters, while the deep water in the main channel of the straits fluctuates between 300 and 600 meters. This layer of density contrast moves with great regularity up and down in the tempo of the tides so that this internal tidal wave is connected alternately with an increase and decrease of the transport of Mediterranean water in to the Atlantic Ocean.

The Danish research ship "Dana" in the period of October 8-10, 1921 in the station 1138 in 35 degrees 59 minutes north 5 degrees 29 minutes west from Greenwich carried out oceanographic series measurements at 3-hour intervals (11), which in a very excellent manner shows the presence of important internal waves. Illustration 3 gives for the period from about October 8 2200 until October 9 2000 the course of the lines of equal density from the surface to the depth of 300 meters. Under that well developed internal waves exist, flowing in regular form, comprising the entire water mass of the Mediterranean Straits, especially in the region of the density contrast layers. For this time the tidal course (water stand curve) at Cadiz (o degrees 12 minutes west) is shown as well as the direction and the strength of the total current (arrows in the upper layers in the middle of the straits). At this time of the observations a neap-tide prevailed. We see at first glance the relation between the tide of the upper surface, the tidal current and the vertical displacement in the region of the density contrast. The tidal process on the upper surface of the Straits goes essentially parallel with the internal waves. An exact analysis of this phenomenon has been undertaken by J. P. Jacobsen and Helge Thomson (12) on the basis of the fluctuations of the isohalines and comparison with the simultaneous tidal curves of Gibraltar. This showed that while in Gibraltar the water stand on October 10 fluctuated around 48.6 cm., the distance of oscillation of the layer (the difference of the highest and lowest position of the 37 0/00 isohaline) amounted to 43 meters and the highest stand occurred .7 moon hours later than at Gibraltar. The coupling of the interior tidal waves with the surface tide may be easily understood in this case, for it is a question of a narrow long stretch of water. The increase of the current in the upper layers, which goes with a decrease of the lower current, causes a sinking of the Sprungschicht*, a decrease of the upper current and an increase of the lower current. The normal tidal flow causes, in this case, periodic displacement of the Sprungschicht with the same period; an internal tidal wave. In such a case the entire process takes place regularly and without essential disturbances.

A further example of an internal tidal wave for the free ocean is as follows: The research ship "Meteor" along the coast of South America north of Cape San Roque, (2 degrees 27 min. South, 34 degrees 57 min. West, Jan. 31 - Feb. 2, 1927 mean depth of water 3910 m) observed at anchor station 254 the mean distribution of temperature, salt content, and density, found in Ill. 4. To 70 meters an almost homogeneous layer was present below which for 40 meters there was a general rapid decrease of temperature of about 13 degrees and a salt content decrease of .8 0/00. The density varied from 23.56 at 60 meters and to 26.62 at 130 meters, so that the density increase in this ideal boundary surface amounted to about 3×10^{-3}. (See Illustration 5). The definite internal wave of half-moon day period was connected with a mean vertical displacement of water of about $7\frac{1}{2}$ meters. We see clearly from Illus. 5 how not only in the top layer (temperature curve in 0 and 50 meters) but also in the lower layer (temperature curves at 150 and 200 meters) the temperature remains essentially constant, (0 meters has a small daily

*While we have used the German here, a good translation might be pycnocline, or layer of density transition.

temperature variation), while at 100 meters depth in the middle of the Sprungschicht it varies more than two degrees with the waves. The curves in Ill. 5 at 100 and 125 meters give the harmonic analysis of the observation curve and show how these curves may be well reproduced by means of a single wave of a period of 12.3 moon hours.

