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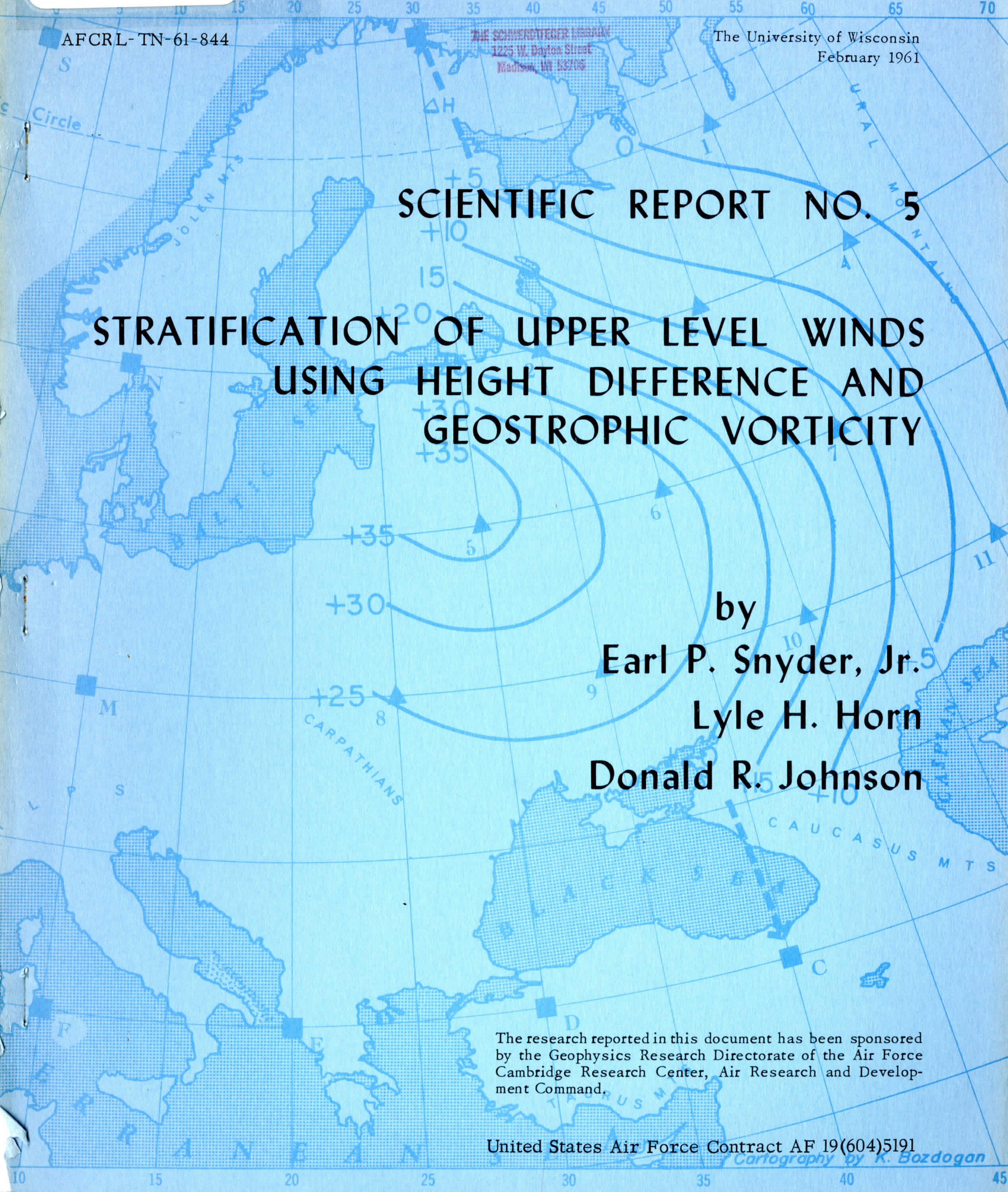

SCIENTIFIC REPORT NO. 5

STRATIFICATION OF UPPER LEVEL WINDS USING HEIGHT DIFFERENCE AND GEOSTROPHIC VORTICITY

by
Earl P. Snyder, Jr.
Lyle H. Horn
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The research reported in this document has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command.

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ABSTRACT

As an experiment in stratified climatology, height difference and geostrophic vorticity have been tested as predictors for the 300 mb. geostrophic winds over eastern Europe. Eight years of winter season data (1950-1957) and data for March 1958 were used in the study. Forecasts using stratified data were compared with those of unstratified climatology. Certain stratifications showed significant improvement over climatology. Further research into this method of forecasting seems warranted by the results of this study.

I. INTRODUCTION

In recent years considerable attention has been given to the problem of preparing improved upper level wind estimates for areas of sparse or unreliable data. Various objective forecasting schemes have been proposed to meet this problem;however the results have more often than not shown negligible skill. In many cases the results have not exceeded the results that can be obtained by using climatological means. In view of these generally poor results, the method of stratified climatology has been investigated as an added tool in meeting the problems of objective forecasting (1). This method is further investigated in this report. An attempt is made to obtain a better understanding of the wind field over areas of unreliable data by the stratification of climatological data.

In objective forecasting, both the dynamical and statistical approaches can be used. Both methods have been aided by the comparatively recent development of high speed electronic computing machines. In this report, statistical methods are used to forecast the 300 mb wind field.

Statistical forecasting may be divided into two classifications, linear and nonlinear. Linear forecasting, where the relationship between two values can be represented by a linear equation, has met with limited success. Results obtained by the Statistical Forecasting Project at MIT (2) in the case of sea-level pressure bear this out. A simple displacement of the pressure systems has been shown to give the same results. In the Statistical Forecasting Project a certain level of skill in forecasting was reached but could not be easily surpassed. This has been true of other studies also (3). This level of skill which is easily reached but difficult to surpass has been called the plateau of predictability. Lorenz (2) points out that nonlinear forecasting may be the answer to surpassing the plateau of predictability.

One approach to nonlinear statistical forecasting is the stratification method which has been discussed by Panofsky (4). The value or values to be predicted are referred to as the predictand while the data used in preparing the forecast are called the predictors. In the stratification method, the first step is to divide the predictor into a number of relatively homogeneous classes. This should be done for a number of predictors. The next step is to test these different predictors to find the one which best explains the properties of the predictand.

Stratification, as pointed out by Cochran (5), may bring about a gain in precision in the estimate of characteristics of the total population. The basic idea is that it may be possible to divide a heterogeneous population into sub-populations, each of which is internally homogeneous.

If this can be done, the different values in the homogeneous sub-population may show a small dispersion about its mean. All values falling within the interval of this sub-population might be given a representative mean value. Ideally, the heterogeneous population is described by a number of sub-populations, each having a mean value and a minimum variance. Because each sub-population has a smaller variance than the total population, a better estimate of the characteristics of the total population is possible. If the homogeneous sub-populations of the predictor are related to homogeneous sub-populations of the predictand, a better estimate of the predictand is also possible.

A preliminary study of stratified climatology of upper winds using height differences as predictors has been done by Cole (6). The study involves the u and v-components of the 500 mb. geostrophic winds at Moscow. Results obtained by stratified climatology showed a 26% reduction over the standard deviation obtained by unstratified climatology in the prediction of the u-component. The v-component showed a 12.5% reduction in the standard deviation. These results were not tested against independent data; however, the magnitude of improvement of the dependent data indicates future possibilities for stratified climatology.

In this report the possibilities of the use of height difference and geostrophic vorticity as predictors of the wind field over eastern Europe are explored. It will be shown that significant improvement of stratified climatology over unstratified climatology can be obtained by the stratification method.

II. DATA AND METHOD OF STRATIFICATION

The data used in this study were the daily 1500 Z components of the geostrophic wind at 300 mb for the five months, November through March for the years 1950-57. These winds were computed in the preparation of the Atlas of 300 mb Wind Characteristics for the Northern Hemisphere (7). In addition, the 1200 Z geostrophic winds for March 1958 were used. The winds were computed from 300 mb height values read to the nearest hundred feet at 450 latitude-longitude grid intersections covering an area extending from 15°N to 75°N latitude. A diamond grid system formed by the intersections of latitudes and longitudes ending in "0" and "5" degrees was used. The eastward and northward components of the geostrophic wind computed from this grid system represent the average conditions over a diamond-shaped area possessing diagonals of 10° of latitude and 10° of longitude. The region covered by this investigation comprises an area of approximately one million square miles of Eastern Europe and selected points in Western Europe and the Eastern Atlantic. See Figure 1.

Since in any stratification scheme it is advantageous to use data from one season, the months of November, December, January, February and March were taken as representative of the winter season in Eastern Europe. This gave a data sample of forty-one months. The data were arbitrarily split into dependent and independent data. The dependent data were used to determine the nature of the stratifications, while the independent data

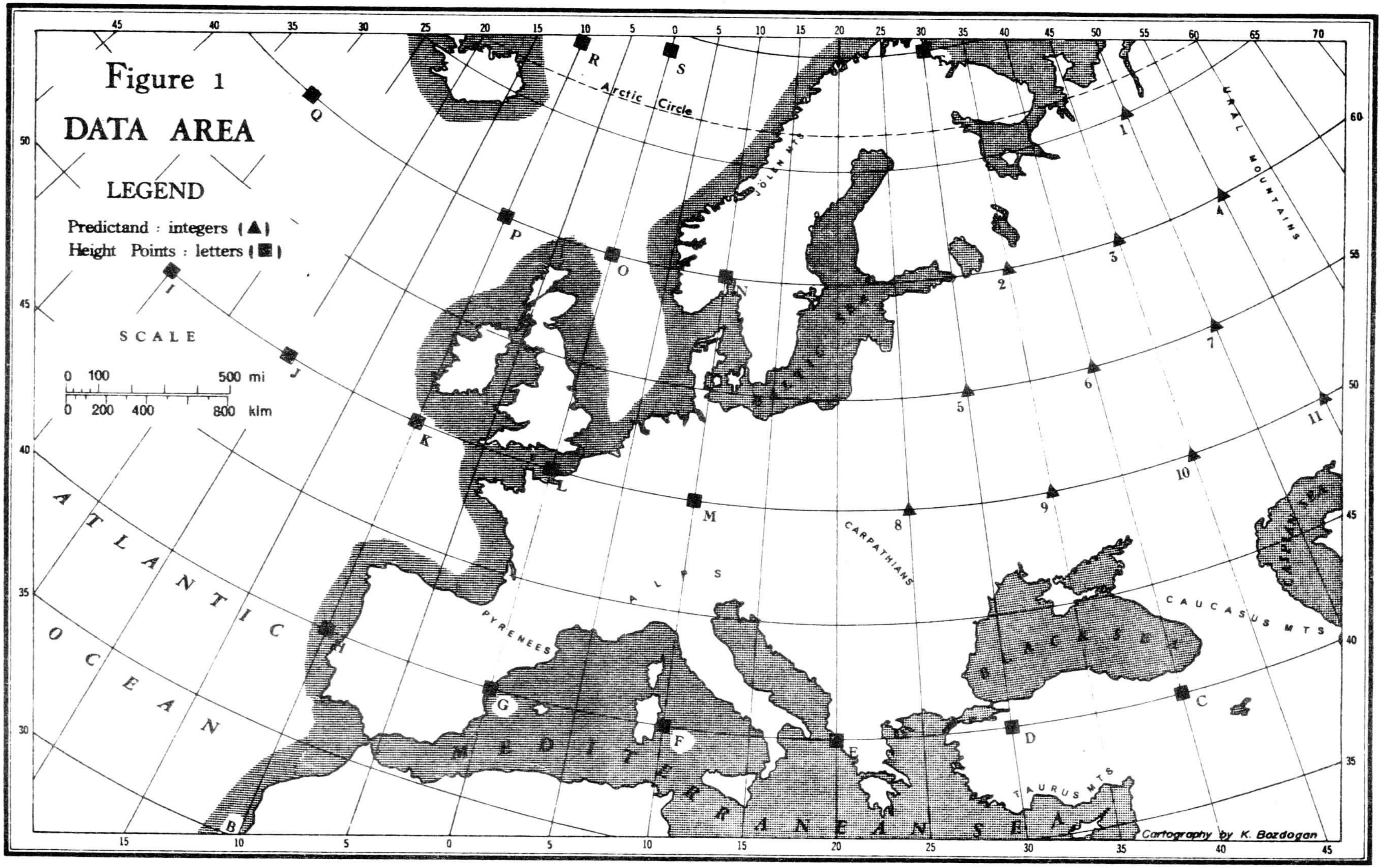


Figure 1
DATA AREA
LEGEND
Predictand : integers (▲)
Height Points : letters (■)
SCALE
0 100 500 mi
0 200 400 800 klm
Arctic Circle
URAL MOUNTAINS
JÖLEN MTS
CARPATHIANS
ALPS
PYRENEES
CAUCASUS MTS
TAURUS MTS
ATLANTIC OCEAN
MEDITERRANEAN SEA
Cartography by K. Bozdogan

