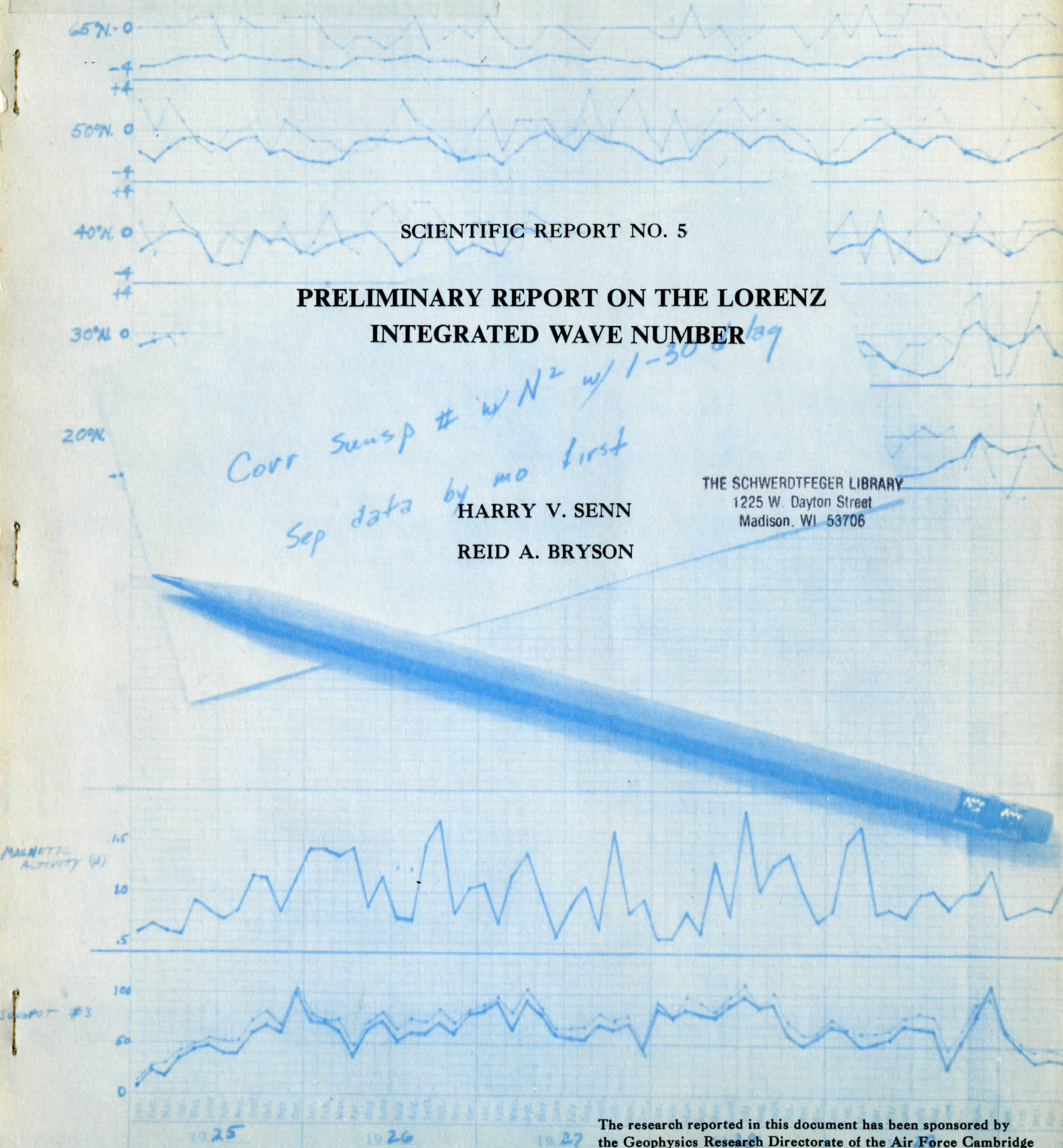
The University of Wisconsin Department of Meteorology Madison

May 1956

SCIENTIFIC REPORT NO. 5

PRELIMINARY REPORT ON THE LORENZ INTEGRATED WAVE NUMBER



HARRY V. SENN

REID A. BRYSON

The research reported in this document has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command

United States Air Force Contract AF 19(604)-992

Scientific Report No. 5

Preliminary Report on the Lorenz Integrated Wave Number

by

Harry V. Senn and Reid A. Bryson

The research reported in this document has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command.

United States Air Force Contract AF19(604)-992
The University of Wisconsin
November 1956

Table of Contents

ABSTRACT

An index, called the integrated wave number, has been suggested by Professor E. N. Lorenz. It is derived from a Fourier series expansion of the pressure profile along a latitude by taking the weighted mean wave number, where the weighting factor is the amplitude of the corresponding sine term. This number was calculated for ten latitudes in the northern hemisphere for each day of ten years in an attempt to find an objective method for stating the pressure system status of the northern hemisphere. Investigations of the wave number by graphical and statistical methods show that while a subjective description of this parameter is difficult, it is highly correlated with the westerly index at certain latitudes as well as with certain measures of solar activity. Suggestions for further research and study include additional investigation into the reasons for the greater significance of the 20° wave number as well as the elusive subjective determination or verification of the computed wave number for a given synoptic situation.

I. Introduction

From the beginning of climatology and of synoptic meteorology there has been a great need for more objectivity in classifying the important features of synoptic situations which recur time and again, as well as those which are important because they occur very seldom. W. C. Jacobs showed how a local climate could be broken down into classifiable types[1]. Other attempts have been made to catalogue whole synoptic maps after typing and/or naming them[2,3]. Although most of the criteria used in these classifying systems are necessarily objective in their sub-element nature, the very large element of subjectivity present in the final typing usually renders them far less universally acceptable than they might otherwise be. This paper reports on a preliminary investigation of the theoretically derived and completely objective Lorenz integrated wave number for the northern hemisphere.

II. Definition of N^2

A. Mathematical Derivation[4]

A Fourier Series describing the pressure profile along a given parallel of latitude takes the following form

$$P_i = \sum_{n=1}^{\infty} A_n \cos\left(\frac{2\pi n x}{L}\right) \qquad (1)$$

where L is the circumference of the parallel, P_i is the difference of the observed pressure at a given longitude i from the mean for the latitude, A_n is the amplitude of the n^{th} term, and n is the wave number, an integer.

After squaring P_i and $\frac{\partial P_i}{\partial x}$ and integrating around the parallel, we may write

$$\frac{\int \left(\frac{\partial P_i}{\partial x}\right)^2 dx}{\int P_i^2 \, dx} = \frac{4\pi^2}{L^2} \frac{\sum_{n=1}^{\infty} n^2 A_n^2}{\sum_{n=1}^{\infty} A_n^2} \qquad (2)$$

where we define

$$\frac{\sum_{n=1}^{\infty} n^2 A_n^2}{\sum_{n=1}^{\infty} A_n^2} \equiv N^2 \qquad (3)$$

N is thus a composite or weighted mean wave number where the weighting factor is the amplitude squared of the corresponding wave. In difference form

$$N^2 = \frac{L^2}{4\pi^2}\left(\frac{\sum_{x=0}^{L} \frac{(P_i - P_{i+1})^2}{(L/36)^2} \frac{L}{36}}{\sum_{x=0}^{L} (P_i - P)^2 \frac{L}{36}}\right) \quad (4)$$

where $dx = \frac{L}{36}$ (10° longitude increments), and $\bar{P}$ is the mean pressure along the parallel, letting

