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Department of Meteorology
University of Wisconsin
February 1960

SCIENTIFIC REPORT NO. 1

AN EXPERIMENT IN THE INTERPOLATION OF PRESSURE CONTOUR PATTERNS IN AREAS OF MISSING DATA

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GENERAL INTRODUCTION

The basic problem of analysis and forecasting on a global scale is the lack of data over large areas of the world. This is quite apparent when one tries to analyze, in detail, the area covered by the Pacific Ocean or portions of Asia.

The purpose of this study is to investigate a method of interpolating the pressure pattern into an area of missing data from adjacent areas of known data. The method used in this study is to describe the known portions of the northern hemispheric 500mb flow by the use of orthogonal polynomials as described by R. A. Bryson and P. M. Kuhn (1956). This method reduces tho 500mb contour pattern to a mathematical formula, which if converted back to the original map will usually contain 95% of the variance of the original 500mb chart. By the use of the variable coefficients of the orthogonal polynomials, the values for these same coefficients can be interpolated for the silent area. This study indicates that a silent area chart can be produced by the method described herein with an average accuracy of 88% or greater.

The basic premise of this approach is that all weather patterns are themselves globally inter-dependent, therefore, the coefficients of the known area must reflect at least the main conditions in the missing area.

The basic data utilized was obtained from the U. S. Navy project AROWA map series. This data, being on IBM cards, made it possible to utilize the IBM 650 electronic computer.

The data contained contour heights for the 500mb surface picked off at selected grid points. The intersections of latitude and longitude ending in zero and the intersections of latitude and longitude ending in five were used as a grid for this deck.

The card deck for February, April, July and October of the year 1951 was used. These months were selected because it was felt they represented the four seasons quite well.

SECTION I
DEVELOPMENT OF THE BASIC METHOD

INTRODUCTION:

The basic principles of orthogonal polynomials will not be covered in detail in this study. The reader may refer to Essenwanger, Horn and Bryson (1958).

One of the earliest if not the first investigator to apply the orthogonal polynomials to weather patterns was Wadsworth (1948). By utilization of the Tschebycheff orthogonal polynomials, Wadsworth mathematically described the surface pressure systems. This basic procedure will be utilized to describe the 500mb patterns. This study will attempt to extend the coefficients of the orthogonal polynomial into an area of unknown data.

FORM & PROPERTIES OF ORTHOGONAL POLYNOMIALS:

The orthogonal polynomials have the following general form:

$$h = \bar{h} + a f(x) + b f(x^2) + c f(x^3) + \cdots$$
$$+ d f(y) + e f(y^2) + \cdots$$
$$+ g f(xy) + j f(x^2 y) + \cdots$$

Where a, b, c, d, etc. are the non-normalized coefficients of the polynomial and f (x), f (y), etc. are the fixed mathematical functions of horizontal distance from an arbitrarily chosen point.

The following two special properties of the orthogonal polynomials should be noted.

1. Each coefficient may be computed independently of all other coefficients.

2. The coefficients are functions of time, height and sector considered.

From these properties it is evident that a comparison of thickness changes, computation of vorticity and zonal and meridional index may be made quickly and easily by use of these coefficients. The properties of the orthogonal polynomials mentioned up to this point are interesting but of no direct use in the determination of the basic pattern that exists in the silent area. The real test lies in the answer of the question, "Do the coefficients of the orthogonal polynomial, at one level and time, vary in a reasonably orderly fashion when computed for various sectors around the northern hemisphere?." This study attempts to answer this question but before continuing further the data collection, computation methods and definitions of various terms should be understood.

GRID SIZE:

For the initial phase of this investigation a 37 x 11 diamond grid of data points was used. This extended from 20°N thru 70°N and was 180° of longitude in width. The data points were at the intersection of the even (0) latitude and longitude lines and at the intersection of the odd (5's) latitude and longitude lines.

The final phase of this investigation used a reduced 6 x 11 diamond grid of data points. This area was 30° in width and extended from 20°N to 70°N. See Appendix A. The reason for the adoption of this smaller area will become apparent in the later sections.

COMPUTATION METHODS: FORMULAE AND TABLES:

Appendix B contains all the data necessary to perform the computations to solve for the coefficients of the orthogonal polynomials. Appendix C contains the tables and formulae necessary for the reconstruction of the basic pressure field from the coefficients.

These computations may at first seem tedious and time consuming; however, by use of electronic computers this time is reduced to minutes. Without the electronic computers it may well take one man several days to obtain the coefficients for one 30° sector and several days more to obtain the reconstructed 30° sector from these coefficients.

DEFINITIONS:

Standard deviation factor - Is an expression of the variance of the pattern considered from the mean, i.e., the root-mean-square of the departures.

Non-normalized coefficients - These are referred to as the U's and are the raw orthogonal polynomial coefficients.

Normalized coefficients - These are referred to as the Z's and are the non-normalized coefficients divided by the standard deviation factor.

SECTION II
FEASIBILITY TEST

INTRODUCTION:

Before going into the development of a special computer program for this silent area analysis method, it was felt that an existing program, using an 180° sector of the northern hemisphere, should be tested. This program was modified to shift 10° east and compute the coefficients for the new 180° sector. By successive 10° shifts the 180° sector was moved around the northern hemisphere back to its original location which produced 36 values of each coefficient. A graphical analysis was made of these 36 successive values. See Graph I.

DATA UTILIZED:

To obtain what was felt to be a representative sample of all seasons, the first three days of February, April, July and October for 1500Z, 1951 were used.

The year 1951 was chosen as the best available historical weather series in IBM card form.

The number of maps computed for the test was limited to 12 due to the relatively long 650 computer time required for each map. Approximately 20 minutes was required to compute the complete set of shifted coefficients using this 180° sector program.

DATA ANALYSIS:

The 36 values of each coefficient were plotted and a smooth curve was drawn between successive points. (See Graph I) All coefficients of the functions f (Y) through f (Y^4) and f (X) through f (X^{12}) were plotted on a separate graph for each function and all three successive days were also plotted on the graph. This gave four groups of graphs, for the four selected months, each chart in a group containing three curves representing the first three days of each month.

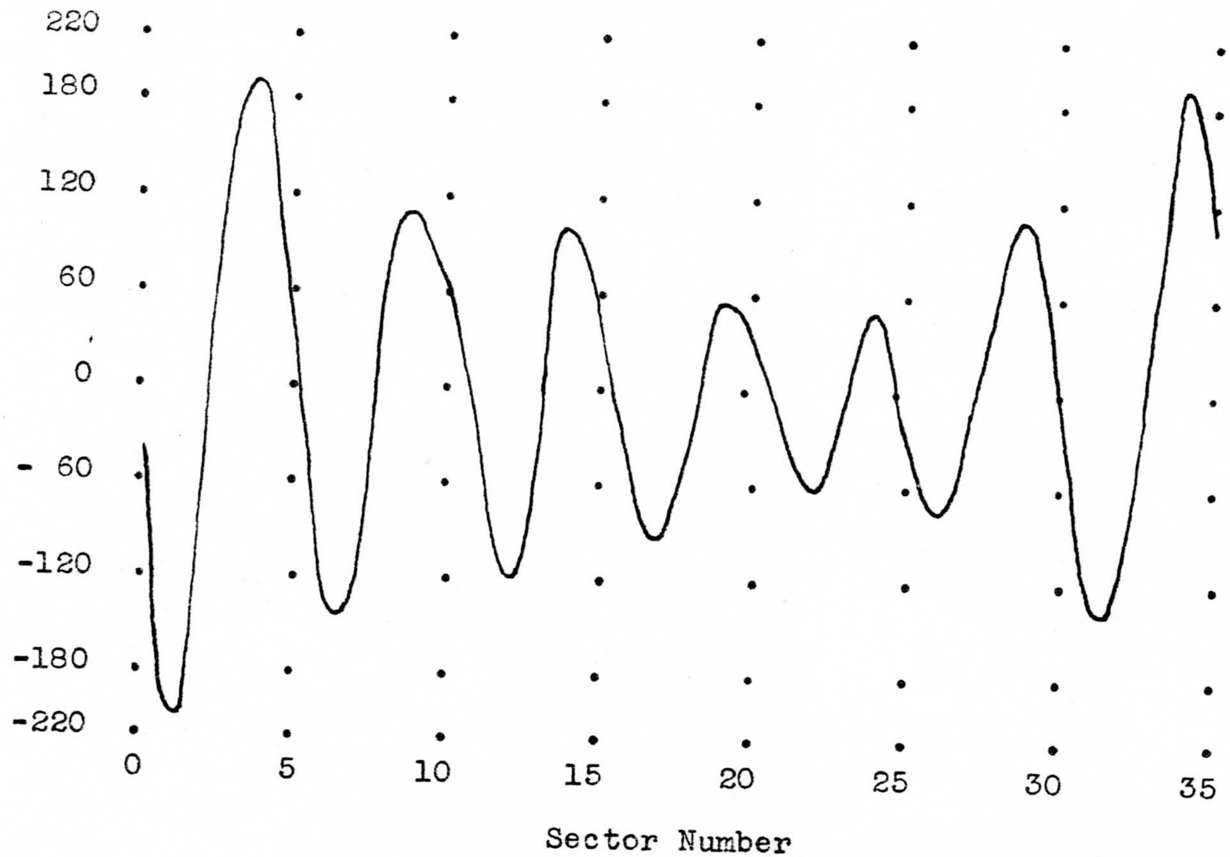
RESULTS:

These graphs were examined for periodicity, characteristic types, and for apparent inter-relationships between the coefficients and the maximum and minimum values observed. From the analysis of the graphs of the 180° sectors, shifted 10 degrees, the following became apparent:

1. The Y (north-south slope) coefficient consistently followed a pattern of a sine or cosine wave of one wave length.

2. The most over-powering term was found to be the north-south slope (Y) contributing more than 90% of the explained variance.