In addition to these pronounced internal tidal waves the observations contain still a large disturbance, which appears around midnight of the January 31 to February 1 period. While in the top layer no trace of this disturbance is to be seen, it embraces the entire Sprungschicht with particular strength in its upper part; at 200 meters depth we notice still a small increase of temperature and a small decrease in salt content, so that the disturbance still has somewhat surpassed the 200-meter level. From the mean vertical structure of the water at the station (see Ill. 4), with plausible assumptions, it may be calculated from what depths the water masses must rise in order to produce at each level the observed disturbances. These values enable us to construct a picture of the current progression through which the disturbance in the thermohaline structure is produced. In so doing we must think not of successive tides but of simultaneous tides. Ill. 6 gives the flow lines of this disturbance and a clear picture of the tidal water displacement with the passage of the disturbance. The course of the disturbance has a striking similarity with the form of those disturbances which J. Sandstrom (13) experimentally produced in stratified water in which he had wind stress take effect upon the water surface by spurts (Ill. 7). It is not to be doubted that at the anchor station 254 buoyance such an underwater wave was produced somewhere in the surroundings of the station by some sort of external disturbance. This then proceeded further along the Sprungschicht and accidentally was included in the series measurements. From where the disturbance came and in what direction it proceeded, cannot be determined from the observations at one station alone; however, a long period of time may pass before such a phenomenon will come accidentally into the observations anew.

The following case leads to a striking phenomenon which will be investigated somewhat closer. The anchor station 369 of the first part of the trip of the North America Expedition of the Meteor (19 degrees 40 min. north, 18 degrees 4 min. W., depth 2140 meters, April 19-21, 1937, 52 repeated series to 200 meters at one-hour intervals) was off the African shelf near Cape Blanco between the Cape Verde Is. and the Canary Islands. In the layer between 30 and 60 meters there was a well defined Sprungschicht of temperature and salt content and the 52-hour measurements showing strong internal waves of tidal character, which were revealed particularly well in the up and down curves of the isotherms. Ill. 8 gives an isothermal representation for the entire period of time. We recognize clearly periodical displacements of the isotherms in the entire layer to a depth of 250 meters in the tempo of a half-moon day tidal wave. In order to demonstrate these repetitions better, the top position of the isotherms are connected in each case by means of a heavy line and the low positions of the isotherms by means of corresponding dotted lines. In addition to the half-moon day waves in the second part of the period, particularly in the region of the maximal Sprungschict, another smaller period appears to be superimposed. With this we will not concern ourselves further here. I would rather call attention to an asymmetrical development of the isotherm pattern which is connected with the passage of the large internal waves. In general the decline of the isotherms is slower than the rise. This appears particularly clear if mean layers are taken (Ill. 9). The rise of temperature from the minimum to the maximum results in 7.2 or about 8 hours, the decline in 4.8 or about 4 hours. In so doing the minima are flat and broad, the maxima development of the Sprungschicht (40 to 60 meters) the assymetry is more strongly developed than in the lower layers. If we will investigate the tidal displacement of water from the pattern of the temperature at a fixed point, it is observed that to a rise of temperature corresponds a lowering of the Sprungschicht and to a fall of temperature corresponds an upward motion of the latter. The pattern of the Sprungschicht is accordingly as in Ill. 10-A. With wave motions it is in general possible to connect the phenomena at a place to the succession in space. We obtain then an approximate representation of the form of the wave profile with the passage of the internal waves at the place of measurements. The form of the waves is then that which is given in Ill. 10-B: A gentle increase from the wave trough to the wave ridge, a steep fall from the wave ridge from the wave trough. This asymmetrical wave profile points to a degeneration of the internal waves, which we see with ordinary surface waves of the ocean before they overturn in the surf. We must therefore perhaps assume that internal waves can degenerate and finally lead to internal surf. We must not, however, presume that this internal surf has the vehemence and rapidity of the surf phenomena of upper surface waves. This internal surf takes place at the small propagation velocity of the internal wave.* It is very important to see how in spite of the very turbulent surf processes at the boundary surface the upper surface can remain completely at rest, as if within the water mass nothing were taking place. Similarly the processes of an internal tidal wave turning into surf at depths must be investigated.

I will give still further examples of this very interesting phenomenon; theyshow that the surf process in the case of internal tidal waves apparently is not something extraordinary. One case concerns the internal waves of the "Meteor" anchor station 366 (10 deg. 16min. N., 16 deg. 38 min. W., v. Greenwich, April 13-14

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* Photo of such internal wave as surf is to be found in the work in Footnote 1, Page 43.