$$S^2 \equiv \Sigma(P_i - \bar{P})^2 \quad (5)$$

$$D_i^2 \equiv \Sigma(P_i - P_{i+1})^2 \quad (6)$$

$$K \equiv \frac{(36)^2}{4\pi^2 \, 35} \quad (7)$$

equation (4) becomes

$$N^2 = K \frac{D_i^2}{S^2} \quad (8)$$

B. Method of Calculation

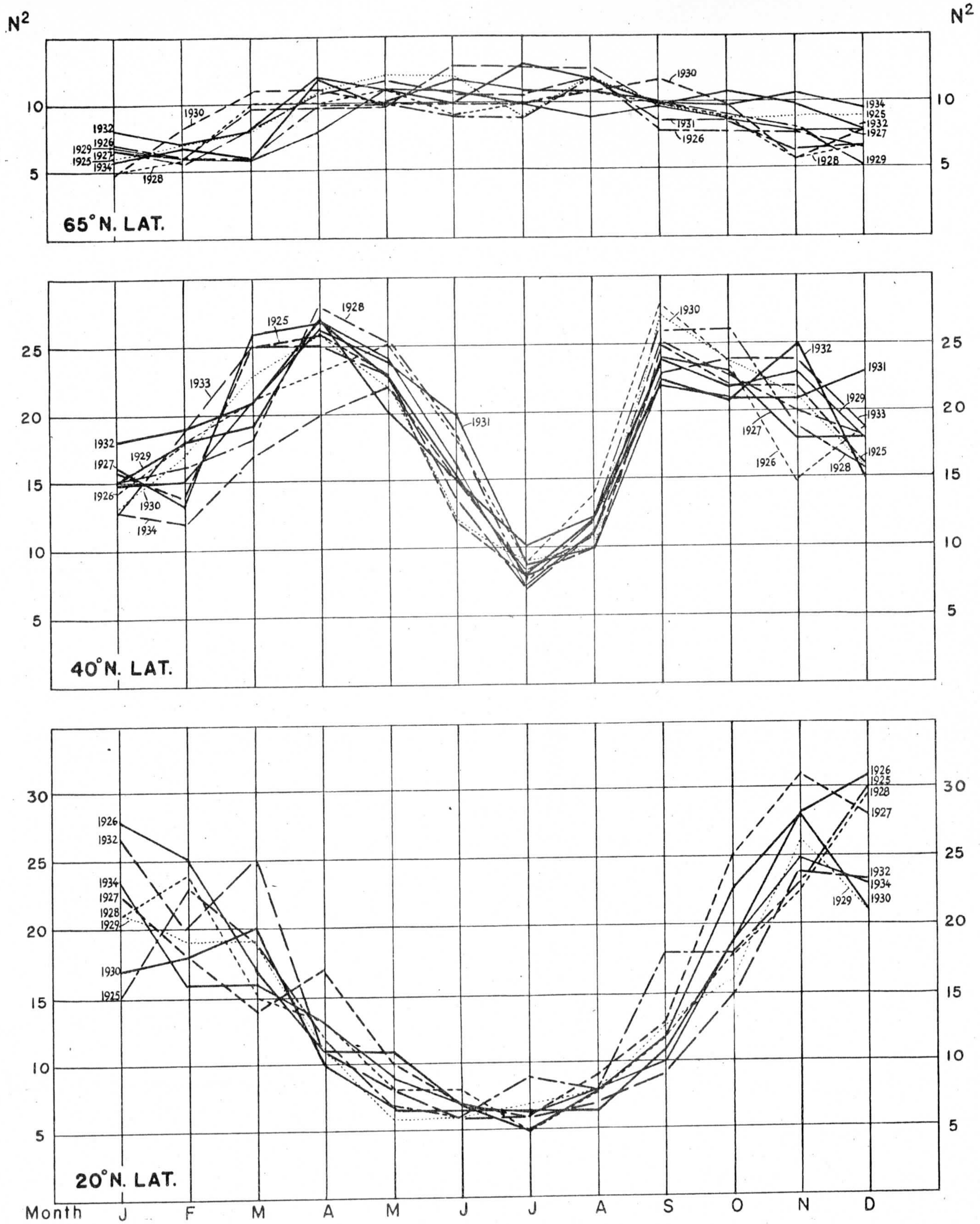
D_i^2 and S^2 were available on IBM cards for the ten years 1925-1934 with the ten latitudes at 5° intervals from 20°N. to 65°N. of surface pressure data[6]. The constant K was computed at .9372 and the deck of N^2 values for each day and each latitude of the northern hemisphere was computed and punched on the IBM 602A. N^2 was also computed by hand for January 5, 15, and 25, 1951 from the AROWA deck of 500 mb data.

Taking the square root of these 36,900 N^2 values would require another program and run through the machine. N^2 and $1/N^2$ are the values needed in the Rossby wave speed formulas[7], and since the variations in N would, in many cases, be detectable only in the decimal values, we found the N^2 values much more useful in both the graphical and statistical methods of investigation subsequently used.

III. Is N^2 Synoptically Meaningful?

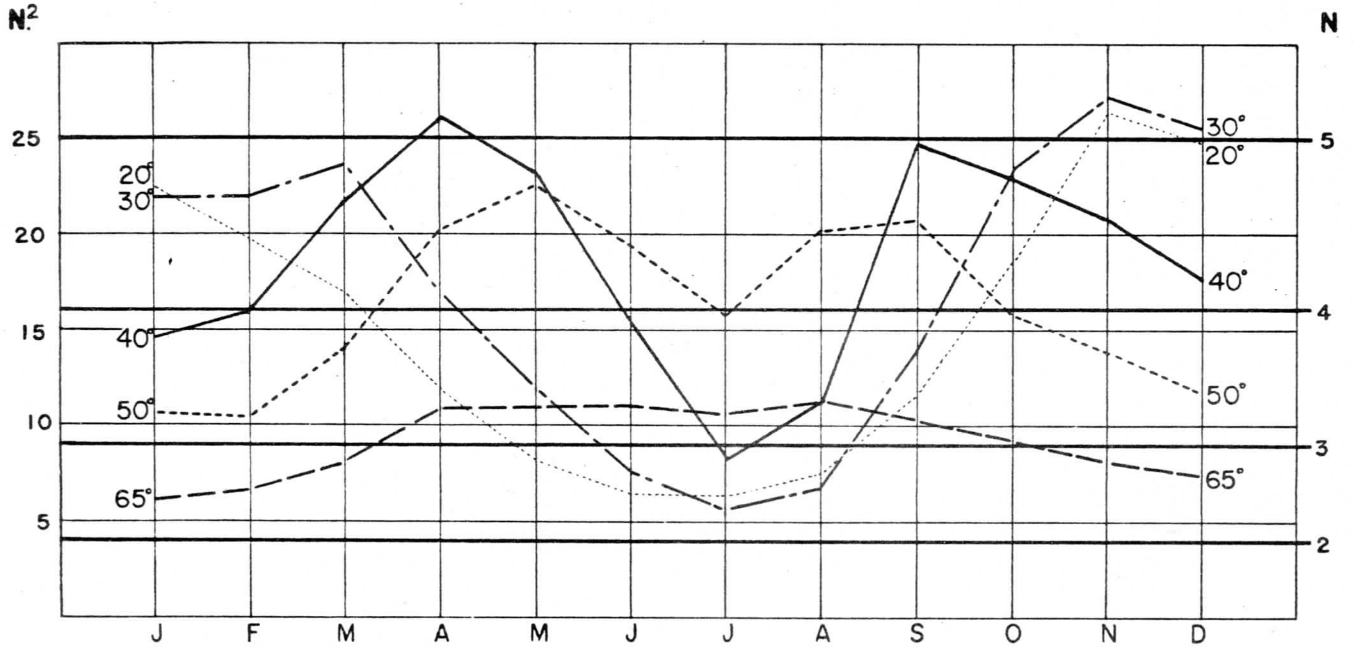
A. Does it act the way we would expect an integrated wave number to act?