3. The X^9 coefficient followed an interesting pattern around the northern hemisphere which approached an amplitude modulated wave. See Graph I. The maxima and minima shifted



Graph I. - Coefficient of f (x^9) term, for 1 February 1951, using 180° sector program.

gradually with time indicating this was not a feature resulting from lack of data.

4. A few of the higher order terms varied rather irregularly. However, they do not contribute greatly to the explained variance of the over-all pattern.

The patterns and consistent movements of the peaks and troughs of the coefficients were encouraging and it was felt that the reconstruction of a silent area by application of this method was feasible. The basic problem was to select an area sufficiently small to give a maximum number of known sectors with a minimum of unknown sectors in the 120° silent area. The choice of 120° for the width of the silent area was based on two considerations; first, this area would cover the Pacific Ocean where the routine data is relatively sparse, second, the area is about the same size as that from central Europe east to Japan, an area from which reliable data may be missing. A second consideration was apparent from the analysis of the 180° sector data. A reliable relationship must be established between the coefficients and their respective changes. This led to the consideration of sectors of various widths.

Based on the fact that the long wave patterns tend toward a 60° wave length it was felt that a 30° width sector would be the best choice. This gave an easily interpretable relationship between the coefficients and the actual weather pattern being described. These relationships will be elaborated in more detail in the next section.

SECTION III
30° SECTOR SILENT AREA ANALYSIS METHOD

INTRODUCTION:

This section deals with the method used to extend the coefficients of the orthogonal polynomial from areas of known data into areas of missing data. The method is based primarily on the findings of the previous analysis of the coefficients and their mathematical relationships to the pattern being described. These relationships, previously mentioned, were used as consistency checks on the interpolations.

METHOD AND RESULTS OF ANALYSIS:

The basic approach was to graph each non-normalized coefficient (17 U's)* as a function of sector number. The first sector (0) began at 100°E and extended to 130°E. There were 35 ten degree shifts made eastward to return to the original sector, numbered from 0-35. In addition to these graphs, graphs of the sector means $(\bar{H})$*, cross product standard deviation factor (S_c)* and the complete standard deviation factor (S)* were plotted. Each shifted sector contains 1/3 new data and 2/3 of the data from the previous sector due to the 20° over-lap caused by shifting a 30° sector by ten degrees.

* See Appendix B

To obtain some feel for the coefficients and to test the basic assumption on which the 30° sector was chosen, i.e., that 30° of latitude is approximately half the wavelength of major systems in mid-latitudes, a complete plot was made of all sectors for the northern hemisphere. The data used for this was the 1st, 2nd and 3rd days of February, April, July and October, 1951.

From this data the following general relationships were obtained:

1. The coefficients of the (X) terms have a definite wavelength of 60° to 80° (6 to 8 sectors).

2. The coefficients of the (Y) term (north-south slope) behave in a predictable manner, having a frequency of two waves for the complete northern hemisphere. This variation was expected since the (Y) term represents the zonal index. The two maxima represent the areas of greatest zonal wind flow, i.e., the areas on the south side of the two major sub-polar lows.

3. The trends of the higher powers of (X) also reflect variations similar to those of the zonal index term. They are found to follow a basic hemispheric sine wave with oscillations of a wavelength of 20° to 30° (2 to 3 sectors) superimposed on this trend.

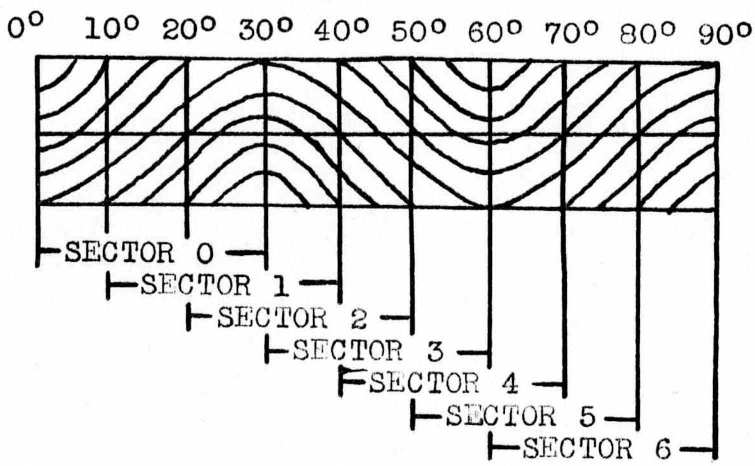
4. The 30° sector chosen proved to be of considerable benefit in the interpretation of the weather pattern being

described by the coefficients. This made it possible to interpolate the simpler terms, X (meridional index), Y (zonal index) and S (standard deviation factor), first and from this relationship interpolate more accurately the higher order terms.

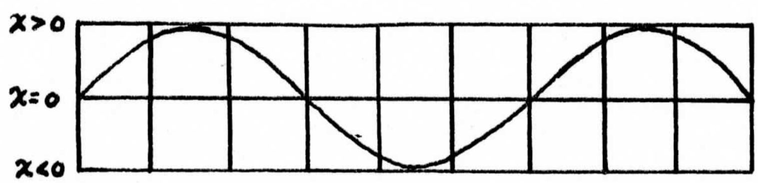
The following diagram provides a highly idealized model of the relationship of the X coefficients to the pattern described. The relationship of the Y terms to each other are analogous to those of the X terms.

DIAGRAM

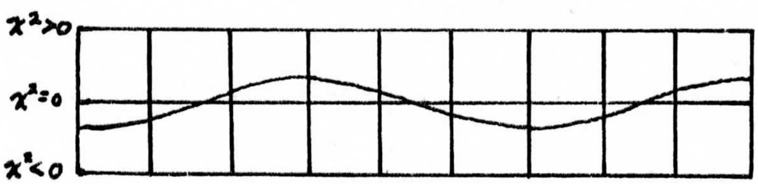
Remarks



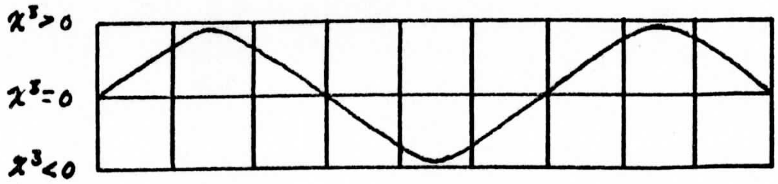
Smooth wave pattern with 60° wave-length



Slope down from west to east arbitrarily taken as negative



Concave downward parabola arbitrarily taken as positive



Slope to the east of the cubic taken as negative.

5. The standard deviation factor (S) was found to be a smooth and easily interpolated curve. The wavelength of this term varied from one cycle for the northern hemisphere in the winter months to three cycles in the summer and fall months.

6. A rapid drop or increase in the standard deviation factor was most often found to be associated with the same type changes in the (Y) coefficient. This is to be expected since the north-south gradient contains a large fraction of the variance.

When an exception to this was found the (X) (east-west) term followed the standard deviation factor and became the stronger coefficient (meridional circulation dominant).

<u>METHOD OF INTERPOLATION:</u>

Except for terms such as the Y (north-south) and S (standard deviation factor) all terms give at least 4 complete cycles of the coefficient outside the silent area. The silent area is found to have at least two cycles to be interpolated. The combination of at least 6 cycles of the coefficients for the northern hemisphere results from the choice of a 30° sector width with the major waves of the pressure pattern having a 60° wavelength.