1937, 60 repeated series at one-hour intervals). Observations were carried out somewhat southwest of the Bissagos Is. upon the broad West African shelf rather close to land. Ill. 11 gives the isothermal representation from degree to degree for the entire period of 5 full periods of half moon-day tides. Here we see again the strong fluctuations in the height of the isotherms, which are most strongly sheared in about 30 meters of depth, since the Sprungschicht of density on the average lies at this depth. Again in spite of the numerous small disturbances which make the curves rather irregular, a clear asymmetry of the large waves. This is clearer in Ill. 12. In the latter all five wave periods are averaged, eliminating the small disturbances. In addition to the asymmetry, which here appears to be more strongly developed than in the former case of anchor station 369 , we note that a clear displacement of the extremes with depth is present. The extremes of the internal waves have a time lag which is larger and larger with depth, particularly in the maximum (the top position of the Sprungschicht.). The lag amounts to several hours. Even this striking phase displacement of the waves with depth is a phenomenon necessarily connected with the phenomenon of internal surf.

A still further increase of asymmetry of the internal tidal wave is shown by the following case, again in the Straits of Gibraltar. The measurements are approximately in the same place as in the case discussed earlier (See Page 46), this time at the time of the spring high water. They were partly made by J. P. Jacobsen and Helge Thomson (12): Dana Anchor Station 35 deg. 47.5 min. N., 5 deg. 21 min. W., April 14, 5:05 to April 15 5:12, 1928, 23 repeated series at approximately one-hour intervals. Ill. 13 is the same type as Ill. 3. It gives for the period of observation the pattern of density (K.G./ Cbm) to 300 meters depth. Below that is shown the simultaneous water stand curve of Cadiz and the total current (the basic current plus the tidal current) for the middle of the

Straits. The internal waves connected with the tides of the sea surface have here degenerated completely and one wave branch stands approximately vertical. It amounts, even in the layer between 70 to 150 meters, almost to a turning over of the waves. To be sure the lower layers are in general still denser than the upper layers; however, there are in two series of measurements several values which clearly show unstable stratification. The internal tidal wave is in this case almost at the point of turning over; an unstable condition. Without doubt the tidal currents, with spring high water, plus the basic currents, which are present in the Straits are the external cause. At this time the inner tidal waves, which otherwise progress calmly and regularly, incline to strong degeneration: this leads naturally to inner surf. The form of this surf process must, as we can perceive from Ill. 13, be like an internal bore. This internal bore proceeds probably the entire length of the Straits and loses itself then in the boundary oceans at large depths, where the Sprungschicht, which is well developed in the Straits, also dissolves into a more constant density increase with depth.

The Straits of Messina provide another interesting case. The smallest cross section of the Straits lies in the northern part, in the Profile Punta Pezzo-Ganzirri; it has a width of less than 4 km and an almost rectangular cross section of only one-third square km., with a mean depth of 80 meters. From this threshold the sea bottom sinks towards the south and the northeast rather gradually, but faster towards the south than towards the Northeast, so that in the north part of the Strait has a cross section of about 7 km length, which always remains below one-half square km. This part of the Strait is the site of the turbulent tidal processes known by the names Scylla and Charybdis (14) Normally the lower layers in the Straits (below about 30 meters depth) are of the heavier Ionian Sea water; above that lies the lighter water of the Tyrrhenian Sea. The latter moves to the south, the former toward the north. This main current distribution corresponds to the density differences of the water masses of the boundary seas. The surface inside the Straits has large fluctuations in the rhythm of the half moon tidal period. The result of the constriction of these Straits and the small density difference of the water layers lying over one another is that these internal tidal waves assume a turbulent form as in the Straits of Gibraltar. The disturbances here reach the sea surface and produce large horizontal vortices which lead to internal surf phenomena.