Figure 1 shows the monthly average N^2 values for each month of the 10 years at latitude 20°N., 40°N., and 65°N., and figure 2 shows the 10 year monthly means for latitudes 20°N., 30°N., 40°N., 50°N., and 65°N. They show that there is very little change in the mean monthly N^2 values from year to year at most latitudes. Consequently, figure 3 shows the 10 year mean monthly values of N^2 with isolines of N. The only real variation on the 12 mean monthly N^2 graphs for each of the 10 years was in the 5 isoline. This was sometimes slightly more extensive in March, April and May, and occasionally appeared in January at latitudes 20° and 25°. The only other feature that varies much is the other 5 isoline extending toward mid-latitudes in November and December. This isoline frequently reaches between latitudes 40° and 45° in September as shown by the dashed line. As we might expect, the middle latitudes have pronounced maxima in both spring and fall, and all latitudes except 65° have maxima in July. The annual range is least at the highest latitudes and increases rapidly as we go south to a point near 30°, decreasingly only slightly at 20°. Now we might have expected N^2 to have the greatest range at about 40° simply because we think of the major storm tracks being completely north of this latitude in summer and well south



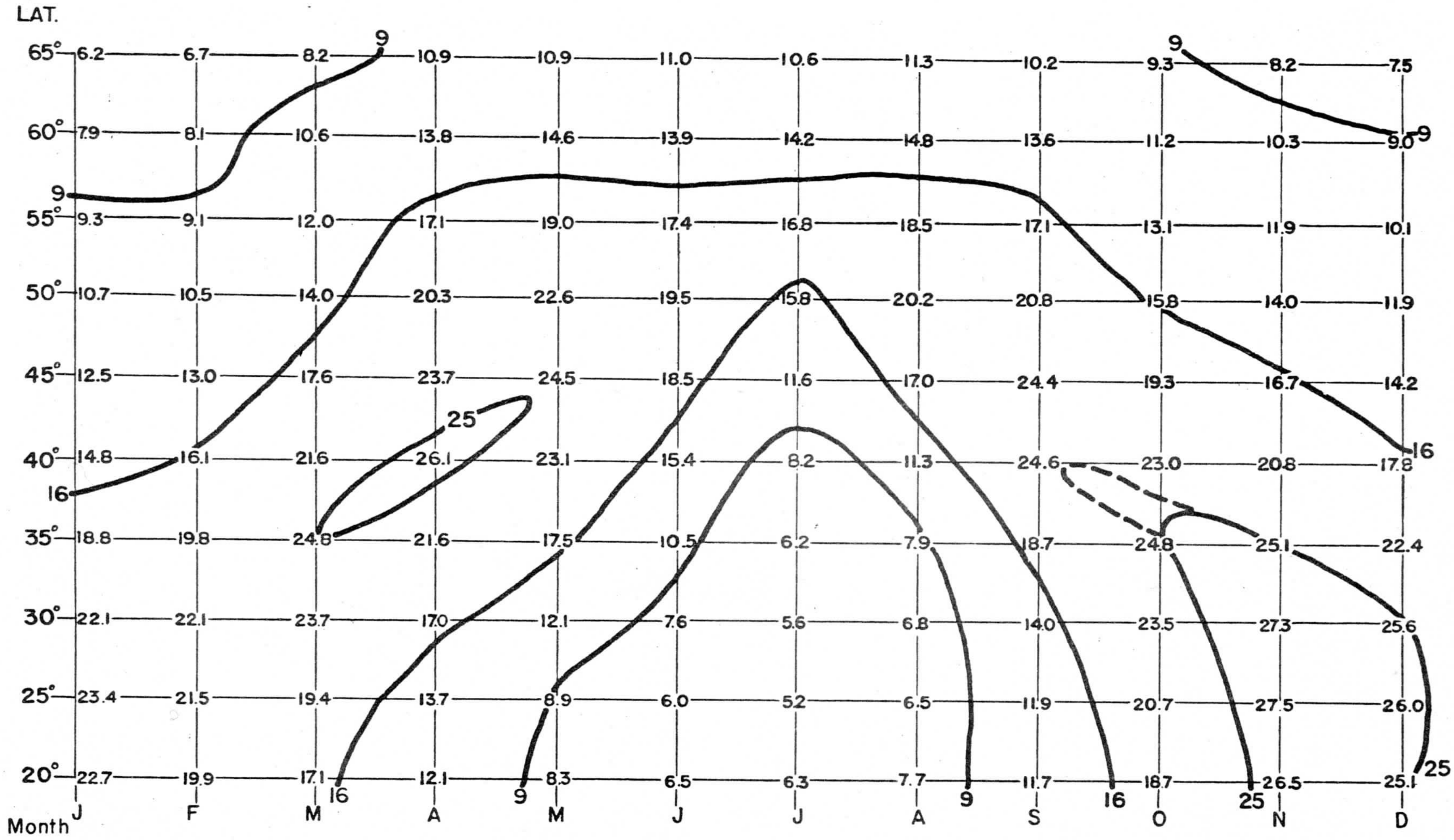
MONTHLY MEAN N²

Figure 1



10 YEAR MONTHLY MEAN N²

Figure 2



10 YEAR MEAN MONTHLY N^2
(all lats.)

Figure 3

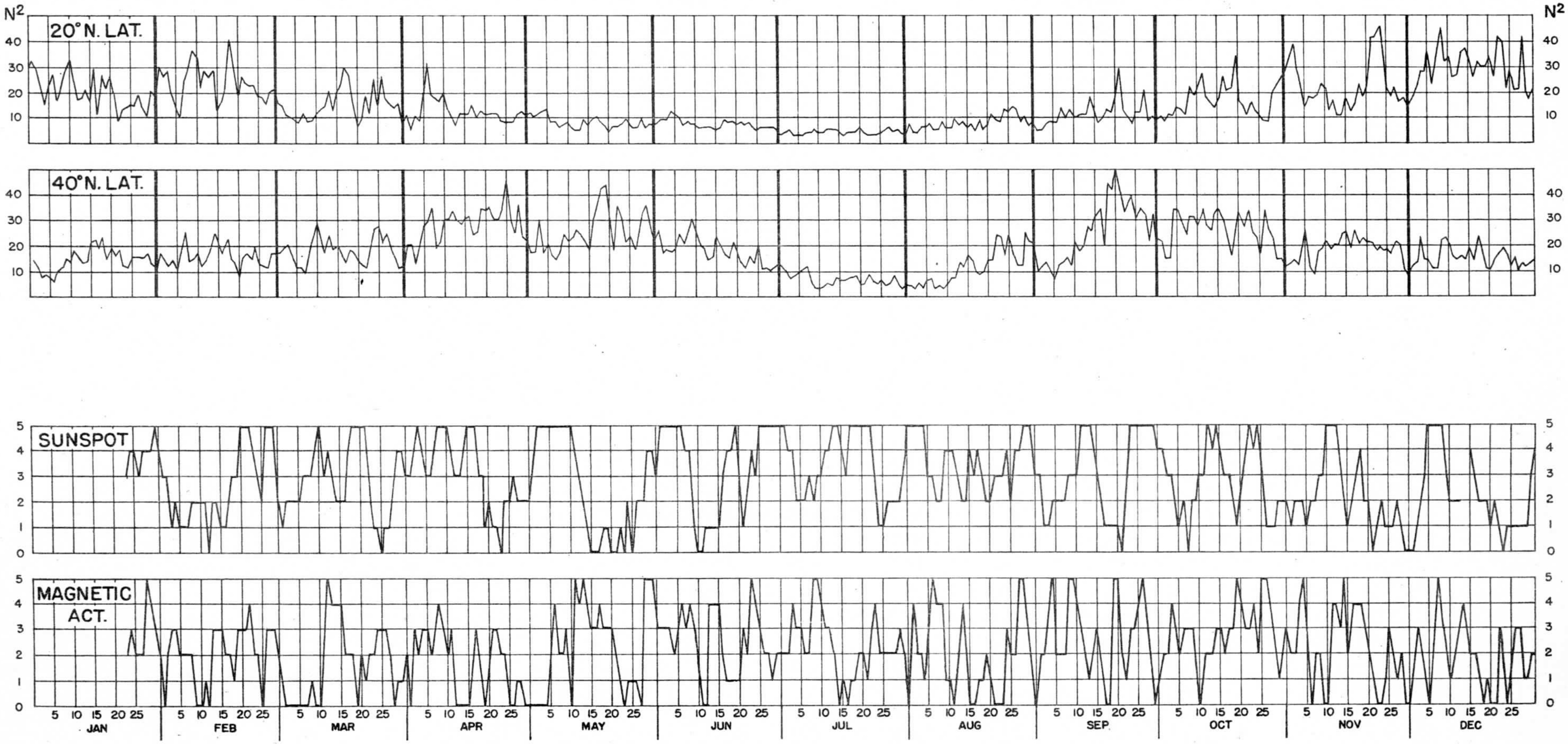
of it in the winter months. If our N^2 is any criterion, this latitude is closer to 30° than to 40° and the typical mean pressure pattern charts are so averaged as to be meaningless for the present purposes. The winter maxima we obtain at the lowest latitudes may mean that N^2 is a good index of the production and transformation of energy which must take place on the largest scale only at those latitudes in winter.