were used for verification purposes. The independent data consisted of the ten months:

November 1950, 1954
December 1951, 1955
January 1952, 1956
February 1953, 1957
March 1954, 1958

By arbitrarily choosing monthly blocks of data rather than using a random selection of daily values for the independent data, the possibility of obtaining both dependent and independent data from the same weather regime was reduced. This should make the verification results obtained from the independent data more meaningfull than they would have been if random daily values had been used. This partitioning of the data gave a total of 302 days with which to test the stratifications obtained from the dependent data. Thirty -one months were left for the dependent data. However, because maps for ten days were not located and because the 300 mb heights for the southern two-thirds of the Eastern Hemisphere during the period December 20, 1954 through February 28, 1955 were missing, the geostrophic winds for these periods were never computed. Thus the dependent data is reduced to a period of 861 days.

In this study, the predictands (i.e., the parameters which are to be predicted) are the eastward (u) and northward (v) components of the geostrophic wind at eleven points in

Eastern Europe. These points are numbered on Figure 1. Two sets of parameters are used (individually and in combination) as the predictors of the geostrophic wind at these points: (1) the 300 mb height differences between selected predictor points and (2) the geostrophic vorticity computed from the Laplacian of the 300 mb heights at the predictor points. Since data may not always be available from the area of the predictand, the predictor points have been selected from outside the area of the predictand points. Because of the general west to east circulation of the atmosphere in middle latitudes the predictor points have been generally chosen from areas located to the west of the predictand area.

In this stratification technique, daily values of the dependent data are partitioned into relatively homogeneous subclasses. After the predictor subclasses have been determined, daily values of the u and v components for the predictand points are also divided into subclasses by use of corresponding data. Thus the predictand dates are partitioned into subclasses which are defined by the predictor subclasses. In this study, the mean values of the predictand subclasses determine a stratified climatology of the wind field over Eastern Europe. If the stratification techniques used are to be useful, the stratified climatology of the wind field should provide the meteorologist with more valuable information than can be provided by a simple unstratified climatology. The success of such a procedure would suggest the feasibility of preparing an atlas depicting the stratified climatology of the wind field over a large area.

Obviously, the success of any stratification depends upon a wise or fortuitous choice of parameters to serve as predictors. As was mentioned above, the 300 mb height differences between certain of the predictor points and the geostrophic vorticity computed from the heights at these points have been used as predictor parameters. The height differences can be used to describe the zonal and meridonal flow in the predictor region, while the geostrophic vorticity describes the nature of the circulation--cyclonic or anti-cyclonic--in this region. Thus the general characteristics of the atmospheric circulation in the predictor area are used to determine the wind field in the predictand area. Height differences between points located in Eastern Turkey, Northern France, Northern Norway, and south of Ireland were computed (See points C, L, T, and K, respectively on figure 1). The geostrophic vorticity was computed from 300 mb heights at points located in the Eastern Atlantic and Western Europe. The geostrophic vorticity was computed from the expressions which are developed below.

The geostrophic vorticity (8) may be expressed as,

$$\zeta_g = gf^{-1}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) + \beta f^{-1} u_g + u_g \frac{\tan\phi}{a} \qquad (1)$$

where ζ_g is the geostrophic vorticity, g is gravity, f is the Coriolis parameter, $\beta = \frac{\partial f}{\partial y}$, ϕ is latitude, a is the radius of the earth, and u_g is the eastward component of the geostrophic wind. For computational purposes, the Laplacian of equation (1) is written in finite difference form,

$$\frac{\partial^2 Z}{\partial x^2} = \frac{(Z_3 + Z_1 - 2Z_0)}{d_x^2}; \quad \frac{\partial^2 Z}{\partial y^2} = \frac{(Z_2 + Z_4 + 2Z_0)}{d_y^2} \qquad (2)$$

where points 1, 2, 3, and 4 indicated by subscripts are described in Figure 2. Using equation (2) and the geostrophic relationship, $u_g = -\frac{g}{f}\frac{\partial Z}{\partial y}$, equation (1) can be written as

$$\zeta_g = \frac{g}{f d_x^2}(Z_3 - Z_1 - 2Z_0) + \frac{g}{f d_y^2}(Z_2 + Z_4 - 2Z_0) - \frac{\beta g}{f^2 d_y}(Z_2 - Z_4) - \frac{g \tan\phi}{a f d_y}(Z_2 - Z_4) \qquad (3)$$

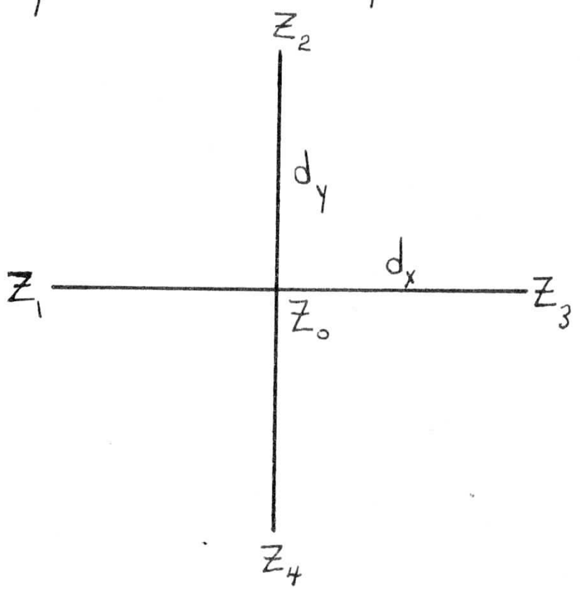
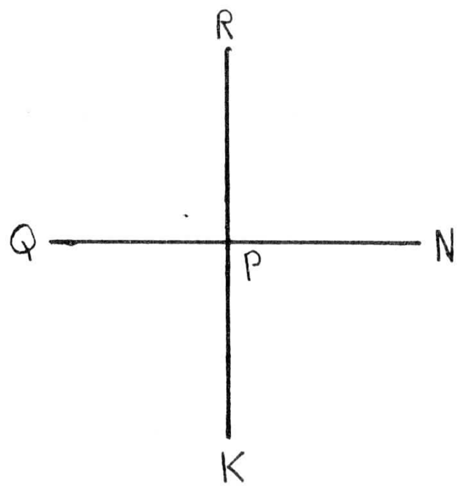


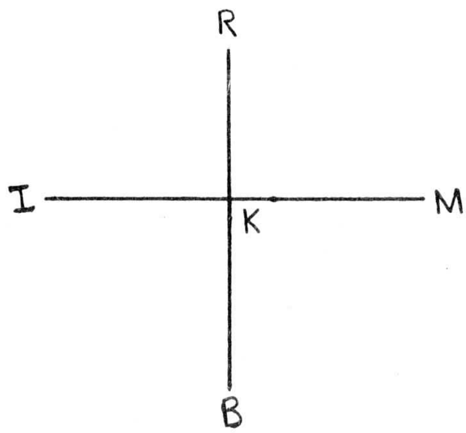
Figure 2. Grid for Geostrophic Vorticity Computation

All the values not enclosed in parentheses in equation (3) are constants which can be computed prior to computing the geostrophic vorticity. The last two terms of equation (3) are sometimes neglected because they are an order of magnitude smaller than the first two terms when ζ_g is computed in regions of large pressure difference. However, because of the large area over which the geostrophic vorticity is computed in this study, these terms have not been omitted. Although the usual definition of vorticity pertains to a small area, in this report the term will be used when considering a larger area. The values of vorticity computed from a large grid serve as a measure of the strength of the major ridges or troughs. Two different scale Laplacians have been used in calculating vorticity. The larger includes the entire west coast of Europe and the smaller an area between Ireland and Iceland. See Figures 3A and 3B for the arrangement of the points used.



d_x = 600 nautical miles
d_y = 600 nautical miles

Fig. 3A. Small Scale Laplacian (Refer to Figure 1 for location of lettered points)



d_x = 775 nautical miles
d_y = 1200 nautical miles

Figure 3B. Large Scale Laplacians (Refer to Figure 1 for locations of lettered points)

To divide the dependent data (heterogeneous population) into homogeneous classes, a frequency distribution of the dependent data for each of the predictors was made. The frequency distributions of the height differences were made by combining height differences in groups of two hundred feet. For example, if the height difference between two points was either six hundred or seven hundred feet, the frequency of occurrence of these values were grouped. This eliminates any bias towards even numbers that may have occurred in reading the maps prior to the computation of the geostrophic winds. The resulting frequency distributions tended to be multimodal. Therefore, it was assumed that the frequency distributions were composed of a number of normal sub-populations. The problem was to find a method which could divide the multimodal frequency distribution into these different normal distributions.

A method similar to that derived by Essenwanger (9) has been used. This method can be referred to as the partial collective method. A partial collective is defined as a portion of a frequency distribution that has been extracted from the entire distribution. The assumptions made are that it is symmetric and approaches a normal distribution. With these assumptions, we will assume that the partial collective represents a homogeneous class.

The equation for the normal curve is

$$P = \frac{1}{\sqrt{2\pi}} e^{-\frac{\tau^2}{2}} \qquad (4)$$

where P is the probability density of the normal curve, $\tau = \frac{X-\bar{X}}{\sigma}$, X is the variate, $\bar{X}$ is the mean of the variate, and σ is the standard deviation. The equation for the ordinates of the normal curve given by Panofsky (4) is

$$Y(\tau) = e^{-\frac{\tau^2}{2}} \qquad (5)$$

Through the use of this equation it is possible to reconstruct a normal curve if only part of the normal curve is given. The method involves the following: (1) inspection of the frequency distribution to find a portion of the curve resembling a normal curve; (2) computation of the standard deviation from this portion of the curve; (3) substitution of the computed standard deviation in the equation for the ordinates of the normal curve and construction of the entire normal curve; (4) subtraction of the normal curve from the original curve which gives a new frequency distribution; (5) repetition of the procedure using the residual distribution. The procedure is repeated until the original frequency distribution has been broken down into its normal curves.