At a 15-day anchor station the research ship, "Marsigli" led by F. Vercelli, carried out a survey of current behavior and structure of the water masses of the Straits, in addition to current measurements and temperature and salinity determinations at various depths at approximately one-hour intervals. This provided information concerning the extraordinarily large temperature and salinity changes which appear at the narrowest points of the straits with the flow of the tides. They point to mighty internal waves through the passage. The thermal and haline structure of the water columns changes from hour to hour, often so much that we doubt whether the observations in each case belong to one and the same vertical. Absolutely simultaneous series measurements would be necessary here. Instabilities often occur which certainly are due to internal surf phenomena. I have in Ill. 14 summarized three cases of such strong temperature changes which show the internal bore. Between 2 and 3 o'clock moontime heavy water advances from the south to the north in the form of a headwave. However warm water flows through the straits. The flow often is so rapid that on the head of the advancing water mass the structure becomes unstable, as in the first and second cases of Ill. 14. It is very characteristic that only in this phase of internal tide does instability take place; never at the phase 180 degrees different (8 to 9 o'clock moontime). This behavior is schematically shown in Ill. 15 in a cross section through the Straits. We recognize that here the internal tidal waves at the time of the spring tide have almost lost the character of a wave and assumed the form of a bore. At 2 to 3 o'clock moontime the process, meteorologically speaking, has the character of a cold front; between 8 and 9 o'clock moontime it has the form of a warm front.

It no doubt exists that internal tidal waves once produced degenerate under certain conditions and undergo profile changes. How do internal waves get to the turbulent surf stage so easily? I would like to point out that we are very poorly informed as to the origin of internal waves and their relation to the general phenomenon of ocean tides. Since its speed of propagation often is less than that of the surface tidal waves important phase displacement of these two types of tidal waves will occur. In general the relation between them must soon be lost. The relationships will be proved only in the general proximity of the place of the origin of the waves; we, however, don't know where these places are. Only this much appears to be established; that the internal waves are of tidal dimensions, are subject to a very strong damping, and the amplitudes therefore must rapidly decrease in all directions (Footnote 3. See V. W. Ekman in the discussion of the Annual Meeting of the International Council for the Exploration of the Sea, Copenhagen 1931) From numerous experiments which N. Zeilon (16) carried out in the laboratory, it may be assumed that internal waves on the continental shelf are produced by the decrease of the water depth taking place on the shore which move along the Sprungschicht onto the shelf toward the continent. Is the resulting degeneration of wave profile of such internal waves a similar process to that for the customary short surface waves of the ocean? If the amplitude of such surface waves gets larger and larger and not small compared to the mean depth of the water, then the propagation is not without form change. As the observations show, the wave profile on the front side become steeper and steeper, the back side gentler, the wave profile becomes asymmetrical. This process may be examined theoretically. Up to a short time ago we could calculate the corresponding successive wave profiles only by stepwise approximations. J. E. Fjedstad succeeded in giving a complete mathematical solution for this profile change. The only simplification which was here necessary was the neglect of the vertical acceleration of the water particles in their orbits, so that instead of the dynamic pressure the static pressure was used. On the other hand, the nonlinearity of the differential equations was completely considered. The solution was obtained not only from the eulerian but also the Lagrangian equations of motion (17). The wave profiles obtained show clearly that in the progress of the waves the asymmetry of the profile always increases until a condition is reached in which the suppositions of the theory are no longer fulfilled. The observations show then what happens. The waves break.

The internal tidal waves will under similar conditions change their profiles, with increase of amplitude and decrease of contrast of the water layers lying over one another. Several cases in which these asymmetrical profiles were observed show large amplitudes. To be sure with internal waves the case is usually otherwise, and the observation place was not upon the shelf. Without doubt both circumstances contributed to asymmetry of the wave profile.

However, there are also cases in which these conditions obviously do not suffice to make the internal surf understandable. It is likely that the seiche waves, even with relatively small water depths, show permanent wave profiles, which even with decreasing water depth do not degenerate except if the water depths are very small and the waves flow directly along the bank. It could be that the internal tidal waves are of the character of such seiche water waves. Then the instability must be based upon other causes.

The conditions under which internal waves occur permit dynamic instability without further ado. For the explanation of this behavior, a simple case here follows: Two water layers of large thickness lie over one another. The upper water mass has the density ρ_2 and moves with a velocity u_2. The lower water mass has the density ρ_1 and moves at the velocity u_1. The theory (18) reveals that the speed of internal waves on the surface of discontinuity is given by

$$c = \frac{\rho_1 u_1 + \rho_2 u_2}{\rho_1 + \rho_2} \pm \sqrt{\frac{g}{\kappa}\,\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} - \rho_1 \rho_2 \left(\frac{u_2 - u_1}{\rho_1 + \rho_2}\right)^2}$$