Figures 4 and 5 of the daily N^2 values at 20° and 40° show that the daily and weekly variations usually have greater magnitude than the monthly variations at both latitudes except in the comparatively quiet summer month of July. If we compare figure 1 with figures 4 and 5, we will see that N^2 has its greatest daily and weekly variations when its monthly mean value is greatest.

Figure 6, a graph of the 10 year daily mean N^2, shows large daily and weekly variations even though we might have expected them to be averaged out. This graph also shows the long, slow and small variations at 60° as well as the maximum annual range and winter maximum of 30° latitude not shown on the previous graphs.

B. Can we verify N^2 subjectively?

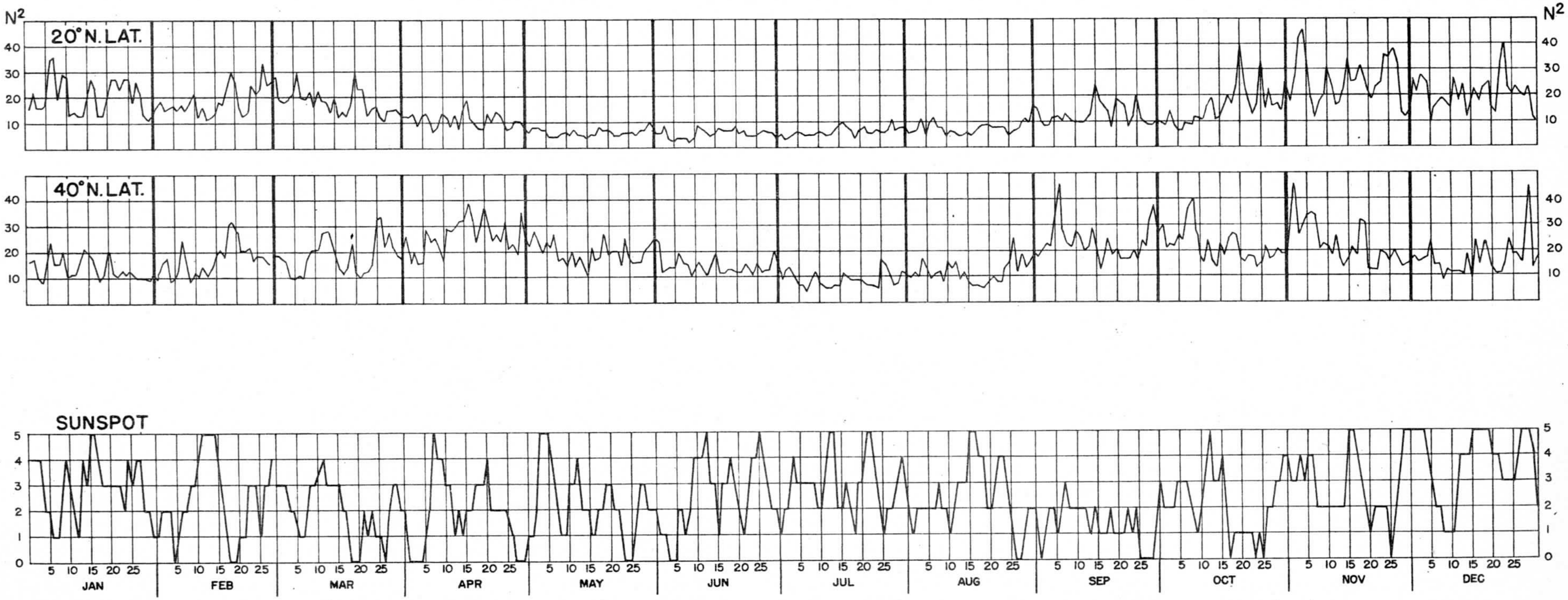
Before we try to verify N^2 we must have some idea of what we are measuring. From equation (3) we can see that there is no consideration given to whether the wave is short



DAILY N^2 VALUES,

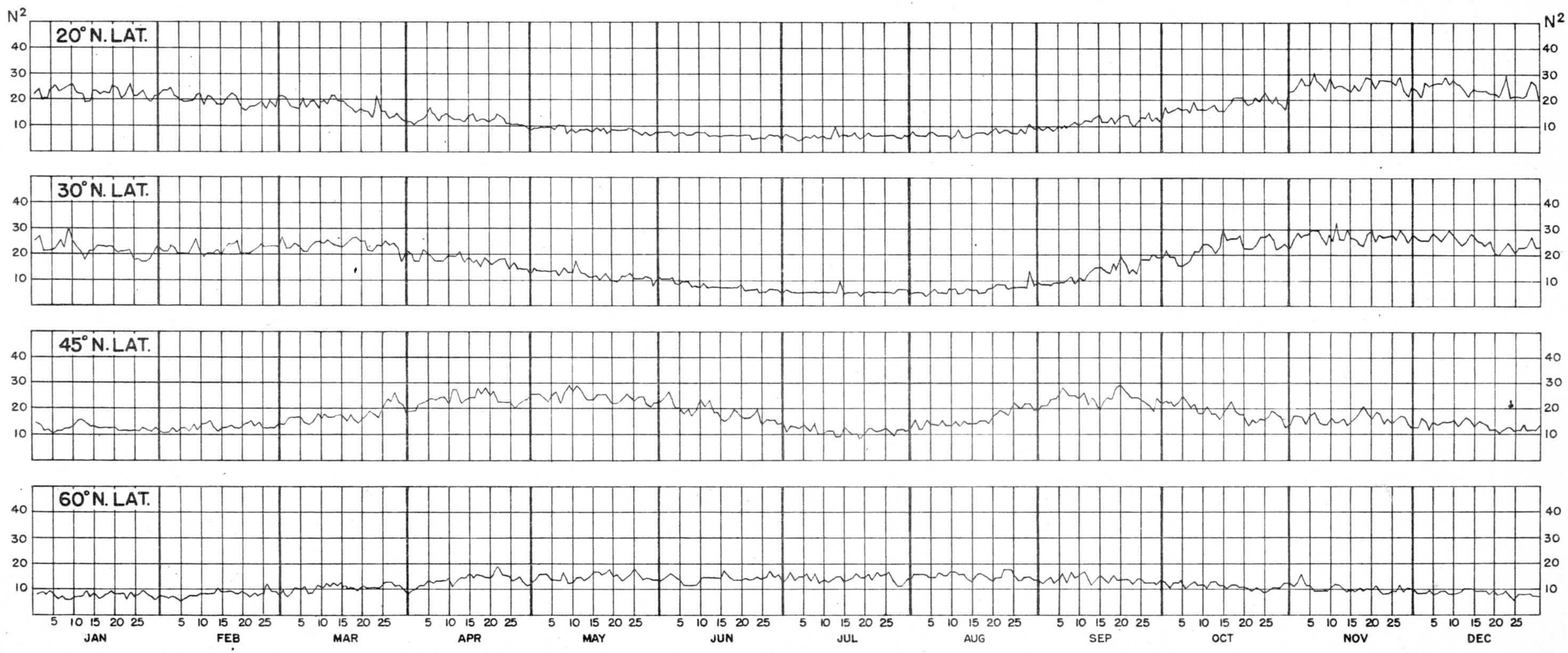
RANKED SUNSPOT NUMBERS, & RANKED MAGNETIC ACTIVITY NUMBERS FOR 1928

Figure 4



DAILY N^2 VALUES & RANKED SUNSPOT NUMBERS FOR 1929

Figure 5



10 YEAR DAILY AVERAGE N^2
1925-1934

Figure 6

or long, and also that a mixture of short waves and long waves will give a value of N^2 the same as would waves of only intermediate length. We might also regard N^2 as a "normalized" value of D_i^2, which in turn is a measure of the north-south kinetic energy along a given parallel.

Representative days from all seasons of most of the 10 years were used in attempts at subjectively typing the synoptic weather map[8], into its N or N^2 classification at each of the 10 latitudes computed. After many failures with simple methods such as counting major highs and/or lows around the latitude circle, setting up criteria as to how many isobars should cross the line each way before a wave was considered, drawing pressure profiles for each latitude circle and computing the mean pressure at a given latitude, and attempting a guess of N by a study of the deviations from it, we spent many more hours comparing the computed values to their corresponding synoptic situations in further attempts to find out whether the analyst using the computations could become so practiced that he could at least make intelligent estimates. However, even this coupled with plastic overlays divided into sections at the latitude circles for the appropriate number of waves for an average date, proved unsuccessful. In an attempt to study simpler patterns and through them gain some insight into the more complicated, we computed N^2 for three days in January, 1951 at 45° from the AROWA 500 mb data. Even at this comparatively simple level, subjective <u>N's</u> varied from 1 to 5, while the computed <u>N^2's</u> varied from only 1.53 to 2.56. Although we finally achieved subjective values of N within 2 of the computed values about one half of the time at latitudes where N had total variations of values from 1 to 7, we found the average deviation for a given month to be 2 or less and concluded that there is no known method of subjectively naming N or N^2.