It was not always possible to divide the frequency distribution into a number of normal curves. In this case, the classes were chosen arbitrarily. This was especially true in the two frequency distributions of geostrophic vorticity which showed only one mode and could not be broken down. However, the distributions of the height difference were in most cases readily divided into normal curves by the partial collective method.

Tables 1, 2, and 3 show the breakdown of the stratifications of height difference, geostrophic vorticity, and height difference- geostrophic vorticity combinations, respectively. The data presented in the tables include:

a) Class number of the stratification (that is, an identifying number of the partial collective).

b) Number of dependent cases in the class.

c) Number of independent cases in the class.

Each of the stratifications is assigned a Roman numeral. Stratification I is a six-class stratification of height differences using as its predictors the combination of the meridional and zonal wind flow between Turkey (Point C), and France (L), and Ireland (K), and Norway (T) (See Figure 1). Stratifications II and III represent separate stratifications of the meridional flow (2 classes) and zonal flow (3 classes), respectively. Stratification IV is a three class stratification of height difference which describes the zonal flow between Turkey and Norway. Stratifications V and VI represent stratifications based on the geostrophic vorticity as depicted by Figures 3A and 3B, respectively. Stratifications VII, VIII, and IX consist of a combination of the above mentioned height differences which describe the zonal wind and the two geostrophic vorticities.

TABLE 1

Stratification I: Meridional Flow Index, Point C (Eastern Turkey) minus Point L (Northern France), vs. Zonal Flow Index, Point K (South of Ireland) minus Point T (Northern Norway)

			K-T			
			$\Delta H \lesssim 900$	$900 < \Delta H \lesssim 1900$	$\Delta H > 1900$	Total
C-L	$\Delta H < 0$	Class	1	3	5	//////
		Dep. cases	72	110	170	352
		Indep. cases	7	34	46	87
	$\Delta H \gtrsim 0$	Class	2	4	6	//////
		Dep. cases	206	190	113	509
		Indep. cases	82	94	39	215

Stratification II: Meridional Flow Index, Point C (Eastern Turkey) minus Point L (Northern France)

	$\Delta H < 0$	$\Delta H \gtrsim 0$	Total
Class	1	2	//////
Dep. cases	352	509	861
Indep. cases	87	215	302

Stratification III: Zonal Flow Index, Point K (South of Ireland) minus T (Northern Norway)

	$\Delta H \lesssim 900$	$900 < \Delta H \lesssim 1900$	$\Delta H > 1900$	Total
Class	1	2	3	///////
Dep. cases	278	300	283	861
Indep. cases	89	128	85	302

Stratification IV: Zonal Flow Index, Point C (Eastern Turkey) minus Point T (Northern Norway)

	$\Delta H \lesssim 900$	$900 < \Delta H \lesssim 1900$	$\Delta H > 1900$	Total
Class	1	2	3	///////
Dep. cases	245	362	254	861
Indep. cases	38	152	112	302

TABLE 2

Geostrophic Vorticity Stratifications

(ζ_g in 10^{-5} sec^{-1})

Stratification V: Small Scale Laplacian (see figure 3A)

	$\zeta_g \leq -1$	$-1 \leq \zeta_g < 2$	$2 \leq \zeta_g < 5$	$\zeta_g \geq 5$	Total
Class	1	2	3	4	////
Dep. cases	324	284	194	59	861
Indep. cases	93	92	82	35	302

Stratification VI: Large Scale Laplacian (see Figure 3B)

	$\zeta_g \leq 0$	$0 < \zeta_g \leq 2$	$\zeta_g \geq 2$	Total
Class	1	2	3	/////
Dep. cases	430	242	189	861
Indep. cases	125	90	87	302

TABLE 3

Height Difference-Geostrophic Vorticity Stratifications
(ΔH in Feet; ζ_g in 10-5 sec^{-1})

Stratification VII: Small Scale Laplacian vs. Zonal Flow Index, Point C (Eastern Turkey) minus Point T (Northern Norway)

			$\Delta H \leq 900$	$900 < \Delta H \leq 1900$	$\Delta H > 1900$	tot.
Small Scale Laplacian	$\zeta_g \leq 0$	Class	1	3	5	////
		Dep. Cases	131	177	114	422
		Indep. Cases	18	61	39	118
	$\zeta_g > 0$	Class	2	4	6	////
		Dep. Cases	114	185	140	439
		Indep. Cases	20	91	73	184

Stratification VIII: Small Scale Laplacian vs. Zonal Flow Index, Point K (South of Ireland) minus Point T (Northern Norway)

			$\Delta H \leq 0$	$0 < \Delta H \leq 1900$	$\Delta H > 1900$	total
Small Scale Laplacian	$\zeta_g \leq 0$	Class	1	3	5	////
		Dep. Cases	21	234	167	422
		Indep. Cases	3	79	36	118
	$\zeta_g > 0$	Class	2	4	6	////
		Dep. Cases	44	279	116	439
		Indep. Cases	11	124	49	184

Stratification IX: Large Scale Laplacian vs. Zonal Flow Index, Point C (Eastern Turkey) minus Point T (Northern Norway)

			$\Delta H \leq 900$	$900 < \Delta H \leq 1900$	$\Delta H < 1900$	tot.
Large Scale Laplacian	$\zeta_g \leq 0$	Class	1	3	5	////
		Dep. Cases	120	197	113	430
		Indep. Cases	16	63	46	125
	$\zeta_g > 0$	Class	2	4	6	
		Dep. Cases	125	165	141	431
		Indep. Cases	22	89	66	177

III. VERIFICATION

In verifying the results of this study, it was first necessary to establish unstratified climatological values. This was done by computing the mean values and variances of the u and v-components at the eleven points (Table 4) from the dependent data of 861 days mentioned in Section II. The u-component is the eastward and the v-component the northward component of the geostrophic wind.

TABLE 4

Climatological Means

Point	$\bar{u}$ (m/sec)	$\bar{v}$ (m/sec)	$\bar{\sigma}_u^2$	$\bar{\sigma}_v^2$
1	9.7	-4.8	137.8	219.9
2	10.8	-3.8	159.8	257.1
3	10.5	-4.3	157.1	246.6
4	11.0	-3.7	154.9	233.2
5	11.0	-3.6	173.7	236.2
6	10.8	-3.8	163.0	238.7
7	11.2	-2.9	162.9	234.4
8	9.2	-0.9	173.1	237.8
9	11.0	-3.5	178.9	201.0
10	12.3	-2.8	169.2	190.7
11	13.7	-0.4	165.1	203.8

The means and variances of the individual classes which were obtained through stratification were computed also. Once the means of the classes were determined, the dispersion or spread around these means was examined both for the dependent and independent predictand data.

The root mean square error $(RMS)_t$ around a given class mean (dependent data) is,

$$(RMS)_t = \sqrt{\frac{\sum_{i=1}^{n_t} (Y_{i,t} - \bar{Y}_t)^2}{n_t}} \qquad (6)$$

where $\bar{Y}_t$ is the mean value of predictand class t, n_t is the number of dependent observations in the class and $Y_{i,t}$ is an observed value in class t. The $(RMS)_t$ values for the individual classes were then compared with the root mean square error $(RMS)_c$ obtained when the unstratified data were used. The unstratified climatological variance is,

$$(RMS)_c = \sqrt{\frac{\sum_{t=1}^{K} \sum_{i=1}^{n_t} (Y_{i,t} - \bar{Y}_c)^2}{N}} \qquad (7)$$

where $\bar{Y}_c$ is the unstratified climatological mean, k is the number of classes into which the unstratified data is partitioned and N is the total number of dependent observations. The comparison in the dependent data for each class t is made using a reduction in error statistic $(RE)_t$,

$$(RE)_t = 1 - \frac{(RMS)_t^2}{(RMS)_c^2} \qquad (8)$$

The reduction in error statistic in the dependent data compares the variance of a given class with the variance of the entire dependent sample. Thus a reduction in error of zero indicates that the variance of the class is the same as the variance of the unstratified data. A positive value of $(RE)_t$ indicates that the variance of the class is smaller while a negative value indicates that the class variance is larger.

In the independent data, the reduction in error statistic tests the success of the stratification by comparing the variance of the independent data about the class mean with its variance about the unstratified mean. Thus for the independent data, the RE for class t is,

$$(RE)'_t = 1 - \frac{\sum_{i=1}^{m_t} (Y'_{i,t} - \bar{Y}_t)^2 / m_t}{\sum_{i=1}^{m_t} (Y' - \bar{Y}_c)^2 / m_t} \qquad (9)$$

where the primes indicate independent data and m_t is the number of independent observations in class t. $\bar{Y}_t$ and $\bar{Y}_c$ are the class means and unstratified mean, repectively, as determined from the dependent data. A negative $(RE)'_t$ indicates that the stratified mean produces poorer results than would be achieved by merely using the unstratified climatological mean. A positive value indicates improvement. Thus the reduction in error statistic reveals the percentage improvement over climatology.

In using both dependent and independent data to measure the success of a given stratification at a predictand point it is desirable to obtain a quantity which describes the reduction in error which is achieved when all classes within the stratification are considered. This mean value of the reduction in error for dependent data can be expressed as,

$$(\overline{RE}) = 1 - \frac{\sum_{t=1}^{K} \sum_{i=1}^{n_t} (Y_{i,t} - \bar{Y}_t)^2}{\sum_{t=1}^{K} \sum_{i=1}^{n_t} (Y_{i,t} - \bar{Y}_c)^2} \qquad (10)$$

Expression (10) also applies to the independent data except, of course, the summation within in a class is from $i = 1$ to $i = m_t$, where m_t is the number of independent observations which fall within class t. The mean reduction in error provides a measure of the comparative success of the different stratifications. For a stratification to be considered successful it must achieve a significant reduction of error in both the dependent and independent data.

The reduction in error values for the individual classes of each stratification are listed in the Appendix. The mean values of the reduction in error (computed from expression 10) are summarized in Tables 5, 6 and 7.