Assuming the frequency in the unknown area was the same as indicated in the known area, it was possible to interpolate the positions of the peaks and troughs. The data being cyclic requires that the number of waves around the hemisphere must be a whole number. If the wavelength of the data in the known area divided into 360° is not an integer, the wavelength of the silent area must be adjusted so the total number of waves is an integer.

The second and possibly the most important check on the frequency of the coefficients in the silent area is through the relationship between coefficients, assuming an ideal model of a 60° wavelength pattern. This relationship consists of the fact that when certain terms, e.g. (Y), are increasing other terms, e.g. (X), should be decreasing to near zero. From this inter-relationship it is possible to check each term against its counterpart to detect errors in the interpolation.

Up to this point the discussion has covered only the interpolation of the frequency or wavelength of the coefficient in the silent area. We now must consider the most subjective part of this interpolation, that of determination of the amplitude or value of the coefficient at various points.

It was found in the analysis, as mentioned previously, that many of the higher order terms tended to have superimposed on them a long wave pattern of one to two cycles which was very useful in this interpolation. By following the trend produced by successive peaks or troughs in the plot of the coefficients, it was possible to interpolate subjectively into the silent area. By use of this technique it was possible to approximate the value of the coefficients at their peaks or troughs.

The normalized coefficients (Z's) are obtained by dividing the non-normalized coefficients (U's) by the standard deviation factor (S). As a result of this relationship the sum of the squares of the Z's gives the explained variance of the pressure surface. This relationship is of prime importance in the development of internal consistency checks on the method of interpolation.

The explained variance of the description of the known areas varies from 85% to 100%, therefore, the interpolated explained variance must fall within the same range. This means that if the explained variance of the interpolated section is greater or less than this range the interpolation must be in error either in one of the non-normalized coefficients (U's) or in the value of the standard deviation factor (S). This consistency check, along with the consistency checks arising from the interrelationships of the various coefficients, is used to determine the existence of an error in the interpolation.

It was found that the cross-product coefficients were the most difficult to interpolate as to frequency and magnitude. However, they were found to contribute less than 6% of the total variance and presented less of a problem than first indicated.

ERROR ANALYSIS OF THE METHOD:

Tables Ia and Ib list the accuracy of the polynomial description based on all data being known for each adjacent sector, i.e., no overlap, in the northern hemisphere for the twenty days considered.

It should be noted that some sectors show low explained variance, one as low as 88%. This is not, however, a real deficiency in the system in that these sectors contain systems of relatively flat pressure gradients and are described well even by this low percentage. The explained variance is found in most cases to range from the mid to upper 90's. Adjacent sectors 0 and 3, sector 0 from 100°E to 130°E and sector 3 from 130°E to 160°E, were reconstructed to obtain an estimate of how good the reproduction was. A comparison of the actual chart to the reconstructed chart may be made by comparing Chart I and Chart II. The explained variance for sector 0 was 99.58% and for sector 3 the explained variance was 97.04%. The orthogonal polynomials describe this pressure pattern in detail.

TABLE Ia

PER CENT REDUCTION BY ADJACENT 30° SECTORS
FOR FEBRUARY & APRIL 1951

Sector No.	Sector Area of Coverage	FEBRUARY 1	FEBRUARY 2	FEBRUARY 3	FEBRUARY 4	FEBRUARY 5	APRIL 1	APRIL 2	APRIL 3	APRIL 4	APRIL 5
17	90°W- 60°W	99.7	98.7	98.3	98.2	98.4	92.8	95.1	94.8	95.2	95.3
20	60°W- 30°W	99.9	99.3	99.1	98.7	99.9	93.1	92.0	97.1	95.4	95.3
23	30°W- 0°	99.8	99.8	99.2	99.1	100.0	96.4	96.4	98.8	98.1	96.7
26	0° - 30°E	93.9	91.6	94.9	98.3	97.6	96.9	96.4	97.6	96.1	96.4
29*	30°E- 60°E	97.2	95.1	95.5	94.8	91.8	94.4	93.8	93.3	95.9	96.1
32*	60°E- 90°E	98.1	98.1	97.9	97.0	96.4	94.4	94.5	97.0	95.8	95.8
35*	90°E-120°E	98.9	98.1	98.9	99.1	98.3	98.2	98.3	99.3	98.5	99.0
2*	120°E-150°E	98.2	99.2	96.7	97.9	97.2	98.6	98.3	98.4	97.7	98.3
5	150°E-180°	98.4	99.0	97.2	98.1	98.3	97.8	98.5	99.1	99.4	99.2
8	180° -150°W	100.0	99.2	99.7	96.6	95.7	96.3	99.1	99.1	99.5	99.1
11	150°W-120°W	98.7	99.6	99.7	99.4	98.2	95.6	95.9	97.2	96.5	97.1
14	120°W- 90°W	96.7	99.8	98.6	98.6	98.7	93.5	96.0	97.1	94.9	96.4

* Percent reduction by the orthogonal polynomials of sectors within which the 120° silent area is located.

TABLE Ib

PER CENT REDUCTION BY ADJACENT 30° SECTORS FOR JULY & OCTOBER 1951

Sector No.	Sector Area of Coverage	JULY 1	JULY 2	JULY 3	JULY 4	JULY 5	OCTOBER 1	OCTOBER 2	OCTOBER 3	OCTOBER 4	OCTOBER 5
17	90°W- 60°W	99.1	98.4	98.7	96.3	97.0	98.7	99.0	98.5	96.8	95.5
20	60°W- 30°W	98.2	97.8	95.7	97.5	98.5	99.0	100.0	99.3	98.7	99.5
23	30°W- 0°	94.5	94.2	96.2	96.4	96.9	96.7	94.4	92.3	98.4	94.9
26	0° - 30°E	95.3	95.5	97.6	99.6	100.0	97.0	92.5	93.2	93.8	94.1
29*	30°E- 60°E	94.5	97.5	98.5	99.7	98.3	99.7	99.9	99.3	97.2	96.2
32*	60°E- 90°E	94.4	93.5	94.1	91.7	93.9	97.2	98.3	98.9	98.1	98.1
35*	90°E-120°E	96.8	97.4	99.5	96.4	97.1	97.1	98.2	98.8	99.6	98.7
2*	120°E-150°E	95.7	88.7	92.9	95.8	96.2	96.2	97.5	98.0	97.9	94.9
5	150°E-180°	98.8	96.0	97.8	96.5	95.8	97.9	98.7	99.2	99.1	99.7
8	180° -150°W	95.4	92.4	97.7	99.3	97.4	99.2	99.2	98.7	98.3	99.3
11	150°W-120°W	94.2	94.5	97.0	92.4	88.0	97.0	96.1	98.5	98.0	97.8
14	120°W- 90°W	96.6	97.2	97.6	95.8	97.2	99.0	98.1	97.2	96.2	94.4

* Percent reduction by the orthogonal polynomials of sectors within which the 120° silent area is located.