C. Can this abstract parameter be useful?

At first it might be concluded that N^2 is entirely too abstract, and that any such wave number which cannot in some way be visualized or subjectively translated to the synoptic chart which it represents, can be of little or no use. However, N^2 gives some interesting, if not very conventional results when applied to Rossby's wave speed formula as we will see presently; and some 2200 correlation coefficients were run between N^2 at various latitudes and values of westerly index, sunspots, magnetic activity, and N^2 itself at other latitudes. Many of these coefficients show significant relationships between N^2 and all of these other variables with some interesting periods when run on a lag correlation basis. Although it is true that lack of subjective analysis limits our understanding of some of these relationships, others are important simply because they show a definite short time relationship between N^2 and other parameters, notably solar activity, which may not have seemed related previously.

IV. How does N^2 correlate with ...

A....N^2 at other latitudes?

A study of figures 2, 4, and 5 shows that although the annual march of N^2 at one latitude may be correlated rather well with that at another, either positively or negatively, the daily graphs show that the correlation is not nearly so high for this shorter period, although it is significant at the 1% level. This is probably due to the fact that we are not measuring just the short or the long wave troughs, and other factors, as previously discussed; and to the fact that changes in N^2, if they are spasmodic in nature would tend to be of varying magnitudes at different latitudes. In other words, we can and often do have a different wave number at one latitude than we do at another only 10° latitude away; and the wave number at a given latitude does not necessarily change smoothly and slowly, but frequently jumps from one magnitude to another very rapidly. This rapid changing from one pattern to another has been demonstrated in laboratory experiments which were recently described with streak-photography by Frenzen[9].

B....westerly index?

Table 1 shows that on an annual basis, the 10 year daily mean N^2 at 20°, 30°, 45°, and 60° are all correlated significantly (at the 1% level or better) with both the mid-latitude westerly index (35^o-55^o) and the high-latitude westerly index (55^o-65^o).

Table 1

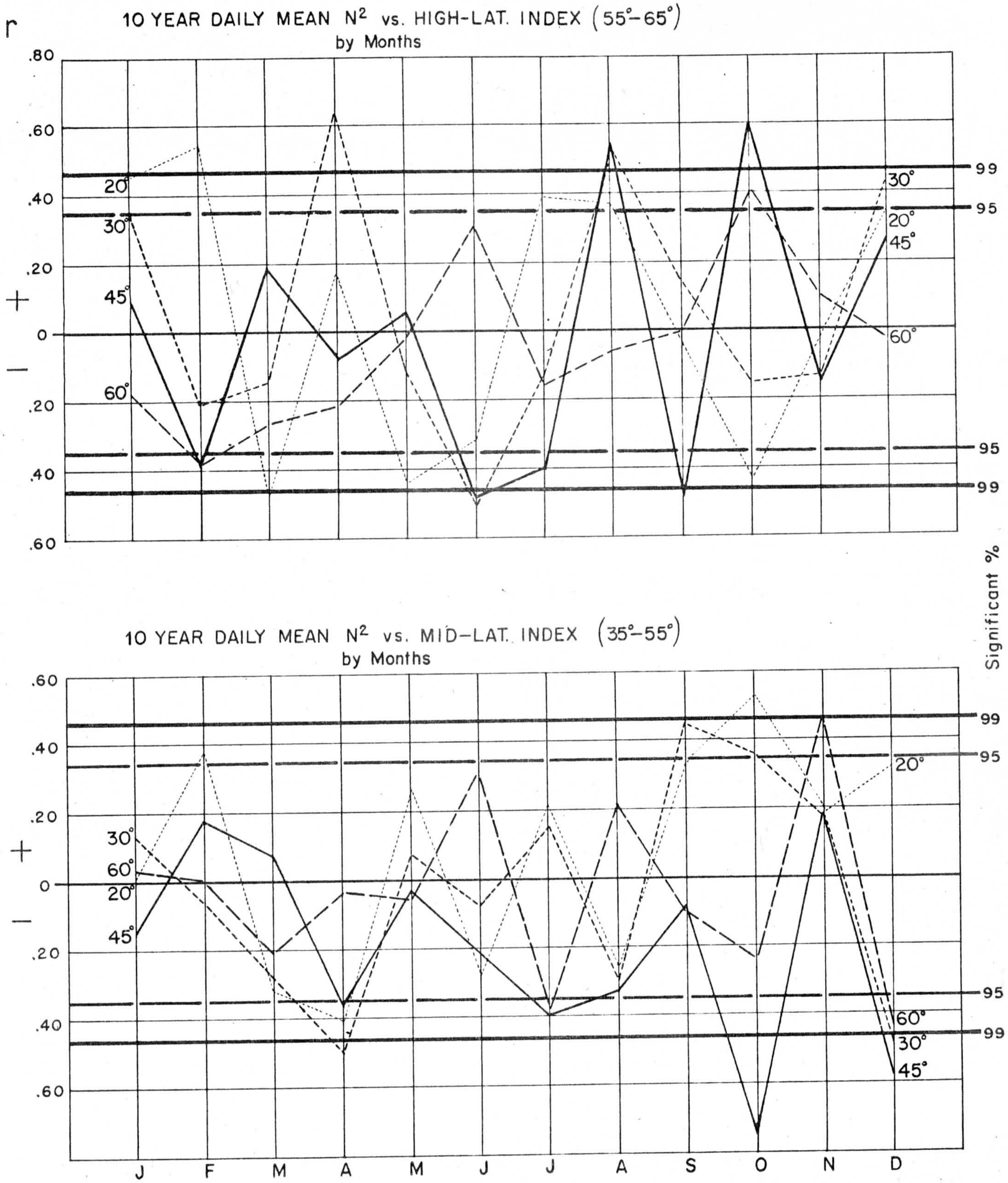
Correlation Coefficients
10 Year Daily Mean N^2 vs. Mid-Latitude and High-Latitude Westerly Index

	20^o	30^o	60^o
Mid-Latitude Index	.68	.58	-.51
High-Latitude Index	.31	.20	-.25

It should be noticed that the correlations are twice as strong between the mid-latitude westerlies and N^2 as the high-latitude westerlies. In all tables and graphs of correlation coefficients a dotted line or circle indicates a correlation significant at the 5% level, while solid indicates the 1% level.

Figure 7 shows that when correlation coefficients are run for the 10 year daily averages grouped by months against the two westerly indices, 35 out of the 96 are significant at the 5% level or better, and 15 of these are significant at the 1% level or better. The apparently random fluctuations from plus to minus coefficients at all latitudes needs much further study and explanation.

The daily $1/N^2$ values were correlated with westerly indices at various latitudes for each month of 1928 (Table 2) and for the years of 1928 and 1929 (Table 3). The 20^o index shows the strongest correlations with $1/N^2$ and <u>all</u>



CORRELATION COEFFICIENTS

Figure 7

latitudes. In fact, it is far better than that between $1/N^2$ and the westerly index at its own latitude. Certainly the great concentration of high coefficients from May through August in Table 2 at all latitudes except 60° is interesting since these are the months of lowest N^2 (highest $1/N^2$) at all latitudes except 60°. Furthermore, there is the persistent and sometimes annoying shift from plus to minus which has plagued most meteorological analysts since Sir Gilbert Walker[10,11] and probably before.