TABLE 5

Mean RE (%) Obtained From Height Difference Stratifications

Stratification I: Pt. C (Turkey) minus Pt. L (France) vs. Pt. K (Ireland) minus Pt. T (Norway)

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	9.1	21.3	17.1	10.1	25.3	21.4	10.8	16.5	16.5	9.5	3.8
Ind u	-1.2	12.4	6.7	6.2	15.3	13.3	5.7	12.4	13.1	8.0	-0.5
Dep v	17.1	11.8	9.5	9.3	7.2	6.5	4.8	24.6	9.6	2.3	7.1
Ind v	5.5	9.0	1.1	2.4	14.5	3.2	0.3	23.7	7.2	-6.2	3.6

Stratification II: Pt. C (Turkey) minus Pt. L (France)

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	1.5	1.3	2.1	0.7	2.4	2.7	0.4	3.8	3.3	0.4	0.0
Ind u	-0.8	0.3	0.9	-0.2	3.0	2.7	0.2	4.6	3.7	0.0	0.1
Dep v	0.1	7.3	0.5	1.0	14.5	2.8	0.9	24.0	7.7	0.2	6.5
Ind v	-0.3	11.0	2.0	0.3	17.1	3.8	-0.2	26.0	9.0	-5.7	5.6

TABLE 5 (Continued)

Stratification III: Pt. K (Ireland) minus Pt. T. (Northern Norway)

	Point 1	2	3	4	5	6	7	8	9	10	11
Dep u	5.5	16.0	11.8	8.3	16.6	13.6	8.3	7.8	8.3	6.8	3.5
Ind u	0.0	9.0	4.0	3.1	8.0	7.0	5.0	4.1	4.8	7.6	2.6
Dep v	16.0	1.8	7.5	9.5	0.4	2.4	4.8	2.1	1.1	2.1	2.6
Ind v	2.3	-2.4	-4.1	3.5	0.0	-2.6	1.1	2.1	-0.5	-2.5	-0.2

Stratification IV: Pt. C (Eastern Turkey) minus Pt. T (Northern Norway)

	Point 1	2	3	4	5	6	7	8	9	10	11
Dep u	17.6	34.7	30.3	16.9	38.9	35.8	18.9	24.2	25.4	13.9	2.3
Ind u	1.2	19.4	7.6	1.8	36.6	27.7	9.4	27.0	31.0	18.4	0.6
Dep v	19.5	10.6	10.9	5.4	6.0	7.0	1.6	3.4	6.7	0.6	0.6
Ind v	20.8	10.0	14.0	6.4	3.0	3.0	3.2	5.4	-1.5	-0.9	0.7

TABLE 6

Mean RE (%) Obtained From Geostrophic Vorticity Stratifications

Stratification V: Small Scale Laplacian (Figure 3A)

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	0.1	-0.1	-0.1	-0.1	0.1	0.2	0.6	0.3	0.9	2.6	1.0
Ind u	0.3	1.9	-0.2	-1.1	0.5	-2.2	-0.8	-0.2	-2.0	-2.0	-0.1
Dep v	0.0	0.9	0.2	0.6	0.0	0.0	0.6	-0.3	0.1	0.1	0.6
Ind v	-1.8	0.4	-4.7	-3.7	-1.2	-4.1	-1.6	-1.4	-1.7	0.0	0.5

Stratification VI: Large Scale Laplacian (Figure 3B)

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	1.1	0.5	0.6	0.1	0.1	0.2	0.3	0.4	1.2	2.5	0.8
Ind u	-1.0	-1.4	-2.1	-0.9	0.4	-1.3	-0.2	-0.5	-2.3	-1.9	1.1
Dep v	0.7	0.4	0.3	2.1	0.9	0.0	1.9	0.3	0.2	0.5	2.6
Ind v	-2.9	0.4	-2.9	-4.0	0.1	-0.6	-2.6	0.3	-1.0	-0.1	0.9

TABLE 7

Mean RE (%) Obtained From Height Difference - Geostrophic Vorticity Stratifications

Stratification VII: Height Difference (Pt. C - Pt. T) vs. Small Scale Laplacian Vorticity

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	17.4	34.7	30.2	17.0	38.7	36.3	19.4	24.0	26.7	15.3	2.5
Ind u	-0.1	18.8	7.3	2.2	36.3	28.4	11.7	26.1	28.9	18.2	0.6
Dep v	20.4	10.5	11.8	6.1	6.0	7.4	3.4	3.3	6.9	1.4	1.5
Ind v	19.1	9.9	10.8	5.4	1.7	1.4	3.7	4.2	-1.5	-0.1	0.5

Stratification VIII: Height Difference (Pt. K - Pt. T) vs. Small Scale Laplacian Vorticity

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	3.0	15.2	9.8	5.0	16.5	13.9	9.5	7.5	8.9	8.3	4.4
Ind u	-0.9	9.8	1.7	2.3	8.5	6.2	4.9	2.1	1.6	6.6	3.0
Dep v	11.9	0.8	3.4	6.9	0.3	0.6	3.3	2.1	0.6	0.8	3.0
Ind v	-1.8	-2.7	-5.4	1.5	-0.5	-4.1	-3.1	0.9	-0.7	-2.0	-3.1

Stratification IX: Height Difference (Pt. C - Pt. T) vs. Large Scale Laplacian Vorticity

	Point										
	1	2	3	4	5	6	7	8	9	10	11
Dep u	18.7	35.1	30.7	16.7	39.0	36.0	19.1	24.1	26.7	16.4	3.1
Ind u	-0.3	17.5	5.1	0.3	36.2	26.8	9.0	26.3	28.6	17.0	2.0
Dep v	20.3	10.9	11.6	7.6	6.6	7.0	3.5	4.2	7.1	1.3	2.5
Ind v	18.7	9.4	11.4	1.1	1.3	1.9	0.5	4.2	-2.6	-1.7	-0.9

IV. RESULTS OF STRATIFICATION

In discussing the results of the stratification, repeated references will be made to the reduction in error results listed in Tables 5,6, and 7 and the Appendix. The reader may also wish to refer to Table 1 - 3 which present the class limits of the different predictors.

A. Height Differences.

In comparing the results of these stratifications using the mean RE of all classes for a given stratification, several interesting points are noted. See Table 5. For example, stratification I, which is based on the combination of the meridional and zonal flow in the predictor region, is more successful than stratifications II and III which are separately determined by the meridional and zonal flow,respectively. This is true in both dependent and independent data. In stratification I, seven of the predictor points show RE's above 0.10 in the independent data, while between them stratifications II and III have only three RE's above 0.10. Except for points 1 and 11 (u-component) and point 10 (v-component), the results of stratification I show improvement over unstratified climatology. The mean RE values obtained for u-component tend to be higher than those obtained for the v-component. This tendency is noted in all stratifications except stratification II where the meridional flow is only used as a predictor. In general, the results of stratifications II and III must be considered unsatisfactory.

Of the four stratifications based on height differences stratification IV, which involves height differences between Turkey and Norway (points C and T), has the highest mean RE's (Table 5). Again the results for the v-component do not measure up to those obtained for the u-component. The results obtained in the dependent data are, of course, more favorable than those obtained in the independent data. An areal description of the mean RE's for the independent data of stratification IV may be found in Figures 4 and 5. The mean RE's for the v-component are highest in the northern portion of the predictand area but fall off to near climatology over the southern portion. On the other hand, the u-component results are most favorable in the southwestern portion of the predictand area with a general decrease toward the east. Along the Ural Mountains the results do not exceed climatology; however, over a large portion of the predictand area the results seem quite satisfactory. These favorable results support the work of Cole (6) who has also used height differences between Norway and Turkey to stratify wind data, but in this case the results have been verified using independent data.

Since the results of stratification IV have been fairly successful, it might be well to examine the reduction in error statistics for the individual classes (Table D, Appendix).* The RE's for all three classes of the u-component

*Table 8 presents the mean values of the u and v components for each of the three classes of the stratification at the eleven predictand points.

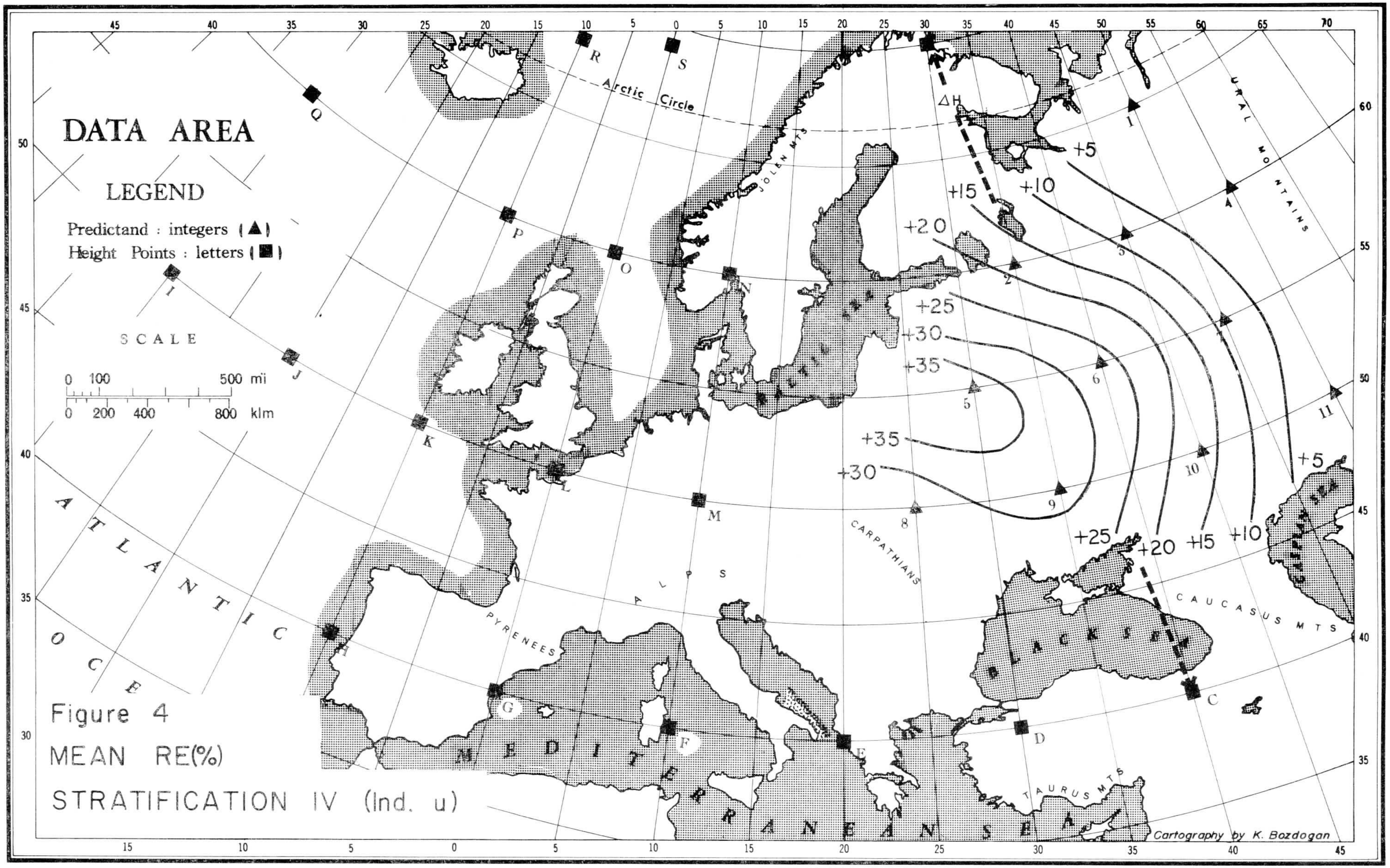


Figure 4
MEAN RE(%)
STRATIFICATION IV (Ind. u)