CHART I
ORIGINAL SECTOR O AND 3, 500 MB CHART
JANUARY 1, 1951

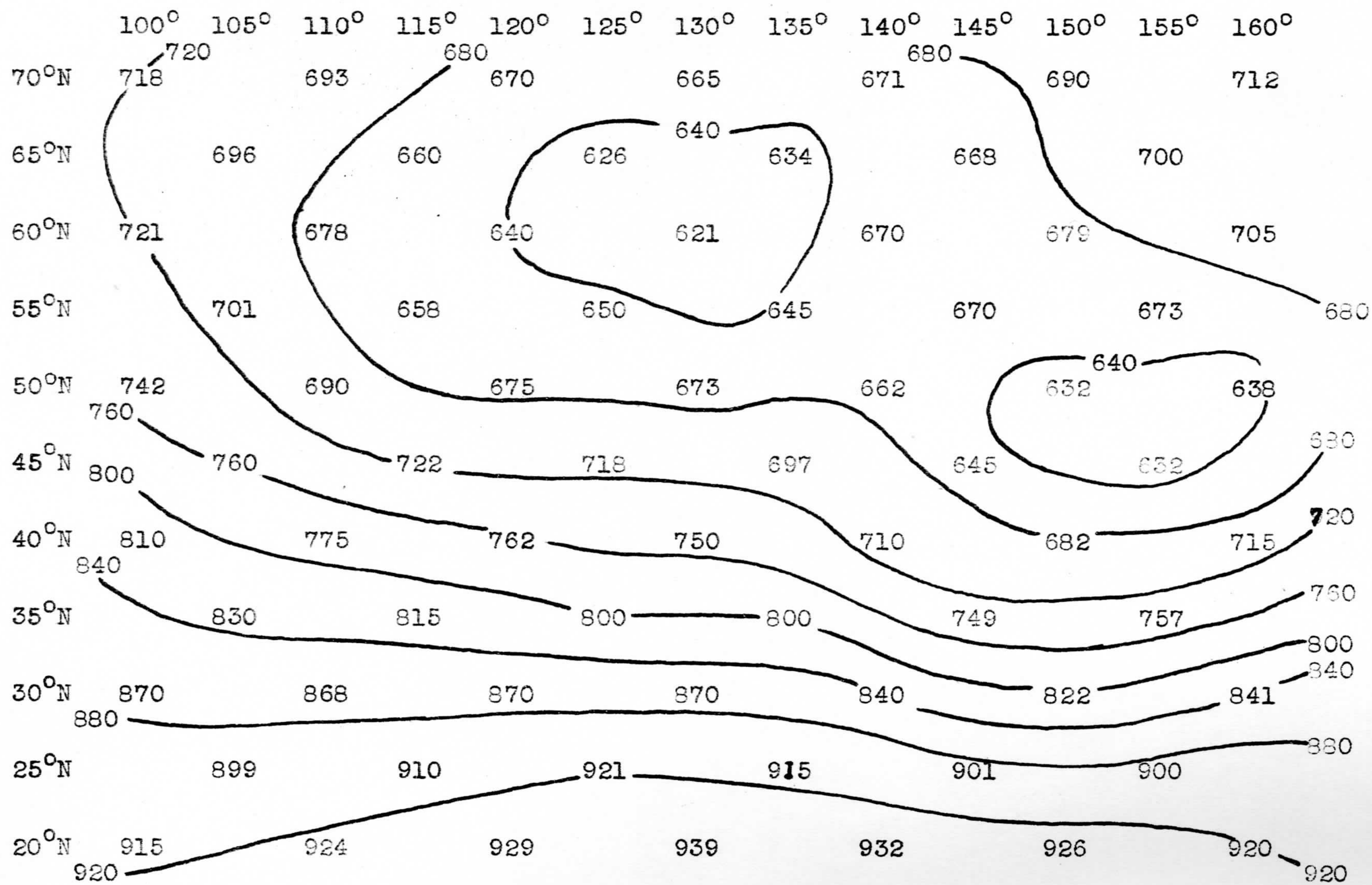


CHART II
RECONSTRUCTED SECTOR 0 AND 3, 500 MB CHART
JANUARY 1, 1951

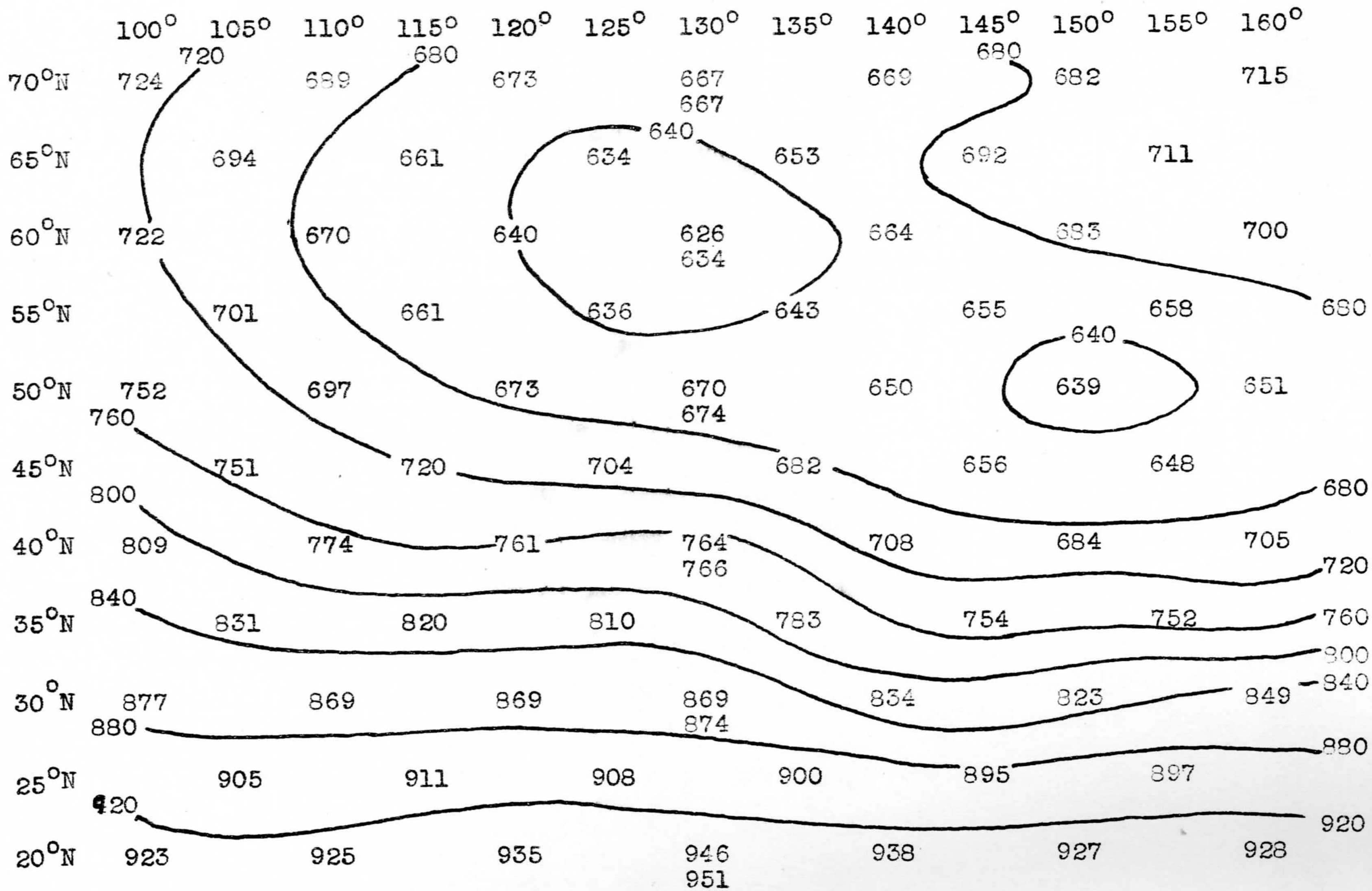


Table II indicates the average accuracy of description in terms of the explained variance for the complete northern hemisphere for each of the twenty days. One significant feature is indicated. That is, the months of predominantly stronger systems (winter) have the greater accuracy of description. Months having weaker systems (summer) were not quite as accurate. Table II also contains the location of the highest and lowest percentage of reduction. The historical weather chart series verified that very strong systems were described better than were the weaker systems.

Since the 4th and 5th days of February, April, July and October 1951 were used for the attempt at interpolating the coefficients, Table III was constructed to indicate the basic accuracy of description of the silent area sector before interpolation. This table in effect tells what error the system must begin with.

As was mentioned in the previous section, the non-normalized coefficients were interpolated as well as the standard deviation factor and the mean height. In order to obtain the normalized interpolated coefficient, it was necessary to divide the interpolated non-normalized coefficients by the interpolated variance. The next step was to obtain some comparison of the interpolated Z's to the actual Z's and, therefore, to the actual map.

TABLE II

EXPLAINED VARIANCE, FOR NORTHERN HEMISPHERE 20°N THRU 70°N, AS DESCRIBED BY THE ORTHOGONAL POLYNOMIALS

DATES	MEAN EXPLAINED VARIANCE, NORTHERN HEMISPHERE	LOWEST EXPLAINED VARIANCE FOR 30° SECTOR		HIGHEST EXPLAINED VARIANCE FOR 30° SECTOR	
		VARIANCE	SECTOR	VARIANCE	SECTOR
1 Feb., 51	98.2%	93.9%	0°- 30°E	100.0%	180° -150°W
2 Feb., 51	98.1%	91.6%	0°- 30°E	99.8%	120°W- 90°W
3 Feb., 51	98.1%	94.9%	0°- 30°E	99.8%	120°W- 90°W
4 Feb., 51	98.0%	94.8%	30°E- 60°E	99.4%	150°W-120°W
5 Feb., 51	97.5%	91.8%	30°E- 60°E	100.0%	30°W- 0°
1 Apr., 51	95.7%	92.8%	90°W- 60°W	98.6%	120°E-150°E
2 Apr., 51	96.2%	92.0%	60°W- 30°W	99.1%	180° -150°W
3 Apr., 51	97.4%	94.8%	90°W- 60°W	99.3%	90°E-120°E
4 Apr., 51	96.9%	94.9%	120°W- 90°W	99.5%	180° -150°W
5 Apr., 51	97.1%	95.3%	60°W- 30°W	99.2%	150°E-180°
1 July 51	96.1%	94.2%	150°W-120°W	99.1%	90°W- 60°W
2 July 51	95.3%	88.7%	120°E-150°E	98.4%	90°W- 60°W
3 July 51	96.9%	92.9%	120°E-150°E	99.5%	90°E-120°E
4 July 51	96.5%	91.7%	60°E- 90°E	99.7%	30°E- 60°E
5 July 51	96.4%	88.0%	150°W-120°W	100.0%	0° - 30°E
1 Oct., 51	97.9%	96.2%	120°E-150°E	99.7%	30°E- 60°E
2 Oct., 51	97.7%	92.5%	0° - 30°E	100.0%	60°W- 30°W
3 Oct., 51	97.7%	92.3%	30°W- 0°	99.3%	30°E- 60°E
4 Oct., 51	97.7%	93.8%	0° - 30°E	99.6%	90°E-120°E
5 Oct., 51	96.9%	94.1%	0° - 30°E	99.7%	150°E-180°