Table 2

Correlation Coefficients
Between 20° Westerly Index
and $1/N^2$ by Months, 1928

		I		III		V		VII		IX		XI	
20°	$1/N^2$	-.24	.07	.23	-.26	.46	.42	.26	.61	-.20	-.03	.07	.46
30°	$1/N^2$	.28	.26	-.21	-.30	.50	.25	-.51	.36	-.40	.04	.28	.36
45°	$1/N^2$	.32	.30	-.07	-.07	-.04	-.44	-.16	.62	.06	-.05	.56	-.39
60°	$1/N^2$	.42	-.29	.23	-.31	.10	.25	.08	.34	-.25	.08	.15	-.40

Table 3 shows that the only place $1/N^2$ fails to correlate significantly with westerly index is at the lower latitudes with mid- and high-latitude indices. Obviously, the best index we can find, as far as correlations between it and $1/N^2$ are concerned, is the westerly index at 20° and not that at the mid and high latitudes as might be assumed.

C....magnetic activity numbers?

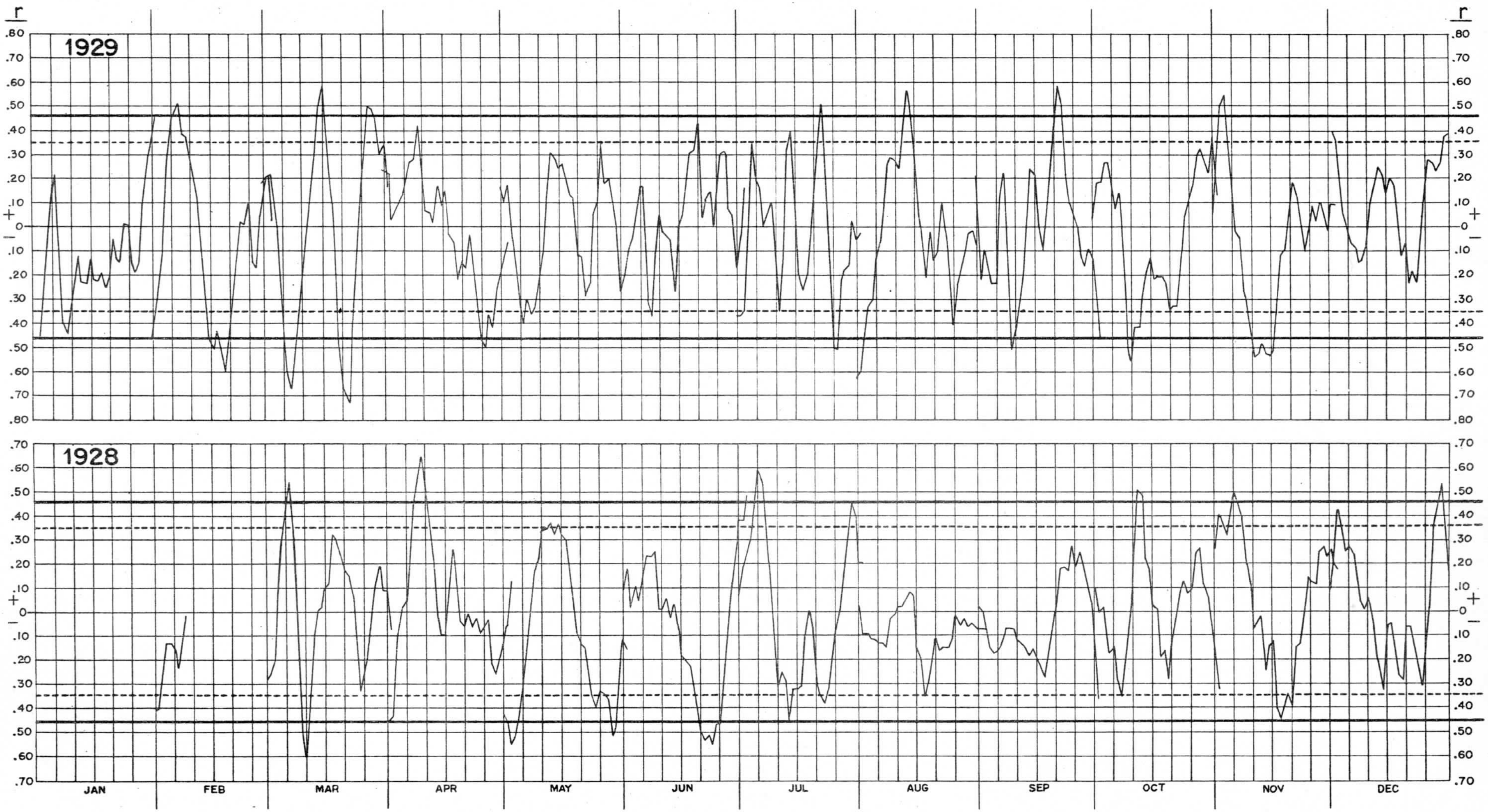
On plotting $1/N^2$ vs. C. (magnetic activity #)[12], there was an apparent relationship between the two when the C value of 14 and 21 days before the $1/N^2$ were plotted against the $1/N^2$ of the date. (A very fast and convenient method of plotting a 2 variable graph on the IBM 416 has been worked out by The University of Wisconsin Numerical Analysis Laboratory).

Table 3

Correlation Coefficients
Between Westerly Index and $1/N^2$ by Years

		$1/N^2$ 20°	30°	45°	60°
1928 Index	20°	.60	.52	-.24	-.46
	30°	-.29	-.30	.14	.24
	45°	-.25	.12	.23	.23
	60°	.01	.04	.22	.13
1929 Index	20°	.64	.60	-.19	-.43
	30°	-.15	-.20	.10	.16
	45°	.05	.00	-.10	-.14
	60°	-.03	.02	-.14	-.14

Table 4 is the result of the correlations run for those two "lag" periods and two latitudes using data from 1928. The sample is rather small, but again the most significant ones are concentrated in the summer months.



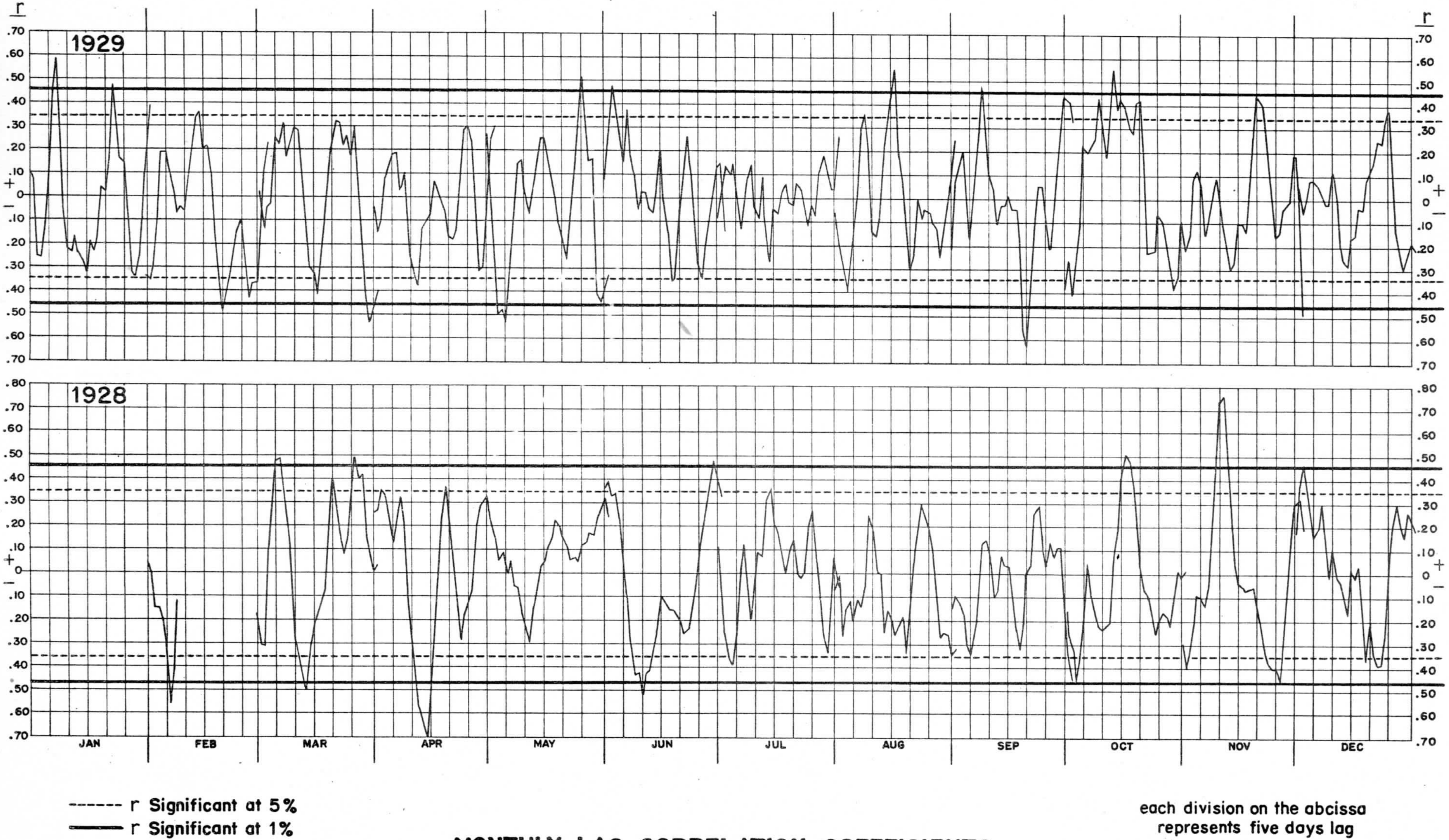
------- r Significant at 5%
——— r Significant at 1%