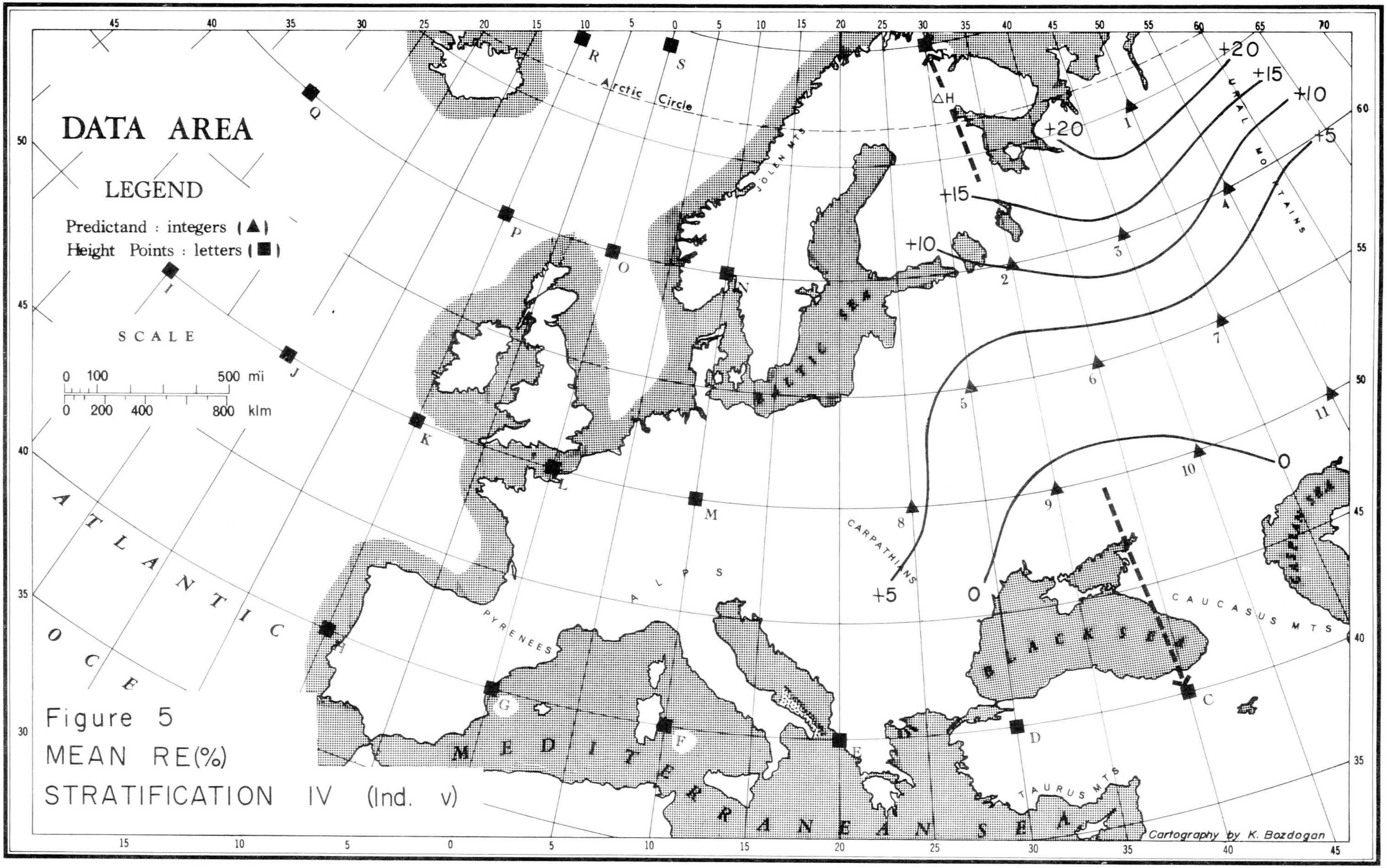


Figure 5
MEAN RE(%)
STRATIFICATION IV (Ind. v)

TABLE 8

Mean Class Values of u and v-Components

Stratification IV: Turkey - Norway

u-Component (m/sec)

Class	Point 1	2	3	4	5	6	7	8	9	10	11
1	2.6	0.2	0.7	3.8	-0.4	0.2	3.3	0.9	2.0	5.9	10.6
2	10.3	11.6	11.2	11.5	11.5	11.2	11.8	8.8	10.9	12.2	14.3
3	15.6	19.7	18.8	17.2	21.2	20.2	17.7	17.8	19.7	18.6	15.7

West Wind - Positive
East Wind - Negative

v-Component (m/sec)

Class	Point 1	2	3	4	5	6	7	8	9	10	11
1	-13.8	-11.3	-11.5	-8.0	-8.7	-9.6	-5.3	-4.9	-8.6	-4.8	1.7
2	-4.5	-3.2	-4.0	-4.4	-3.6	-3.4	-3.3	-0.8	-3.2	-2.4	-1.1
3	3.5	2.5	2.3	1.3	1.3	1.3	0.1	2.7	1.2	-1.6	-1.4

South Wind - Positive
North Wind - Negative

(dependent data) show considerable improvement over climatology, with some RE's showing as much as 40 per cent improvement over climatology. In the independent data the results for classes 1 and 3, which represent weak westerlies (or easterlies) and strong westerlies, respectively, generally compare favorably with the dependent results for these classes. The only significant exception is found in the class 1 results along the Ural Mountains (points 4, 7, and 11). Class 2, which represents westerly flow of intermediate strength, fails to show any improvement over climatology in the independent data. However, this lack of improvement may be due to the fact that the class 2 mean is about equal to the unstratified mean at all eleven points. This results in low values of the reduction of error in the independent data while still giving high values in the dependent data. Thus it seems that stratification is most useful when the classes represent the more extreme values of the data. When the independent observations consist of values which are near the climatological mean the reduction in error statistic shows little or no improvement over climatology. This does not mean that the stratified data necessarily fails to provide any advantage over the unstratified data. For example, given a successful stratification the investigator's prior knowledge of the associated error distribution of each class permits him to have more confidence in his prediction.

An inspection of Table D2 reveals that v-component results of stratification IV are not as satisfactory as those of the u-component. Classes 1 and 2 which represent strong northerly and weak northerly winds, repectively, show little improvement over climatology. In fact,the class 1 independent data has RE's which are predominantly negative; however, class 3 (weak southerly winds) shows significantly high RE's both in the dependent and independent data. When examining the v-component results of stratification IV, the reader should keep in mind that the stratification is based on height differences between Turkey and Norway and thus is a measure of the zonal flow. In general, the results of all the height stratifications indicate that the v-component of the geostrophic wind can best be predicted using the v-component in the predictor region, and similarly, the u-component is best predicted by the u-component over the predictor area. Tables D1, D2 and 5 reveal that the success of stratification IV varies considerably between the eleven predictor points. For example, the mean RE's (Table 5) of the v-component results, independent data, vary from -1.5 at point 9 to +20.8 at point 1. When the individual classes are considered the variations are even more pronounced. In class 1 the independent data v-component results vary from -40.0 at point 6 to +15.4 at point 1 (Table D2). Variations such as these indicate that even though the RE averaged over the eleven points may be small, (or even negative), valuable information concerning an individual predictand point may be obtained. Similarly, the results presented in tables D1 and D2 reveal that a stratification may provide important information concerning a given class of data although the mean RE of all classes does not appear to be significant. A careful examination of the tables in the Appendix will provide the reader with such information. The reader may also wish to investigate the number of independent observations in each class when investigating the significance of the individual class RE's.

B. Geostrophic Vorticity

Table 6 presents the mean RE's attained through stratifications V and VI, which are based on the geostrophic vorticity computed from the small scale and large scale Laplacians, respectively. It is readily apparent that these stratifications afford no improvement over unstratified climatology. The results are consistently poor for all predictor points. An examination of Tables E1 and E2 reveals that only in class 4 (stratification V) of the dependent data do the individual class RE's appear significantly positive. Class 4 is associated with strong cyclonic motion as measured by the small scale Laplacian, which covers an area of the eastern North Atlantic Ocean. However, since the RE's of the independent data for this class are either small or negative, the stratification cannot be considered satisfactory.

These results indicate that little or no improvement over unstratified climatology is attained through the use of vorticity alone. Little difference is noted between the large scale and small scale computations of vorticity. However, as will be shown in the next section, vorticity may be combined with height differences to obtain successful stratifications.

C. Combination of Height Differences and Geostrophic Vorticity

Stratifications VII, VIII and IX are based on a combination of height differences and geostrophic vorticity. Stratification VII combines the zonal flow between Turkey and Norway with the vorticity determined from the small scale Laplacian, while stratification IX combines this same zonal flow with the vorticity determined from the large scale Laplacian. On the other hand, stratification VIII combines the zonal flow between Ireland and Norway with the vorticity as measured by the small scale Laplacian. The results of these stratifications are listed in Table 7 and in Tables G, H and I of the Appendix. These stratifications each contain six classes (see Table 3).

The mean RE's attained by stratification VII (Table 7) are quite satisfactory. An areal description of the independent results may be found in Figures 6 and 7. The results of this stratification are quite similar to those of stratification IV. Since both these stratifications involve height differences between Turkey and Norway, it seems likely

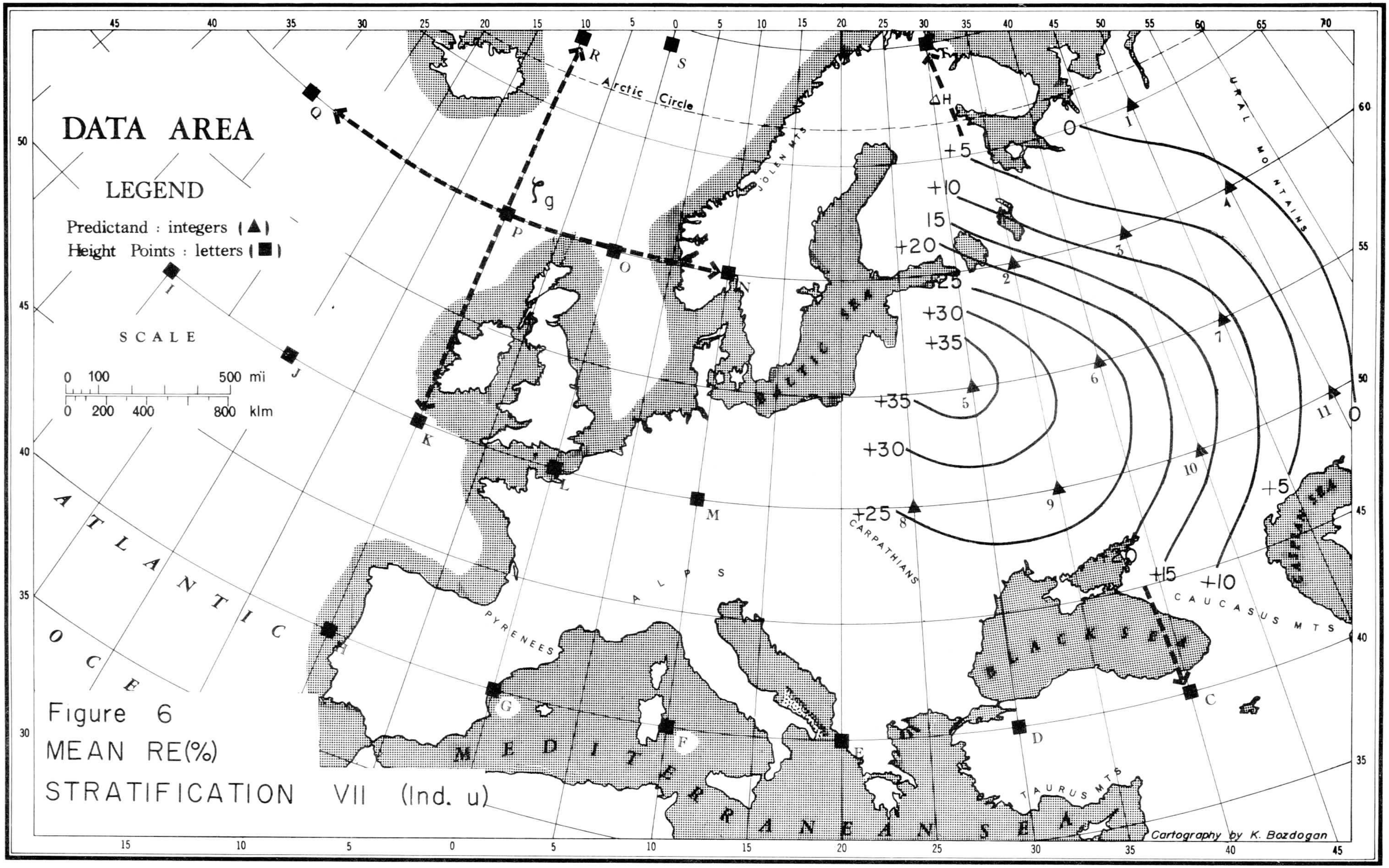


Figure 6
MEAN RE(%)
STRATIFICATION VII (Ind. u)