The expression used for this determination of the accuracy is as follows:

$$R = \left[\frac{17\Sigma Z'Z - \Sigma Z'\Sigma Z}{\sqrt{[17\Sigma(Z')^2 - (\Sigma Z')^2][17\Sigma(Z)^2 - (\Sigma Z)^2]}}\right]^2 \cdot \Sigma(Z^2)$$

Where: Z' = is the interpolated normalized coefficient

Z = is the calculated normalized coefficient

17 = is the number of coefficients

R = the lower limit of accuracy of the interpolated coefficients

The equation is a product, the first part being the explained variance of the computed normalized coefficients by the interpolated normalized coefficients. The second part of the product is the explained variance, of the actual pattern, by the computed normalized coefficients. The product of the two parts gives the lower limit of accuracy of the interpolated coefficients. The equation is based on the assumption that the interpolated coefficients are farther from the true coefficients which would describe the pattern perfectly than are the computed coefficients.

Table IV indicates the average and extremes of the accuracy of interpolation for the eight day sample. The lowest accuracy, 88%, was again found to be in the summer months and was expected due to the relatively flat pressure gradients that exist. The highest accuracy obtained,

TABLE III

ACTUAL EXPLAINED VARIANCE FOR SILENT AREA 30° E THRU 150° E BASED ON DATA BEING KNOWN

Dates	Mean Explained Variance for 30°E thru 150°E	Lowest Explained Variance for 30° Sector		Highest Explained Variance for 30° Sector	
		Variance	Sector	Variance	Sector
4 Feb., 51	97.2%	94.8%	30°E- 60°E	99.1%	90°E-120°E
5 Feb., 51	95.9%	91.8%	30°E- 60°E	98.3%	90°E-120°E
4 Apr., 51	97.0%	95.8%	60°E- 90°E	98.5%	90°E-120°E
5 Apr., 51	97.3%	95.8%	60°E- 90°E	99.0%	90°E-120°E
4 July, 51	95.9%	91.7%	60°E- 90°E	99.7%	30°E- 60°E
5 July, 51	96.4%	93.9%	60°E- 90°E	98.3%	30°E- 60°E
4 Oct., 51	98.2%	97.2%	30°E- 60°E	99.6%	90°E-120°E
5 Oct., 51	97.0%	94.9%	120°E-150°E	98.7%	90°E-120°E

TABLE IV

RANGE OF ACCURACY OF INTERPOLATION OF THE 120° SILENT AREA

Dates	Average For Complete Silent Area	Lowest Accuracy For Individual 30° Sector: Accuracy	Lowest Accuracy For Individual 30° Sector: Sector	Highest Accuracy For Individual 30° Sector: Accuracy	Highest Accuracy For Individual 30° Sector: Sector	Remarks
4 Feb., 51	90.52%	80.23%	30°E- 60°E	98.53%	90°E-120°E	
5 Feb., 51	84.97%	72.71%	30°E- 60°E	91.80%	120°E-150°E	
4 Apr., 51	88.31%	74.51%	30°E- 60°E	95.36%	120°E-150°E	
5 Apr., 51	91.87%	87.84%	30°E- 60°E	94.76%	120°E-150°E	
4 July, 51	89.68%	78.72%	60°E- 90°E	95.38%	30°E- 60°E	Actual explained variance low
5 July, 51	76.31%	64.42%	60°E- 90°E	86.74%	30°E- 60°E	
4 Oct., 51	93.23%	88.07%	60°E- 90°E	97.08%	90°E-120°E	Actual explained variance high
5 Oct., 51	93.51%	91.10%	120°E-150°E	95.03%	60°E- 90°E	

93%, was for the month of October and is due to the high percentage of reduction found for the silent area during that particular time.

The overall accuracy of this method (88.55%) as indicated in Table IV seems acceptable for such a large area. It is believed, however, that further refinements can be obtained through the use of climatological models of the coefficients for the silent area. This approach is possible by utilizing the information available with electronic computing machines.

SECTION IV

MACHINE APPLICATION OF METHOD

In operational meteorological forecasting, the limiting factor that determines the usefulness of any mathematical method is the time required to perform the calculations. Even if a system could be developed to provide a perfect (100%) forecast for 24 hours in advance, it would be of little immediate practical value if it took a week to perform the computations. This problem is, however, greatly reduced by the use of the electronic computers now in existence.

The existing program used on the IBM 650 computer takes approximately $4\frac{1}{2}$ minutes to calculate the coefficients of the orthogonal polynomial for 36 overlaping 30° sectors. This information may be provided in many output forms. The most desirable is the presentation of the data in the form of plotted graphs.

Utilization of the silent area analysis method described in this study is quite within the existing electronic computer capabilities. The 20 graphs ready for interpolation may be obtained within five minutes of the input of the data. The most time consuming portion of the operation would be the manual interpolation of the coefficients into the unknown area. From the results of this study it appears that an individual thoroughly familiar with the polynomials could perform this interpolation within a one hour period. During this interpolation of these values for the silent area the interpolated data could be punched on cards as it became available. The ideal situation would be to eliminate the manual interpolation completely and have the computer perform this operation, however, this would require a purely objective interpolation technique. Attempts to develop such a technique are discussed in the following section.

The resulting deck of cards containing the coefficients for adjacent known sectors and adjacent interpolated sectors could be fed back into the computer. The computer can then produce a reconstructed chart for the northern hemisphere within twenty minutes of the receipt of this data. These charts would appear in the same form as the "Numerical Analysis Center Charts," so often seen in publications.

The point of input of the raw station data was purposely omitted. This could be even more time consuming than the interpolations if it were not for the fact that it is possible to perform this operation on a computer. It is possible to feed into the computer known station 500mb height data and have this checked for accuracy and converted to 500mb heights at the diamond grid intersection previously discussed. The time required for this conversion would be approximately thirty minutes.

From this discussion it can be seen that the final silent area analysis, in chart form, could be obtained within two hours after the receipt of the final upper air data from the known area. This time estimate could be greatly reduced by eliminating the manual interpolation.

SECTION V

ATTEMPTED OBJECTIVE TECHNIQUES

In an attempt to arrive at a purely objective technique for interpolation the following approaches were used.

The cyclic nature of the coefficients suggests the use of harmonic analysis techniques in developing an objective interpolation method. However, the standard harmonic analysis technique can not be directly applied due to a portion of the data being missing. This condition necessitates the adoption of an approximation of this missing data before the harmonic analysis technique can be applied.

The first approximation of the missing data was taken as the mean of the known data. Based on this, the first harmonic was computed and the additional harmonics were computed from 18 of the successive values of the known points. The use of the mean of the known data as an approximation of the missing data resulted in phase angle errors of the first harmonic of greater than 50° and amplitude errors of as much as 50% of the true amplitude. These errors were too large for this method to be considered as satisfactory.

The second approximation of the missing data was obtained by using the values of the sectors 180° from the missing sectors. This was accomplished by taking the departure from the mean, changing its sign and adding it to the mean to give the approximation for the point 180° from known data point. The mean is defined as the mean of the known points that are not opposite the unknown area. The additional harmonics were computed by use of 18 consecutive points of the known data. This method of approximating the unknown data points yielded errors of the phase angle of the first harmonic of 30° or less. The computed amplitude was in error by as much as 30% of the true amplitude. Although this accuracy is still poor, a definite improvement in the computed phase angle was obtained.

The periodogram method was also attempted but the results were discouraging. The problem of extending the analysis into the area of missing data was still basically unchanged.

These three attempts at interpolating the coefficients objectively were mentioned in order that the reader may know what has been tried. A means of producing a purely objective technique may be some time in development or may never be developed but this in no way invalidates the usefulness of this method of describing an area of missing data. In fact, the high average accuracy, 88%, that is obtained by the subjective technique indicates the usefulness of this approach.