each division on the abcissa represents five days lag

MONTHLY LAG CORRELATION COEFFICIENTS

BETWEEN SUNSPOT ACTIVITY (0-32 DAYS PREVIOUS TO DATE) & N^2 VALUES AT 40° N. LATITUDE

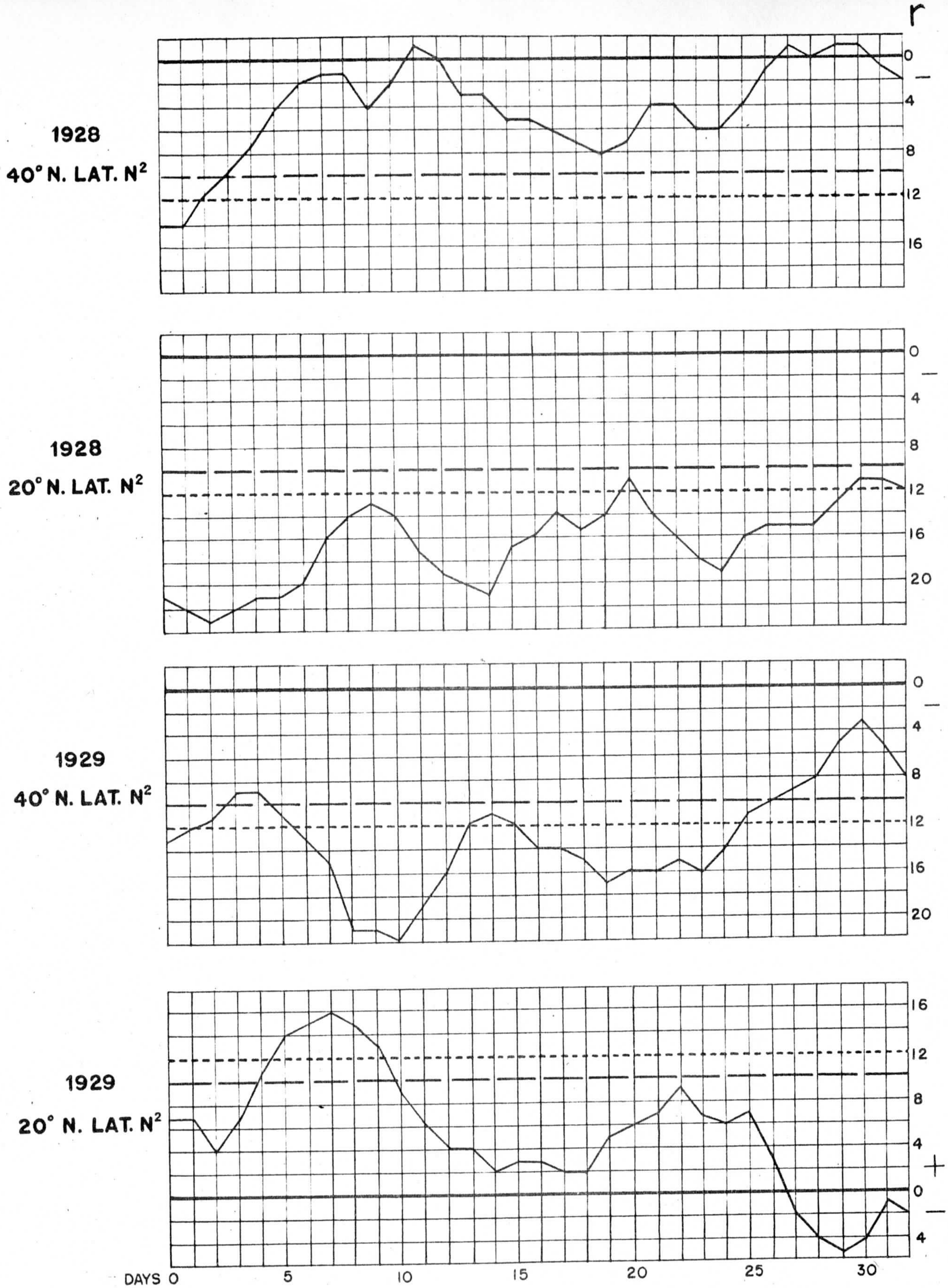
Figure 8



MONTHLY LAG CORRELATION COEFFICIENTS

BETWEEN SUNSPOT ACTIVITY (0-32 DAYS PREVIOUS TO DATE) & N^2 VALUES AT 20° N. LATITUDE

Figure 9



——95% confidence level
------99% " "

YEARLY CORRELATION COEFFICIENTS between SUNSPOT ACTIVITY (0-32 DAYS PREV. TO DATE) and N^2

Figure 10

In an attempt to eliminate an apparent 10-11 day cycle in N^2, a 10 day running mean of N^2 centered on the 5th day was run and then subtracted from the appropriate N^2 value before correlation with 11 and 14 day lags of C#.

On an annual basis none of the coefficients obtained were significant, and on a monthly basis, (30 cases), only one of 24 was significant at the 5% level. See Tables 5 and 6.

D....sunspot index numbers?

Table 4

Correlation Coefficients
Between 20° $1/N^2$, 45° $1/N^2$ and
14 and 21 Day Lags of C#

	Jan	Feb	Mar	Apr	May	Jun
45° $1/N^2$ v. 21 D.L. C#	.32	-.13	.16	.42	.13	.11
20° $1/N^2$ v. 21 D.L. C#	-.02	-.15	.19	.40	.26	-.52
20° $1/N^2$ v. 14 D.L. C#	-.33	.05	.08	-.28	-.06	.18
	Jul	**Aug**	**Sept**	**Oct**	**Nov**	**Dec**
45° $1/N^2$ v. 21 D.L. C#	.28	-.20	-.28	-.36	-.18	.12
20° $1/N^2$ v. 21 D.L. C#	-.10	-.47	.25	.01	.05	.16
20° $1/N^2$ v. 14 D.L. C#	.45	.07	-.31	-.09	.09	.13

Figures 8, 9, and 10 give over 1600 correlation coefficients between 40° N^2 vs. from 0-32 days lag C# for both the annual and monthly correlations, (365 and 30 cases respectively), for 1928 and 1929; and those for

20° N^2 vs. the same C#s for the same periods. There are considerably more significant cases at 40° than at 20°, but in either case, one can see why little relationship between the two variables has been shown in the past. Most of the strongest relationships show up not on the same day as the sunspots occurred, or even at the same relatively constant period of lag, (except during certain short seasonal periods) but at varying periods throughout each month. A further complicating fact is the constant shift from plus to minus coefficients in a somewhat rhythmic fashion, but at varying intervals both between and within the months.

Considerably less than half of the monthly coefficients are positive. However, all the correlation coefficients (on an annual basis) at 40° N that were over .01, were negative. The annual coefficients deserve much further study because of the apparent great shift in importance of the sunspots from 20° to 40° from 1928 to 1929.