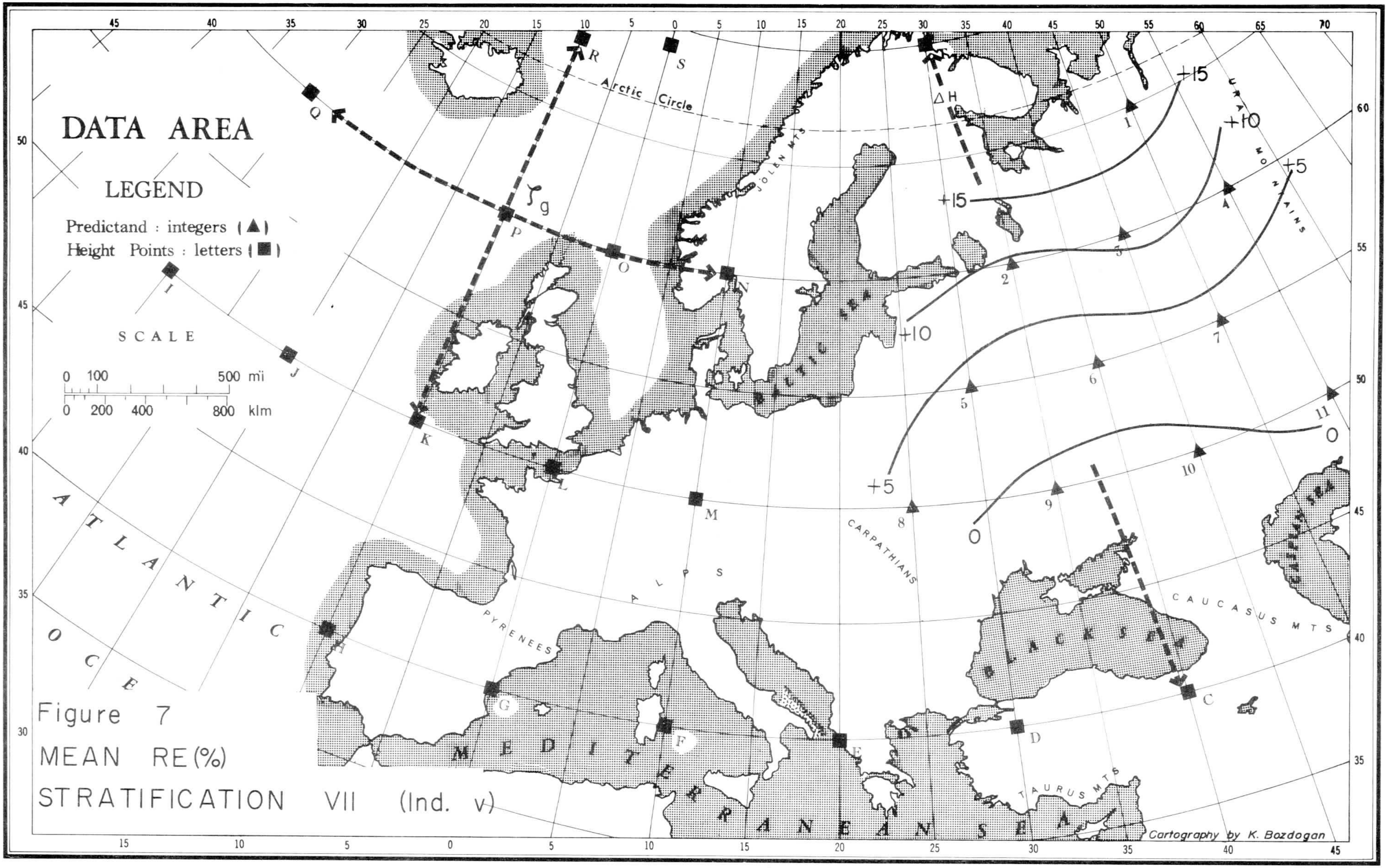


Figure 7
MEAN RE(%)
STRATIFICATION VII (Ind. v)

that the height differences play the more significant role in the success of stratification VII. Again the u-component results are most successful with mean RE's of 18 to 36 per cent in the western portion of the predictand area. These high values decrease to near zero along the Ural Mountains. The v-component results (Figure 7) are likewise similar to those of stratification IV, with relatively large mean RE's over the northern portion of the predictand area but with results near unstratified climatology over the southern portion.

An examination of the RE's attained by the individual classes of stratification VII reveals that in the u-component, dependent data only seven of a possible sixty-six RE's (six classes, eleven predictor points) were negative (See Tables G1 and G2). Each of the six classes of the stratification had, at least one predictand point, an RE of above 35 percent. As in the case of the mean RE's, the lowest values were found along the Ural Mountains. Also as previously, the individual class RE's of the v-component results were not as high as those of the u-component results. This applies to both dependent and independent data. The reader may examine the results in more detail by studying Tables G1 and G2 along with Table 9 which lists the mean values of the u and v-components for each of the classes of stratification VII.

Table 7 reveals that stratification IX which combines the zonal flow between Turkey and Norway with vorticity computed from the large scale Laplacian has results quite

TABLE 9

Mean Class Values of u and v-Components

Stratification VII

u-Component (m/sec)

Class	Point 1	2	3	4	5	6	7	8	9	10	11
1	3.2	-0.3	0.7	5.2	-0.3	1.2	4.7	1.4	3.9	7.8	11.0
2	2.0	0.7	0.6	2.2	-0.5	-1.0	1.6	0.4	-0.2	3.7	10.2
3	10.3	11.9	12.0	11.9	11.9	12.6	13.0	8.8	12.1	13.5	15.3
4	10.3	11.3	10.5	11.1	11.1	9.9	10.8	8.7	9.8	11.0	13.2
5	15.3	18.7	18.2	17.3	20.8	20.8	18.1	18.1	21.7	20.5	16.9
6	15.9	20.6	19.2	17.2	21.5	19.7	17.4	17.6	18.0	17.1	14.6

West Wind - Positive
East Wind - Negative

v-Component (m/sec)

Class	Point 1	2	3	4	5	6	7	8	9	10	11
1	-12.4	-11.8	-9.6	-6.6	-9.7	-8.5	-1.8	-5.0	-8.0	-2.5	3.8
2	-15.6	-10.9	-13.8	-9.8	-7.8	-10.9	-9.4	-4.9	-9.6	-7.6	-0.8
3	-2.5	-3.9	-2.7	-2.5	-3.6	-2.4	-2.2	0.0	-2.2	-1.7	0.0
4	-6.3	-2.6	-5.3	-6.2	-3.6	-4.4	-4.3	-1.6	-4.2	-3.0	-2.1
5	4.5	2.3	3.8	2.6	-0.2	3.0	1.8	1.1	2.2	-0.6	0.7
6	2.6	2.7	1.1	0.4	2.6	0.0	-1.3	4.1	0.4	-2.3	-3.0

South Wind - Positive
North Wind - Negative

similar to those of stratification VII. Evidently, the size of the Laplacian has little effect on the results. Table 7 also reveals that the results of stratification VIII, which combines height differences between Ireland and Norway with the small scale Laplacian vorticity, are not particularly satisfactory. Obviously, a stratification based partially on height differences in an area far to the west of the predictand area does not succeed as well as one based on height differences across the predictand area. Tables H1 and H2 reveal great variations in the RE's between the individual classes and predictor points. For example, in the independent data of Table H1 (u-component) RE's range from 80.1 to -149.9 percent. Again the reader may examine the tables in more detail to gain useful information concerning particular types of conditions at specific predictor points.

V. SUMMARY AND CONCLUSIONS

The reduction in error statistics obtained in this study indicate that a stratified wind climatology can provide the meteorologist with information which is superior to that provided by unstratified climatology. The size of the independent data sample was sufficiently large to make these results significant. Of course, in any investigation of this type the degree of improvement over unstratified climatology depends upon a wise or fortituous choice of parameters. The following statements may be made and conclusions drawn concerning the results of this study:

1. The stratifications based on height difference were more successful than those based on geostrophic vorticity. The geostrophic vorticity alone cannot be considered as a suitable parameter for stratification; however, in combination with the height differences the vorticity can be used as a useful parameter. The success of the combination of height differences and vorticity suggests that Grosswetter Types may be useful stratification parameters. Grosswetter types consider both the strength of the zonal (or meridional) wind components and the clyclonic (or anticyclonic) nature of the flow.

2. As would be expected the closer the predictor area is to the predictand area the better are the results attained. For example, the height differences between

Norway and Turkey produced much better results than those further to the west. In this study the heights at predictor points located in Norway and Turkey represent conditions on either side of the predictand area. Such a choice of predictor points is obviously advantageous.

3. The stratifications used here indicate that better results can be attained for the u-component of the geostrophic wind than for the v-component. The difficulty in determining the positions at troughs and ridges is, no doubt, the cause of the poorer results for the v-component. The fact that height differences which measure the meridional flow in the predictor region serve as a better predictor of the v-component than do the height differences which measure the zonal flow indicates that it is important to use predictors which somehow determine the position of the major troughs and ridges.

4. In this study the eleven predictand points cover a large section of Eastern Europe. As would be expected, there is considerable variation in the success of the stratification techniques between the various predictand points. Although there is a general tendency for the results to be poorest in the eastern portion of the

predictand region, the reader may find that even in the most remote sections of the predictand region certain classes of a given stratification provide valuable information. Furthermore, one would hardly expect a single predictor to be highly correlated with all eleven predictand points. Yet in this study it has been shown that certain stratifications have produced favorable results over extensive areas. Such results lend encouragement toward the eventual production of an atlas of the stratified wind climatology of extensive areas.

5. In general, the extreme classes of a given stratification produced better reduction in error results than did the middle classes of the stratification. The mean values of the middle classes tend to be near the unstratified climatological means, so forecasts of these classes give results which are equivalent to the unstratified climatological means. However, it has been pointed out that the stratified results are of value because they permit the forecaster to put greater confidence in his prediction than he could if he had used unstratified climatology. By an examination of the reduction in error statistics for individual classes, the reader will discover some unusually high values for certain types of conditions in the predictor region. An investigation of these conditions may lead to improved stratification techniques.