SECTION VI
CONCLUSIONS

This paper has covered the basic concept of silent area analysis by use of the orthogonal polynomials and a discussion of one possible method of interpolation. The approach presented here is based on a small sample of data but the results are encouraging.

It is felt that this system or one similar may prove to be not only feasible but of considerable operational use in the field of aeronautical meteorology. The time consuming manual calculations have been eliminated by the use of electronic computers to the basic time required to manually plot and analyse the northern hemisphere 500mb chart.

The average accuracy of the representation of the interpolated silent area was found in this test to be 88%; however, the range of the error for individual 30° sectors was from as low as 64% to as high as 98% reduction of the basic sector. Further refinements, mentioned previously, should be considered in order to raise this lower limit. Part of this low reduction is due to the flat pressure gradient and part due to errors made in the interpolation of the unknown or silent area coefficients. Better results might be obtained by the use of more objective interpolation schemes. To become an operational system subjective error must be minimized.

There appears to be little doubt that for a smaller silent area sector, say less than 60° in width, that even a direct linear interpolation would yield good results. The terms that are highly variable (higher order terms) contribute less than 6% where the more conservative low order terms contribute more than 82%, therefore, the low order terms are the true mainstay of the system. From these terms one may see the changes in the zonal and meridional circulations occurring around the northern hemisphere and thereby add one additional tool to aid in analysis of a silent area.

By averaging the interpolated coefficients for the silent areas over 30 day periods, it is possible to build up a climatological map for each area of missing data. The errors made in the daily analysis over such a period should tend to cancel out, producing a monthly climatological pressure pattern with an accuracy greater than the accuracy of the daily interpolated silent areas as shown in Table IV. The average interpolated map should be more accurate than an average map obtained by a single interpolation of the unknown area from an average map of the known area. This method of combining daily interpolations of the coefficients need not be restricted to one pressure level but may also be used at various levels. This, in turn, would yield a vertical climatological distribution of the coefficients which could be utilized to extend the present climatic data both vertically as well as horizontally.

BIBLIOGRAPHY

1. University of Wisconsin Scientific Report No. 11, "Half-hemispheric 500 MB Topography Description by Means of Orthogonal Polynomials," part 2, Specification by Oskar Essenwanger, Lyle H. Horn and Reid A. Bryson, AF Contract No. AF 19(604)-992.

2. University of Wisconsin Scientific Report No. 4, "Half-hemispheric 500 MB Topography Description by Means of Orthogonal Polynomials," part 1, Computation by Reid A. Bryson and Peter M. Kuhn, AF Contract No. AF 19(604)-992.

3. Wadsworth, G. P. "Short Range and Extended Forecasting by Statistical Methods," Air Weather Service Tech. Rept., 105-38, 1948.

4. Fisher, R. A. and Yates, F. Y., Statistical Tables for Biological, Agricultural and Medical Research, London; Oliver & Boyd, 1943.

APPENDIX A

EXAMPLE 30° DIAMOND GRID

	100°	105°	110°	115°	120°	125°	130°
70°N	i=1 o j=1		i=3 o j=1		i=5 o j=1		i=7 o j=1
65°N		i=2 x j=2		i=4 x j=2		i=6 x j=2	
60°N	i=1 o j=3		i=3 o j=3		i=5 o j=3		i=7 o j=3
55°N		i=2 x j=4		i=4 x j=4		i=6 x j=4	
50°N	i=1 o j=5		i=3 o j=5		i=5 o j=5		i=7 o j=5
45°N		i=2 x j=6		i=4 x j=6		i=6 x j=6	
40°N	i=1 o j=7		i=3 o j=7		i=5 o j=7		i=7 o j=7
35°N		1=2 x j=8		i=4 x j=8		i=6 x j=8	
30°N	i=1 o j=9		i=3 o j=9		i=5 o j=9		i=7 o j=9
25°N		i=2 x j=10		i=4 x j=10		i=6 x j=10	
20°N	i=1 o j=11		i=3 o j=11		i=5 o j=11		i=7 o j=11

APPENDIX B

COMPUTATION METHOD FOR 30° SECTOR

A. Terms utilized:

X thru X^5	Y^2: $X-X^2$
Y thru Y^4	Y^3: $X-X^2$
Y: $X-X^2$	Y^4: $X-X^2$

B. Computation of orthogonal polynomial for 30° (longitude) sectors utilizing 39 grid points or pieces of data.

1. Compute mean of the entire sector

$$h = \frac{1}{39}\sum_{i=1}^{39} h_i$$

2. Compute standard deviation factor

$$S^2 = \frac{77}{39}\sum_{i=1}^{39} (h_i - \bar{h})$$

3. Compute the 7 column sums (C_i), one for each 5° of longitude across the 30° sector.

4. Compute the column means.

$\bar{C}_i = \frac{C_i}{6}$ For i= 1, 3, 5, 7 Longitude ending in zero, as 100, 110, etc.

$\bar{C}_i = \frac{C_i}{5}$ For i= 2, 4, 6 Longitude ending in fives, as 105, 115, etc.

5. Compute the eleven row sums (r_j), one for each 5° of latitude from 20°N thru 70°N.

6. Compute row means (R_j).

$R_j = \frac{r_j}{4}$ For j= 1, 3, 5, 7, 9, 11 For latitude ending in zero, as 70°, 60°, etc.

$R_j = \frac{r_j}{3}$ For j= 2, 4, 6 For latitude ending in fives, as 65°, 55°, etc.

7. Compute T_s

$$T_s = \sum_{i=1}^{7} \bar{C}_i \alpha_{si}$$

Where: s = power of X: X to X^5

α_{si} = table values for east-west terms

8. Compute T_t

$$T_t = \sum_{j=1}^{11} R_j \beta_{tj}$$

Where: t = power of Y: Y to Y^4

β_{tj} = table values for north-south terms

9. Compute the five non-normalized coefficients for the powers of X: X-X^5.

$$U_s = T_s \sqrt{\frac{11}{\Sigma \alpha_s^2}}$$

Where: S = power of X

10. Compute the four non-normalized coefficients for the powers of Y: Y-Y^4.

$$U_t = T_t \sqrt{\frac{7}{\Sigma \beta_t^2}}$$

Where: t = power of Y

11. Compute the normalized coefficients (Z's) by dividing the non-normalized coefficients (U's) by the standard deviation factor (S).

C. Computation of the 8 cross product terms (XY, X^2Y, XY^2, X^3Y^2, XY^3, X^2Y^3, XY^4, X^2Y^4).

1. Compute the mean height of all grid points at even (0) intersections of latitude and longitude.

$$\bar{h}_c = \frac{1}{24} \sum_{i=1}^{24} h_i$$

2. Compute the standard deviation factor.

$$S_c^2 = \sum_{i=1}^{24} (h_i - \bar{h}_c)^2$$

3. Compute P_{ti}

$$P_{ti} = \sum_{j=1}^{6} h_j \beta_{tj}$$

Where: t = power of Y t = 1,2,3,4

i = column index

i = 1,2,3,4

j = row index

j = 1,2,3,4,5,6

Multiply each column of h's by each column of cross product table values and obtain the sum of these six products. There are 4 columns of h's used and 4 columns of table values. This gives a total of 16 sums of the products (P_{ti}).

4. Compute Ts,t

$$T_{st} = \sum_{i=1}^{4} P_{ti} \alpha_{is}$$

Where: i = column index

α_{is} = cross product table values

s = power of X s = 1,2

Example:

$T_{1,1} = P_{1,1}\ (3) + P_{1,2}\ (1) + P_{1,3}\ (-1) + P_{1,4}\ (-3)$
$= T_{x,y}$

$T_{1,2} = P_{2,1}\ (3) + P_{2,2}\ (1) + P_{2,3}\ (-1) + P_{2,4}\ (-3)$
$= T_{x,y^2}$

5. Compute the non-normalized coefficients (U's).

$$U_{st} = T_{st} \frac{1}{\sqrt{\sum \alpha_s^2 \cdot \sum \beta_t^2}}$$

6. Compute the normalized coefficients (Z's) by dividing the non-normalized coefficients (U's) by the

cross product standard deviation factor (S_c).