The first expression represents the advective velocity and is a type of mean value of the two velocities of the two water masses. Beside this mean velocity is the velocity of waves with the wavelength

$$\lambda = \frac{2\pi}{\kappa}$$

which the roots of the expression give. The first member under the root gives the speed of the internal waves in a stationary system (See Page 42), therefore the speed of pure gravity waves. The second member with a negative sign is the speed of pure inertia waves. The gravity wave term is always positive if ρ_1 is greater than ρ_2, that is if the heavy water is under the lighter water. In nature this is always the case. Pure gravity waves are therefore always stable. The inertia wave term is, on the other hand, always negative; it therefore acts always in a manner to weaken the static stability of the gravity waves. This weakening can produce dynamic instability. This occurs if:

$$(u_2 - u_1)^2 > \frac{g}{\kappa}\,\frac{\rho_1^2 - \rho_2^2}{\rho_1 \rho_2}$$

that is, for a certain wave length and the density contrast on the discontinuity, with sufficiently large velocity shear on the boundary surface between the two media.

In the cases considered the thickness of the two water masses was small, and this means a further evaluation of the expression on the righthand side of the equal sign in the above relation as a result. If h_1 and h_2 are the thicknesses of the lower and upper water layers, we obtain the inequality in the form:

$$(U_2-U_1)^2 > \frac{g}{\varkappa}(\rho_1-\rho_2)\left[\frac{1}{\rho_1 \operatorname{ctgh} \varkappa h_1} + \frac{1}{\rho_2 \operatorname{ctgh} \varkappa h_2}\right]$$

If we also simplify so that h_1 equals h_2 equals h is assumed, and we consider that $\varkappa h_1$ is a very small quantity, then we can with satisfactory exactness set $\frac{1}{\operatorname{ctgh} \varkappa h} = \operatorname{tgh} \varkappa h = \varkappa h$ and obtain the inequality $U_2 - U_1 > \sqrt{gh \frac{\rho_1^2-\rho_2^2}{\rho_1\rho_2}}$

With this inequality the following table may be set up for the various values of the density differences ρ_1 minus ρ_2 and of the layer thickness to indicate the velocity difference which must be surpassed for the dynamic instability of the internal tidal waves to occur.

The velocity difference (cm/sec.) must be larger than:

$\rho_1-\rho_2$	10^{-4}	5×10^{-4}	10^{-3}
h = 150 m	52	117	166
100 m	47	96	135
50 m	30	68	96
25 m	20	48	68

From these values we see that relatively small velocity shear in the two water layers lying over one another is sufficient to produce instability of the internal tidal waves, which <u>must</u> necessarily then lead to internal surf. Since the main currents in the straits always are opposed in the water masses lying over one another, the inclination of internal waves to degenerate and turn into internal surf must be particularly large. That is the case in the straits of Gibraltar and in the Straits of Messina as we have seen. V. Bjerknes has calculated the wave form and streamline in such instable internal waves for other purposes and in Ill. 16 two such cases are reproduced. In the upper diagram both water masses have a similarly directed translation velocity; by the lower diagram, the main currents are directed oppositely; in both the velocity of the upper water is larger, as is always the case in nature. We see at once that cases of asymmetry predominate, as found with internal tidal waves. We must actually assume in the cases of the current stratification which almost always take place in the ocean, that asymmetrical wave forms are due to dynamic instability. Naturally the instability cannot maintain itself for a long time. In further progress the waves develop into mighty internal turbulences. With dynamic instability finally the boundary layer rolls up into a vortex of the same sign as the shear between the two unequally moving layers. Ill. 17 gives the progressive changes which an unstable wave passes through (Footnote 3, See L. Rosenhead: The Formation of Vortices from a Surface of Discontinuity. Proceedings of the Roayl Society A 134, 170.) The final result is certain that a mixed layer is formed by the vortex in which a more or less constant transition of density and of velocity from the lower to the upper layer occurs. That the Sprungschicht of temperature and salinity on the shelves rapidly decreases in intensity towards the coast, and is missing in a broad strip near the coast, is probably because the internal tidal waves nearing the coast with smaller and smaller water depths become so dynamically unstable that the Sprungschicht is destroyed. This process is the basic condition for the production of cold water rising near the coast.