BIBLIOGRAPHY

1. Christensen, Walter I., Reid A. Bryson and Charles W. Hutchins, "An Experiment in Stratified Climatology of Upper Winds", Scientific Report No. 2, AFCRC-TN-201, Department of Meteorology, University of Wisconsin, February 1960.

2. Lorenz, Edward N., "Prospects for Statistical Weather Forecasting", Final Report, AFCRC-TN-59-224, Department of Meteorology, M.I.T., January 1959.

3. White, R.M., "Pressure-Height Predictability as a Function of the Type and Amount of Initial Data", Journal of Meteorology, 13, 40-45, 1956.

4. Panofsky, Hans A., Some Applications of Statistics to Meteorology, The Pennsylvania State University, 1958.

5. Cochran, William C., Sampling Techniques, John Wiley and Sons, Inc., 1953.

6. Cole, A.E., "Review of Preliminary Study Conducted to Investigate the Feasibility of Developing a Stratified Climatology of Upper Winds", Geophysics Research Directorate, Air Force Cambridge Research Center, Cambridge, Massachusetts, date unknown.

7. Lahey, James F., Reid A. Bryson, Harold A. Corzine, Charles W. Hutchins, and Alison Lathbury, "Atlas of 300 mb Wind Characteristics for the Northern Hemisphere", Final Report, Part I, AFCRC-TR-59-270, Department of Meteorology, University of Wisconsin, March 1960.

8. Haltiner, George J. and Frank L. Martin, Dynamical and Physical Meteorology, McGraw-Hill Book Co., Inc., 1957.

9. Essenwanger, O., Berichte der Deutshen Wetterdienstes, No. 10, Bad Kissingen (1954).

APPENDIX

TABLE A1

Stratification I: Class RE (%) u-Component

Upper Row: Dependent Data
Lower Row: Independent Data

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pt.
1	9.8	1.8	14.9	36.8	36.0	17.6	19.1	43.2	22.9	2.4	0.0	18.6
	35.0	29.9	65.3	69.7	-2.5	-9.1	-45.2	-82.5	-29.2	-37.9	-51.0	-5.2
2	-16.8	3.7	-1.9	-15.3	17.2	14.2	-2.2	22.6	12.2	-4.5	-10.6	1.7
	-12.3	1.3	-7.6	-12.4	-6.6	-2.7	-1.9	-4.6	-2.2	1.5	1.1	-4.2
3	10.3	0.2	0.8	7.7	28.2	30.6	14.9	15.5	16.2	24.3	34.3	16.6
	-15.3	-1.7	-11.5	-1.5	-3.1	-6.7	-9.8	-0.5	-3.7	-10.7	-16.1	-7.3
4	16.0	37.9	20.9	7.8	35.1	26.5	14.7	22.6	26.2	23.4	-5.2	20.5
	7.1	6.8	9.9	7.4	12.9	6.8	14.0	10.8	8.0	0.9	0.2	6.8
5	13.7	32.6	37.3	25.5	21.5	34.5	31.7	-2.7	10.2	8.9	24.7	21.6
	3.9	11.1	2.5	3.7	3.3	-0.2	3.1	1.7	0.9	12.5	8.8	4.7
6	40.0	44.6	35.6	26.8	23.1	3.2	-9.0	12.2	18.1	6.7	-9.5	17.4
	-1.5	48.5	33.4	27.9	53.7	50.2	33.6	50.9	49.7	41.4	-9.2	34.4

TABLE A2

Stratification I: Class RE (%) v-Component

Upper Row: Dependent Data
Lower Row: Independent Data

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	11.3	40.8	14.0	25.3	32.0	14.7	-21.3	9.3	40.1	-28.2	-54.5	6.4
	-101.1	22.6	-19.0	-16.7	13.4	8.9	-20.4	44.9	7.4	-11.6	5.4	-6.0
2	31.2	18.3	22.2	24.9	13.3	22.2	17.8	25.6	19.9	28.4	10.9	21.3
	-16.8	-10.0	-28.7	4.6	10.6	-11.6	-15.0	18.5	4.6	-30.7	3.1	-6.5
3	12.0	17.1	-4.1	5.4	23.8	-6.0	-11.9	4.5	-0.6	-13.1	2.3	1.7
	1.4	1.6	-1.8	2.8	14.2	-4.6	18.2	31.7	-3.6	20.2	24.7	9.5
4	14.7	7.7	-0.4	6.6	13.1	14.8	10.8	27.3	17.1	16.9	16.4	13.2
	10.3	31.2	14.0	0.3	27.0	20.3	1.6	20.0	20.9	1.2	0.1	13.4
5	7.0	10.2	5.9	-2.0	13.5	-7.7	4.8	27.0	-17.5	-17.4	21.1	4.1
	14.8	-7.2	4.0	3.5	12.6	0.3	14.8	28.7	2.6	5.5	9.5	8.1
6	23.3	-10.5	22.8	10.4	12.9	-3.5	8.2	47.8	13.0	-1.5	11.7	12.2
	31.2	15.5	21.1	12.7	11.3	4.9	7.4	33.5	7.4	0.1	4.3	13.6

TABLE B1

Stratification II: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predict- and Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	5.5	-3.6	7.7	13.5	-1.7	6.5	8.9	-6.4	-6.2	-4.0	13.8	3.1
	-8.8	-4.0	-5.9	-4.6	-3.1	-4.2	-3.6	-3.2	-2.8	-3.7	2.8	-3.7
2	-0.9	4.8	-1.6	-7.9	5.4	0.3	-5.2	11.1	10.2	3.6	-9.4	-0.9
	3.0	2.3	3.8	2.3	5.8	5.4	2.3	8.8	7.4	2.0	-4.1	3.5

TABLE B2

Stratification II: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predict and Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-10.0	16.3	-5.4	-7.7	20.5	-6.2	-11.1	15.8	-2.7	-22.8	-1.8	-1.4
	0.0	-5.1	-3.6	4.9	13.3	-4.0	9.5	30.9	-0.9	4.1	15.7	5.9
2	6.9	1.2	4.8	7.2	10.6	9.3	9.3	29.9	15.0	16.3	12.4	11.2
	0.0	18.4	4.3	-1.0	9.4	9.3	-3.8	22.7	14.4	-2.3	2.1	6.7

TABLE C1

Stratification III: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-12.2	0.6	-1.1	-3.9	17.0	7.4	-1.2	20.9	5.9	-6.8	-8.1	1.7
	-13.2	-0.9	-12.5	-16.0	-14.7	-11.3	-7.7	-15.9	-11.7	-0.3	1.2	-9.4
2	9.1	13.6	5.5	5.6	13.6	14.5	11.1	3.4	9.0	18.9	9.1	10.3
	4.3	2.9	6.3	8.3	1.5	2.5	2.8	-1.5	-1.9	-3.2	-3.5	1.7
3	19.4	33.9	31.7	23.7	19.9	19.4	15.3	0.1	10.7	8.0	9.7	17.4
	8.5	26.7	13.6	11.5	26.0	23.2	16.1	17.8	18.3	25.7	13.9	18.3

TABLE C2

Stratification III: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave.11 pts.
1	24.4	19.9	17.3	24.9	11.4	16.1	7.7	11.1	16.9	13.2	-8.9	14.0
	-24.6	-22.3	-41.5	3.3	-2.9	-22.5	-15.2	6.3	-9.6	-16.6	2.7	-13.0
2	11.5	-3.6	-4.4	2.1	-3.1	3.9	1.8	-13.7	4.1	5.2	5.9	0.9
	3.2	5.8	6.2	0.9	3.6	10.1	5.5	2.1	9.0	4.9	0.1	4.7
3	13.1	-9.8	11.0	2.7	-5.8	-11.9	5.9	10.6	-16.8	-11.2	11.3	-0.1
	22.8	4.0	11.0	10.4	-0.2	0.7	11.3	0.5	-0.3	2.4	-1.4	5.6

TABLE D1

Stratification IV: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	3.4	20.9	15.4	6.0	43.7	31.2	14.6	41.5	16.0	3.9	8.1	18.6
	4.6	41.5	17.1	-13.8	51.6	30.3	-8.2	11.3	42.5	26.0	-9.0	17.6
2	7.7	37.3	30.0	11.9	32.0	40.3	23.9	9.8	26.0	16.8	4.9	21.9
	2.3	-0.7	1.1	1.5	-0.7	-0.7	0.1	0.6	0.1	0.1	-0.3	0.3
3	46.1	44.7	45.5	34.9	44.5	34.1	16.6	28.6	34.4	20.4	-6.1	31.2
	-0.3	30.7	14.9	9.9	52.3	48.5	27.9	46.5	49.9	36.0	5.5	29.3

TABLE D2

Stratification IV: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	18.5	33.0	11.1	2.9	23.1	16.8	-20.9	-2.9	16.5	-7.4	-17.7	6.6
	15.4	-11.8	-21.5	7.2	-9.8	-40.0	-9.0	13.4	-36.6	-19.2	6.6	-9.6
2	15.1	0.1	9.9	0.9	-7.7	1.6	13.7	-2.1	-6.9	5.8	1.6	2.9
	-0.3	1.9	0.3	0.9	0.0	1.0	-1.0	0.1	0.8	1.1	-1.3	0.3
3	27.4	4.8	13.1	14.7	9.6	6.0	6.7	17.9	17.3	1.9	17.6	12.5
	40.4	28.3	33.6	14.5	10.8	16.4	10.6	10.4	7.6	2.7	2.6	16.2

TABLE E1

Stratification V: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-8.6	1.4	-0.3	-1.9	-2.1	-4.8	-3.2	-9.2	-11.6	-3.7	-1.7	-4.2
	-0.4	-6.8	-1.1	0.5	-0.3	1.3	3.4	-0.9	1.8	4.0	2.6	0.4
2	2.2	-0.3	-4.5	-5.1	-9.3	-1.4	-3.3	-4.6	1.6	-5.9	-12.9	4.0
	-1.6	0.2	-0.1	0.8	-0.4	0.4	0.7	3.6	3.2	2.5	1.0	0.9
3	2.0	-14.3	-5.7	5.4	8.3	8.3	11.4	11.9	15.4	21.5	25.3	8.1
	3.8	3.0	-0.1	-3.4	2.2	-6.7	-2.6	-1.0	-7.9	-7.4	2.8	-1.6
4	36.4	44.9	46.1	21.8	35.1	14.8	9.8	41.6	22.9	12.2	-10.5	25.0
	15.9	15.8	14.4	2.2	8.7	1.0	-6.7	0.0	-0.4	-13.8	-11.2	2.4

TABLE E2

Stratification V: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	1.4	-0.6	4.8	3.5	-0.3	6.9	2.1	5.7	10.4	3.2	11.7	4.4
	2.1	-2.2	2.0	1.1	-1.1	2.0	5.4	0.0	3.5	5.6	1.6	1.8
2	-8.9	-1.1	-6.4	-10.6	-3.0	2.0	-3.8	-10.7	-9.0	6.4	1.9	-3.9
	-2.0	-2.8	-3.2	-1.2	-0.6	-1.4	1.8	-0.3	0.6	0.0	0.1	-0.8
3	1.3	2.9	-7.8	8.0	6.4	-16.9	-3.2	3.5	-3.9	-15.8	-9.7	-3.2
	-4.3	11.0	-10.5	-12.1	2.0	-5.9	-9.7	-0.4	-5.1	-3.3	3.7	-3.1
4	35.3	11.6	39.0	18.3	-0.3	14.1	29.8	9.1	5.5	9.3	-27.9	13.1
	0.1	1.3	-2.2	0.9	-2.4	-18.4	-2.1	-1.9	-6.4	0.5	-1.3	-2.9