D. Orthogonal polynomial tables as computed by Fisher & Yates (1943).

1. North-south table (11 points) β

Y	Y^2	Y^3	Y^4
- 5	15	- 30	6
- 4	6	6	- 6
- 3	- 1	22	- 6
- 2	- 6	23	- 1
- 1	- 9	14	4
0	- 10	0	6
1	- 9	- 14	4
2	- 6	- 23	- 1
3	- 1	- 22	- 6
4	6	- 6	- 6
5	15	30	6
$\Sigma\beta^2$ = 110	858	4,290	286

2. East-west table (7 points) α

X	X^2	X^3	X^4	X^5
3	5	1	3	1
2	0	- 1	- 7	- 4
1	- 3	- 1	1	5
0	- 4	0	6	0
- 1	- 3	1	1	- 5
- 2	0	1	- 7	4
- 3	5	- 1	3	- 1
$\Sigma\alpha^2$ = 28	84	6	154	84

3. North-south cross product table (6 points) β'

Y	Y^2	Y^3	Y^4
- 5	5	- 5	1
- 3	- 1	7	- 3
- 1	- 4	4	2
1	- 4	- 4	2
3	- 1	- 7	- 3
5	5	5	1
$\Sigma\beta_c^2$ = 70	84	180	28

4. East-west cross product table (4 points) α'

	X	X^2
	3	1
	1	- 1
	- 1	- 1
	- 3	1
$\Sigma\alpha_c^2 =$	20	4

APPENDIX C

RECONSTRUCTION FORMULA AND TABLES

A. The basic equation used to obtain the height of any particular grid point is as follows:

$$h_{ij} = \bar{h} + \sum_{\substack{s=1 \\ t=0}}^{5} U_{st}\,\alpha'_{si} + \sum_{\substack{t=1 \\ s=0}}^{4} U_{st}\,\beta'_{tj} + \sum_{t=1}^{4}\sum_{s=1}^{2} U_{st}\,\alpha''_{si}\,\beta''_{tj}$$

Where: i = column index

j = row index

s = exponent of X

t = exponent of Y

α' = east-west reconstruction table values

β' = north-south reconstruction table values

α'' = east-west cross product reconstruction table values

β'' = north-south cross product reconstruction table values.

B. Reconstruction table values.

1. North-south table (11 points) β'

Y	Y^2	Y^3	Y^4
-.180	.194	-.173	.134
-.144	.077	.035	-.134
-.108	-.013	.127	-.134
-.072	-.077	.133	-.022
-.036	-.116	.081	.089
0	-.129	0	.134
.036	-.116	-.081	.089
.072	-.077	-.133	-.022
.108	-.013	-.127	-.134
.144	.077	-.035	-.134
.180	.194	.173	.134

2. East-west table (7 points) α'

X	X^2	X^3	X^4	X^5
.171	.164	.012	.073	.033
.114	0	-.012	-.170	-.132
.057	-.099	-.012	.024	.164
0	-.132	0	.146	0
-.057	-.099	.012	.024	-.164
-.114	0	.012	-.170	.132
-.171	.164	-.012	.073	-.033

3. North-south cross product table (11 points) β''

Y	Y^2	Y^3	Y^4
-.598	.546	-.373	.189
-.478	.177	.259	-.670
-.359	-.109	.522	-.567
-.239	-.314	.496	-.080
-.120	-.436	.298	.378
0	-.477	0	.558
.120	-.436	-.298	.378
.239	-.314	-.496	-.080
.359	-.109	-.522	-.567
.478	.177	-.259	-.670
.598	.546	.373	.189

4. East-west cross product table (7 points) α''

X	X^2
.671	.500
.447	-.125
.224	-.500
0	-.625
-.224	-.500
-.447	-.125
-.671	.500

Appendix D

IBM 650 Program for Computing Polynomials for 30° Longitude Sectors, Shifting Sectors by 10° Longitude after each Calculation

The 650 program described here is designed to compute the coefficients for five powers of X and four powers of Y. The coefficients are computed for a 7 x 11 diamond grid whichencompassesa 30° sector of longitude from 20° North latitude to 70° North latitude. The sector is shifted 10° at a time from West to East for 35 shifts, or a total of 36 sectors, a new set of coefficients being computed for each sector.

There are 44 data cards per hemispherical map. An Arowa deck with the following format was used in this study.

Column	Contents
1-6	Date
7, 8	Time
9, 10	Latitude
11	Card number
12-38	Nine pieces of data (3 digits each) 500 mb heights are in tens of feet, first digit omitted.

Cards are "read in" according to latitude and card number in the following order: 701, 601, 501, 401, 301, 201, 702, 602, 502, 402, 302, 202, 703, . . . , 204, 651, 551, 451, 351, 251, 652, . . . , 254. The first two digits are latitude and the last is the card number. The program stores the data in locations 904 through 1299.

The output of the program consists of 144 cards of data per hemispheric map, punched in the following format:

Column	Contents
1, 2	Year
3, 4	Month
5, 6	Day
7	Level (2 was used to indicate 500 mb)
8	Card number 1 - East-West, North-South U's 2 - East-West, North-South Z's 3 - Cross product U's 4 - Cross product Z's
9, 10	Sector (00 - 100° East to 130° East, 01 - 110° East to 140° East, etc.)
11-14	$\bar{h}$ or $\bar{h}_c$
15-18	S or S_c
19-54	Nine pieces of data (4 digits each) for cards 1 and 2. The terms are in the following order: x, x^2, x^3, x^4, x^5, y, y^2, y^3, y^4.
19-51	Eight pieces of data (4 digits each) for cards 3 and 4, columns 52-54 are zero. The terms are in the following order: xy, xy^2, xy^3, xy^4, x^2y, x^2y^2, x^2y^3, x^2y^4.

The program is optimized and requires 4.75 minutes to compute the coefficients for the 36 sectors of one hemispheric map. The program begins at location 0000. The storage entry switches should be set to 70 1951 9999 +. A 533 board wiring diagram is included. There are two programmed stops. A stop with 01 9999 9999 in the program register display indicates an incorrect card number. A stop with 01 9999 7777 in the program register display indicates an incorrect date. In either case remove the cards from the deck which represent the day in which the error occurred and restart the program at location 0000.

A 533 board wiring diagram is included.

Donald E. Coon

IBM 650 Program for computing polynomials for 30° longitude sectors, shifting sectors by 10° longitude after each calculation

Location	Instruction	Location	Instruction	Location	Instruction	Location	Instruction	Location	Instruction
2479	7019589801	9419	5900011957	5465	4619951958	7950	5900011997	1953	7019879801
2479	1919511989	9219	4619908001	9765	6019571991	7955	1519581992	1996	8800051993
2479	3000041995	9819	6979511996	5665	2440001998	7956	4819931999	1957	5800011997
	6900030006	6	2490010012	12	6900150018	18	2400210024	24	8100080030
30	8200010036	36	8900050042	42	6900450048	48	2400010004	4	6900070010
10	2400130016	16	6900190022	22	2400250028	28	6900310034	34	2400370040
40	6900430046	46	2400490002	2	6900050008	8	2400110014	14	6900170020
20	2400230026	26	6900290032	32	2400350037	37	7090300087	87	6900900093
93	2400370140	140	6590390047	47	3000030055	55	3500030063	63	2090020070
70	6090010027	27	3000030085	85	6580020143	143	3500030051	51	1090020009
9	2190010067	67	6090390075	75	1190010025	25	4400790080	79	6080010135
135	3000030193	193	2090020050	50	6090390057	57	3000030065	65	6580020073
73	1690020081	81	4500840185	84	199999999	185	0199997777	80	6992380137
137	2449030056	56	4000490001	1	4800540105	54	5800010060	60	8100080011
11	5300470117	117	6090010125	125	1100780033	33	2190010037	49	5200060155
155	5000010080	100	6990300106	106	2450830049	105	6090010113	113	3000010069
69	6580020077	77	1601300013	13	4500660167	66	6090010023	23	1000760131
131	2190010039	39	5200010095	95	5100080035	35	8900050037	150	6900450098
98	2400010104	104	6900190072	72	2400250066	167	5300040123	123	6090010181
181	1001340089	89	2190010097	97	6902000053	53	2400010154	154	6901070110
110	2400130116	116	6901190122	122	2400250128	128	6000490103	103	1101560061
61	2100490052	52	6000110115	115	1100680173	173	2100110064	64	6502170071
71	6900230126	126	2200230176	176	6000350139	139	1101560111	111	2100350095
250	6780010157	157	2190000165	165	8100090121	121	2492390178	178	4002310082
231	5000010121	82	2790300187	187	2807500062	62	0807500112	112	2790300267
267	2807000162	162	2807100172	172	807000212	212	2790300317	317	2907200222
222	6901750228	228	2402810184	184	6900780331	331	2490290237	237	6990000243
243	2490030099	99	6901020205	205	2490040161	161	6501140169	169	6902720225
225	2202720275	275	6902780381	381	2202780431	431	6902340287	287	2202340337
337	6901900293	293	2201900343	343	6900960149	149	2401520255	255	6900580211