E. General discussion of correlation coefficients obtained

According to probability theory[13, 14] the 99% confidence level is attained for 365 cases with correlation coefficients in excess of approximately .12, and the 95% level with coefficients in excess of approximately .10. With monthly correlations containing about 30 cases the coefficients must be approximately .35 and .46 to provide the confidence levels of 95% and 99% respectively. In all of these figures, theory demands that our samples of N^2 and westerly indices

Table 5

Yearly Correlation Coefficients Between
A = (10 Day Running Mean N^2)-(N^2)
and C, with 11 and 14 Day Lags
for 1928 and 1929

Year	20°(A) v.14C	20°(A) v.11C	30°(A) v.11C	45°(A) v.14C	45°(A) v.11C
1928	-.003	.003	-.041	-.093	-.045
1929	.036	.067	-.093		

Table 6

Monthly Correlation Coefficients Between
A = (10 Day Running Mean N^2)-(N^2)
and C, with 11 and 14 Day Lags
for 1928

Month	45°(A) v.14C	20°(A) v.11C
Jan.	-.378	.046
Feb.	.030	-.113
Mar.	-.058	-.101
Apr.	-.302	.261
May	.006	.063
June	.321	.253
July	.285	-.317
Aug.	.169	.276
Sept.	-.303	-.041
Oct.	-.180	.230
Nov.	.154	-.202
Dec.	-.016	-.071

are random samples from the near infinite populations of these elements so that they are truly representative cases and not unduly influenced by outside factors not usually found in the total population of elements. This may not always be the case, but we assume that it is fulfilled nearly enough so that we may use them as random samples.

The C# was also correlated with the ranked R_z (sunspot #) and the correlation is rather weak when taken by days for each month. However, when the days are used for annual correlations, 1929 is significant at the 1% level while 1928 was not significant at all. This is in direct contrast to authors[15] who point out a fairly good correlation between the two.

We have little way of knowing whether we may be getting good correlations between secular or annual trends of magnetic activity or sunspots and our various N^2 parameters. However, no annual cycles for either have been reported, although there certainly is a very prominent cyclical quality to N^2. Conversely no monthly cycles can be shown for N^2 while there is a fairly good cycle in sunspots of approximately a month due to the 27 day rotation period of the sun.

It might be pointed out that most of the significant correlation coefficients in figures 7 and 8 are in runs up to 7 in length and that these may be an indication that if these trends are each counted as one significant coefficient, we would have very few of them. However, from the argument above, it is highly improbable that cycles in both variables are being correlated. To prove the importance of the coefficients obtained, an analysis was made of the average length of the runs of significant coefficients as well as the length of plus and minus runs, and when an average run is taken and then the number of significant runs considered, there are over 5 times as many significant runs as we might expect to find if the entire sample were made up of completely random sub-samples from the original population of elements.

It is interesting to note that N^2 is more closely related to the westerly index, as well as to magnetic activity and sunspots, during the summer months than at any other general period of the year.

From our figures 8 and 9, it appears that if we consider just the plus coefficients or just the minus, one can see a general cyclical trend of from 10-15 days with an average very close to 13 days between one peak and the next.

Figure 10, showing the annual correlations between 0-32 days previous sunspot activity and N^2 values, shows that at 40° there was a significant relationship between only 0, 1, and 2 days previous sunspot activity and N^2 of date in 1928. In 1929 all days from 0 through 26, (except 3 and 4 which are very close to the 5% level) are significantly correlated. It is interesting to note that at 20° the situation is almost exactly reversed for the same periods of time. Although the figures for 1929 at 40° seem to show the most significant relationship at about 9 and 19 days, it is obvious that the relationships between N^2 and solar activity as measured by sunspots is a very complex one.

All of the significant correlations at both 20° and 40° are negative for 1928; but for 1929 all of the significant ones are negative at 20° and the few that are significant at 40° are positive. While the relationship between the variables shown by these coefficients may not be as strong as the others, if there were no relationship one would expect the coefficients to vary from plus to minus in a random or irregular fashion. Since they vary in a smooth, almost predictable fashion, we think that although the relationship may be progressively weaker or stronger as the case may be, it still exists, despite the fact that we can't place quite 95% or 99% confidence in its not being due to chance.

V. Conclusions and Summary

Although no way has been found to subjectively state what N^2 is for a given synoptic situation because of its peculiar qualities in averaging the effects of long and short waves, it has properties which seem to render it a valid and useful number. Although daily and weekly values vary considerably, monthly means vary little from year to year; and variations with latitude are what we might expect except for the apparent southward extension of the zone of maximum range of N^2 which seems to lie nearer 30° than 40° and above as we might have expected.

$1/N^2$ correlated very well with westerly index as it should, but the best correlations of $1/N^2$ at <u>any</u> latitude are between the index at <u>20°</u> and that particular value, instead of the index at the same latitude as $1/N^2$ as one might have expected. This indicates that more importance should be given to both N^2 and the westerly index at 20° as affecting perturbations as far north as 50° instead of relying exclusively on the most commonly used mid-latitude index.

The correlations between N^2 and C# are extremely interesting, since a large percentage of the coefficients surpass the 95% and 99% confidence levels in a very peculiar cyclical fashion, suggesting something of a 13 day cycle in some aspects of both solar activity and N^2.

VI. Suggestions for further research

The use of N^2 suffers most from the lack of possibility of subjective interpretation and since we believe no methods can be invented to accomplish this goal with the present N^2, the number should be altered in some way as to subtract out the short wave effect or long wave effect so that some sort of subjective representation is possible. This would be desirable for most purposes.

Appendix A includes a listing of all IBM card decks available as well as their listings and the listing of other data run in the course of this project.

References

1. Jacobs, W.C., "Wartime Developments in Applied Climatology," Meteor. Monographs, Vol.1, No. 2, 1947.

2. Elliott, R.D., "Extended Range Forecasting by Weather Types," Compendium of Meteorology, Am. Meteor. Society, Boston, 1951.

3. Bryson, R.A., Kuhn, P.M., Lahey, J.F., et. al., Scientific Reports now in progress, University of Wisconsin.

4. Wahl, E., Verbal communications, February, 1955.

5. Sokolnikof, I.S. and E.S., Higher Mathematics for Engineers and Physicists, McGraw-Hill, pp.61-82, 1941.

6. Wadswort, G.P., Computation of D_1 and S from the M.I.T. Surface Pressure Deck.

7. Rossby, C-G., and Collaborators, "Relation between Variations in the Intensity of the Zonal Circulation of the Atmosphere and the Displacements of the Semi-Permanent Centers of Action", Jour. of Marine Res., 2:38-55, 1939.

8. U. S. Weather Bureau, "Daily Series, Synoptic Weather Maps, Northern Hemisphere Sea Level and 500 mb Charts," var. Years.

9. Frenzen, P., "Westerly Flow Past an Obstacle in a Rotating Hemispherical Shell", Bulletin of the Amer. Met. Soc., Vol. 36, No. 5, May, 1955.

10. Walker, Sir Gilbert, "Correlations in Seasonal Weather V. Sunspots and Temperature," Mem. Indian Meteor. Dept., 21: pp 61-90, 1915.

11. Huntington, E., Earth and Sun, Yale University Press, 1923.

12. Mitra, S.K., The Upper Atmosphere, The Asiatic Society, India, 1952.

13. "Significance of Correlation Coefficients", Air Weather Service Bulletin, September, 1954.

14. Brooks, C.E.P., Carruthers, N., Handbook of Statistical Methods in Meteorelogy, London, pp. 109-110, 1953.

APPENDIX A

IBM Cards Available for Further Study, and Listings

Data and IBM Decks Available for Further Study

	Cards	Listings
1925-1934 Original D_i^2 and S^2	x	
" " Computed N^2, Daily	x	x
" " Monthly Mean N^2 with Sunspot and C#	x	x
" " 5 Day Running Sum N^2	x	
5 Day Running Mean N^2, 192902 and 192802	x	
" " $1/N^2$ plus Index, C# lags (not all lats.)	x	
" " 10 Day Running Mean N^2 (not all lats.)	x	x
10 Year Daily Mean N^2 with mid and high lat. Index	x	x
Difference Between 10 Day Running Mean N^2 and N^2 of Date, Several C# lags, R_z (not all lats.)	x	
C# Lag Deck (0-31 Days) and 20^o Index, 1928 only	x	
Correlation Coefficients from figures 7 and 8	x	x
R_z Lag Deck (0-32 Days) and 20^o, 40^o Indices, C# of Date, and 20^o, and 40^o N^2, 1928 and 1929	x	
Correlation Coefficients from Figure 6 and Including over 600 of 20^o Index v. C# Lags		x