211	2401640367	367	8100050223	223	8300030129	129	8800010235	235	6027050059
59	1069030207	207	2127050108	108	6047460101	101	1069030257	257	2147460199
199	6069030307	307	1980010481	481	1547140164	164	2047140417	417	4001200171
120	5000010226	226	5800010235	171	4200740325	74	8100050180	180	5200010226
325	8802170531	531	5000010387	387	5200010393	393	6901460249	249	2401640235
300	5100050206	206	4001090160	109	5000060226	160	5300030166	166	4202190170
219	8000010375	375	5200040226	170	6990000276	276	2490050132	132	2490060038
38	2790490443	443	0907000202	202	2790490357	357	2907580210	210	8300030216
216	8100060322	322	6090050179	179	1027490153	153	2427750328	328	2190050285
285	6090060493	493	1027170221	221	2190060229	229	4001820083	182	4203350086
335	5000010041	41	5200010322	83	6590050091	91	3500020147	147	6403500261
261	3100010467	467	2090070124	124	6090050581	581	1980010305	305	3500010311
311	6403500361	361	2090020118	118	6090060425	425	3500010631	631	1190020189
189	1900920163	163	3500010269	269	6403500411	411	6080020319	319	6903720475
372	2190080279	279	8100050385	385	6900880141	141	2400448001	44	0000000148
148	6527050159	159	3500030517	517	6402200681	681	3100010437	437	2027050158
158	4004610262	461	5000010567	567	4102700044	270	6527470151	151	3500030209
209	6403120273	273	3100010329	329	2027470044	262	8100040168	168	6902710141
400	6527100215	215	3500030323	323	6403260487	487	3100010543	543	2027100213
213	4002660617	266	5000010422	422	4105250044	525	6527500355	355	3500030263
263	6403160127	127	3100010133	133	2027500044	86	6990050142	142	2490090198
198	6990060204	204	2490100335	617	8100040373	373	8300060379	379	8900340435
435	6068100265	265	1947490369	369	1527220177	177	2027220575	575	4203780429
378	5200010284	284	5800010435	429	4002320183	232	5000010138	138	8300060284
183	8100030239	239	8300100145	145	8900430201	201	6068540259	259	1947101331
1331	1527261381	1381	2027260479	479	4202820233	282	5200010188	188	5800010201
233	4001360537	136	5000010192	192	8300100188	537	8100080593	593	6027261431
1431	1928950315	315	3100040227	227	2092380186	186	6490080299	299	3100010405
405	2092280214	214	6592380423	423	3100070191	191	2092380450	450	4002030254
203	5000010593	475	2490111481	1481	4404859011	485	2190140643	643	6901960349

IBM 650 program for computing pplynomials for 30° longitude sectors, shifting sectors by 10° longitude after each calculation

349	2490120455	455	6590140313	313	3500020419	419	6490120283	283	1580010289
289	6402420253	253	2090130260	260	1690120667	667	6780020625	625	1602420197
197	4605000251	251	6990130407	407	2490120455	500	6090139011	254	6590070511
511	3500040321	321	6990300277	277	2290300334	334	6990200240	240	2290200246
246	6590080303	303	3100020561	561	3500040371	371	6990310327	327	2290310384
384	6990210281	281	2290210238	238	6590010195	195	3000030353	353	3500030611
611	1590290469	469	2090390376	376	6905290332	332	2402810434	434	7190300484
484	6590390241	241	1500780333	333	2090290021	21	7190200421	421	6901740377
377	2400210224	224	6590291531	1531	1500780383	383	2090390290	550	6900150218
218	2400210274	274	6090291581	1581	3000020587	587	6580020245	245	1602480403
403	4502560000	256	6590290363	363	1603660471	471	2090390428	290	6590090247
247	3500020453	453	6403061317	1317	3100010473	473	2090070230	230	6090090637
637	1980010661	661	3500021367	1367	6403061417	1417	2090020324	324	6090101631
1631	3500020687	687	1190020295	295	6901520475	152	2190080309	309	6903620365
365	2401520505	505	8300231311	600	8100140356	356	8300020412	412	6927420345
345	2427410094	94	4202970298	297	5200010503	503	5000010412	298	4003010252
301	8300020457	457	5000020412	252	8300050208	208	6990000264	264	2407300433
433	2407341337	1337	2407380291	291	2407420395	395	8000000351	1311	8100030351
351	8900050272	272	6049271681	1681	1968601731	1731	1527300535	535	2027300278
278	6049271781	1781	1968661387	1387	1527340339	339	2027340234	234	6049271831
1831	1968720693	693	1527381343	1343	2027380190	190	6049271881	1881	1968780399
399	1527420347	347	2027420445	445	4803480449	348	5800010304	449	4203020553
302	5000010258	258	8900050304	304	5200010272	650	8100030406	406	8300150462
462	8900030268	268	6047420397	397	1968820603	603	1527530507	507	2027530456
456	6047420447	447	1968860557	557	1527571361	1361	2027570310	310	4804130314
413	5800010519	1300	6502720427	427	1502800585	585	6902720675	675	2202721325
1325	6902780382	382	2202780432	432	6902341437	1437	2202341487	1487	6901901393
1393	2201900650	553	6502720477	477	3000041537	1537	8080020495	495	5102160401
401	4013000555	555	6503080585	314	4215170318	1517	8900030523	523	5000010519
519	5200010268	318	8100070374	374	6027571411	1411	1929030573	573	3100020482

IBM 650 program for computing polynomials for 30° longitude sectors, shifting sectors by 10° longitude after each calculation

482	2092370340	340	6490080653	653	3100010359	359	2092270368	368	6592370527
527	3100070545	545	2092370354	354	4006070358	607	5000010374	358	6990000364
364	2490380320	320	2490280254	428	6590030635	635	8280010341	341	1503121567
1567	2090030424	424	1605770532	532	4606850236	685	6902880391	391	2401440497
236	6903890292	292	2401440497	497	8100061303	1303	2790570408	408	0807120474
474	2790570579	579	2807110623	623	6907160569	569	2407150418	418	6907170370
370	2407160619	619	6907740627	627	2407471350	1350	6907750478	478	2407480451
451	2790570506	506	0807700582	582	2790571587	1587	2807430605	605	6990001461
1461	2407460499	499	2407490352	352	2407141617	1617	2407170420	420	2490050426
426	2490060144	144	5000011400	1400	6049280483	483	1980010657	657	1507140669
669	2007141667	1667	6007460501	501	1049280533	533	2107460549	549	6027631717
1717	1149040409	409	1049280583	583	2127050458	458	4015110512	512	8100040468
468	6590041375	1375	8280010632	632	1503160521	521	2090040528	528	1606821637
1637	4603900441	390	6914430296	296	2401440547	441	6901940597	597	2401440647
1511	5200010144	547	6049190673	673	1980010697	697	1507170571	571	2007170470
470	6007491353	1353	1049191323	1323	2107490402	402	6027681373	1373	1149040459
459	1049191423	1423	2127100463	463	4015111767	1767	8100081473	1473	6990000629
629	2427260679	679	4013320170	1332	5000010338	338	2427570629	1450	6047121817
1817	1980010491	491	1507141319	1319	2007141867	1867	6007460551	551	1047121917
1917	2107460599	599	6027630518	518	1149040509	509	1047120568	568	2127050458
647	6047391493	1493	1980010618	618	1507170621	621	2007170520	520	6007491403
1403	1047391543	1543	2107490452	452	6027681523	1523	1149040559	559	1047391593
1593	2127100463	58	2047140417	146	2047140300	288	5000011400	1443	5000010547
389	5000011450	194	5000010647	15	7190200421	174	7190200550	175	2290210238
529	2290210434	88	0000000148	271	400	45	4800540105	200	4800540150
7	4500660167	107	4500660250	19	4400790080	119	4400790100	31	7090300087
96	2190080309	362	2190080600	90	7090300067	43	5200060155	5	5300470117
17	1000760131	29	8900050037	114	6049270000	308	0049090000	78	0000000100
130	4000000000	76	0000000501	134	447	92	0000000077	156	0000010000
68	0000080000	1500	0000000401	102	216	3	0000000701	350	0000000039

IBM 650 program for computing polynomials for 30° longitude sectors, shifting sectors by 10° longitude after each calculation

196	0000070000	242	0000000002	220	4	312	0000000006	326	0000000003
316	0000000005	248	3500000000	366	299	306	0000000024	280	0000060000
577	0000000195	682	0000000385	217	15000000	553	6501140477	495	5001920601
601	6502720677	677	3000041687	1687	5180020401	1951	0100001951	1951	0100001951

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WIRING DIAGRAM FOR 30° LONGITUDE SECTOR POLYNOMIALS