TABLE F1

Stratification VI: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-8.9	8.3	3.6	1.4	-2.0	6.8	6.2	-8.3	-0.1	1.1	0.3	0.8
	-6.1	-4.1	-5.5	-2.9	-1.1	0.8	2.1	-1.3	2.6	5.4	4.7	-0.5
2	7.0	-4.3	-8.8	-1.7	-5.3	-7.0	-0.8	0.3	0.0	11.5	8.4	-0.1
	1.6	0.6	0.9	4.2	2.9	-0.7	-1.2	-0.4	-5.7	-7.7	-2.0	-0.7
3	17.3	-10.1	6.6	0.5	12.8	-4.7	-10.9	21.5	6.7	-4.6	-6.5	2.6
	6.6	2.4	3.0	0.3	1.5	-2.9	-0.8	4.6	-5.7	-3.1	1.6	0.7

TABLE F2

Stratification VI: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	5.9	-0.1	7.2	-3.1	-4.0	0.9	-0.4	-15.3	-0.9	-6.5	2.5	-1.3
	2.9	-4.1	2.8	-1.4	-3.9	1.5	5.4	-1.0	3.1	5.5	-0.1	1.0
2	-18.5	-8.1	-16.5	-2.4	-1.0	-5.3	8.5	10.8	-10.1	0.8	1.7	-3.6
	-0.7	1.1	-4.9	-3.4	0.4	-4.2	-6.1	0.6	-3.4	-4.8	-1.6	-2.5
3	14.3	13.2	7.1	20.8	15.8	5.8	-0.2	23.5	17.2	17.1	4.8	12.7
	-12.4	12.2	-6.3	-5.3	10.2	2.3	-8.1	5.3	1.3	-1.9	6.4	0.3

TABLE G1

Stratification VII: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-7.2	16.7	5.3	-2.0	40.5	23.5	10.3	33.0	10.9	5.0	1.2	12.5
	5.5	39.8	-5.0	-50.3	46.1	1.3	-47.1	1.0	23.6	-4.9	-31.4	-1.9
2	16.0	26.1	27.0	18.2	47.4	41.5	22.7	51.5	26.9	8.1	16.1	27.4
	2.5	49.5	67.2	53.2	61.2	70.3	44.5	19.3	59.1	53.9	5.8	44.2
3	-3.3	31.1	21.9	5.9	24.9	37.7	24.4	-5.1	17.6	17.4	15.3	17.1
	1.3	0.5	5.9	4.8	-2.2	2.8	8.7	1.3	-1.6	1.0	3.2	2.3
4	18.1	43.3	38.4	17.9	39.0	44.9	24.8	24.1	35.4	18.0	-3.9	27.3
	3.5	-0.6	0.0	0.1	0.0	3.3	1.5	-0.1	1.4	2.6	1.3	1.2
5	42.1	47.3	48.2	32.0	24.6	11.6	-2.8	13.1	14.3	6.5	-16.8	20.0
	7.3	23.9	8.3	-2.5	39.8	35.4	21.1	34.7	45.4	38.6	6.4	23.5
6	49.5	43.6	43.6	37.3	60.9	52.7	23.5	41.3	54.2	34.7	4.0	41.3
	-6.9	33.1	18.8	15.8	56.8	52.3	30.7	51.1	46.8	31.0	3.0	30.2

TABLE G2

Stratification VII: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	14.1	35.7	8.1	8.6	27.2	13.2	-13.9	-6.6	10.7	0.5	-14.6	7.5
	22.3	-6.9	-37.1	6.3	-22.4	-45.5	9.0	7.9	-46.6	3.5	18.3	-8.3
2	26.0	30.8	18.5	-0.5	19.7	23.1	-15.8	1.0	25.6	-9.4	-16.8	7.2
	-14.8	-22.4	-8.7	-2.6	2.9	-27.6	5.8	18.4	-5.2	-29.9	-0.4	-7.7
3	14.5	-5.5	12.9	3.7	-15.2	14.3	18.2	-9.1	5.0	5.1	15.9	5.4
	-4.1	-0.1	-3.6	-1.2	0.0	3.4	1.8	0.8	6.4	5.4	1.2	0.9
4	18.8	5.9	8.5	1.1	-0.6	-9.8	10.4	5.1	-17.3	6.9	-11.0	1.6
	0.1	4.9	-3.7	3.0	0.0	-1.6	-3.7	-1.3	-1.0	-0.3	-2.8	-0.6
5	40.6	2.4	27.1	18.9	10.4	12.9	18.2	19.9	27.6	9.7	27.9	19.6
	45.8	19.7	38.6	19.4	8.2	17.8	20.2	6.9	15.7	11.0	-2.8	17.3
6	18.0	7.3	3.8	12.5	10.9	2.6	-0.6	18.5	9.5	-3.4	12.6	8.3
	37.0	33.5	29.3	11.9	12.0	14.0	5.5	12.1	4.9	0.4	5.9	15.1

TABLE H1

Stratification VIII: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-2.4	8.0	6.2	-0.4	56.2	37.9	23.5	55.5	8.0	-8.7	32.7	19.7
	29.7	57.4	52.9	64.5	80.1	77.3	53.7	-11.6	62.4	64.9	-0.4	48.3
2	17.6	3.1	-4.4	-11.3	25.1	12.9	-4.5	52.6	21.3	18.9	23.2	14.0
	-48.2	52.9	34.4	64.8	-149.9	7.3	54.9	-102.5	-42.9	46.6	38.5	-4.0
3	-16.3	-3.7	-16.3	-10.8	3.1	4.0	7.3	-7.1	-4.2	6.0	-1.1	-0.6
	-7.2	-13.3	-15.8	-5.8	-7.4	-9.7	-7.5	-6.3	-5.4	-3.4	-3.4	-7.7
4	2.3	15.5	13.7	4.0	20.9	16.1	7.7	19.6	17.2	10.5	-0.8	11.5
	0.4	0.2	0.6	-0.5	-1.5	-0.7	-0.1	-1.3	-4.6	-2.2	0.8	-0.8
5	14.0	36.4	36.9	22.3	8.0	4.0	2.0	-2.9	0.2	7.0	9.7	12.5
	6.8	7.8	9.0	12.3	2.7	16.3	20.0	7.4	9.0	24.2	17.3	12.1
6	27.1	31.7	24.4	26.1	38.2	42.2	35.6	4.4	27.6	12.1	11.8	25.6
	9.8	38.5	17.1	10.5	38.0	23.8	11.5	23.1	19.4	20.6	9.0	20.1

TABLE H2

Stratification VIII: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	69.8	67.2	62.1	70.8	52.4	41.9	52.2	65.5	27.0	46.2	-10.0	49.6
	9.0	-28.6	-88.8	48.8	7.9	-22.6	-18.8	-8.1	-1.9	-11.3	-4.6	-10.8
2	26.4	36.8	22.5	21.6	8.7	39.0	11.2	-5.1	19.5	22.7	-7.8	17.8
	-125.6	-51.8	-72.4	-62.8	-10.0	-56.8	-30.0	-12.2	-26.3	-21.9	25.1	-40.4
3	2.1	4.1	0.0	1.6	-0.5	9.2	-1.3	-17.0	13.7	6.3	-0.6	1.6
	-4.2	-12.0	-2.4	-0.1	-2.9	-1.4	4.2	4.6	3.2	5.4	-0.3	-0.5
4	13.9	-0.2	-7.4	10.0	3.9	-3.1	1.4	7.6	1.4	3.0	2.2	3.0
	-7.2	4.4	-11.0	-4.5	6.1	-0.8	-13.3	4.2	1.8	-4.8	-3.1	-2.6
5	18.3	-15.5	12.4	9.7	-8.0	-1.8	15.4	14.1	-8.8	-1.5	25.3	-1.1
	17.2	-2.1	4.6	15.7	1.2	1.6	19.2	1.6	2.6	9.0	8.2	7.2
6	5.6	-1.5	9.5	-7.0	-2.6	-22.0	-7.8	5.6	-21.9	-24.6	-8.9	-6.9
	27.3	9.7	14.9	3.7	-1.3	-1.6	6.4	-0.6	-2.0	-1.1	-5.4	-4.5

TABLE I1

Stratification IX: Class RE (%) u-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	-2.3	27.0	12.9	3.6	37.9	39.0	31.7	37.9	22.0	16.7	6.4	21.6
	-112.0	12.1	-60.7	-78.6	50.4	12.5	-26.9	16.3	31.9	14.1	-10.5	-13.8
2	13.1	16.6	19.2	8.3	49.4	26.1	0.7	45.2	15.1	-0.5	10.0	18.5
	22.1	49.2	41.9	28.0	52.3	46.9	9.2	6.1	49.6	34.3	-7.0	30.2
3	-1.4	46.1	33.7	15.4	24.3	43.7	25.8	-10.2	17.9	9.2	2.6	18.8
	-2.5	-0.3	0.2	0.4	-1.8	-3.4	1.7	2.2	-6.2	-1.5	3.4	-0.7
4	22.1	27.5	26.3	8.1	41.3	36.8	22.1	33.9	38.8	30.4	10.0	27.0
	9.1	-1.4	0.8	2.4	0.8	0.1	0.0	0.0	-2.9	0.5	1.1	1.0
5	48.0	46.2	47.5	28.3	28.5	15.6	-1.3	17.0	25.1	17.1	3.6	25.1
	-0.4	32.2	19.8	9.4	45.1	48.7	33.9	42.3	57.1	44.9	18.1	31.9
6	44.8	45.1	45.8	41.1	59.4	49.2	31.2	38.9	43.1	27.6	-10.5	37.8
	-1.1	27.6	6.7	8.7	57.2	48.4	22.1	49.4	44.4	27.7	0.4	26.5

TABLE I2

Stratification IX: Class RE (%) v-Component

Upper Row: Dependent
Lower Row: Independent

Class \ Predictand Point	1	2	3	4	5	6	7	8	9	10	11	Ave. 11 pts.
1	11.0	38.8	-8.6	-5.0	27.0	8.6	-36.4	-29.5	14.1	-16.6	-14.5	-1.0
	3.1	7.3	-11.7	-3.2	1.7	-40.0	7.8	10.7	-22.7	1.3	18.5	-2.5
2	28.6	28.3	33.5	16.0	20.1	26.1	3.8	22.2	23.5	5.4	-18.1	17.2
	24.7	-56.3	-38.8	20.0	-36.1	-41.9	-8.4	18.8	-56.4	-32.3	0.4	-18.8
3	19.8	-7.3	20.2	-2.8	-18.8	3.6	14.1	-18.3	-14.8	-0.8	-1.4	-0.6
	-7.5	-1.9	-2.0	-14.7	-3.4	1.2	0.0	-0.4	2.1	4.2	-0.1	-2.0
4	11.5	10.5	-1.5	12.3	7.6	-0.8	16.0	17.7	2.7	14.5	8.7	9.0
	-1.4	5.6	-2.6	-8.1	1.8	0.5	-9.5	1.8	-0.2	-1.2	-10.6	-2.2
5	38.3	7.5	33.8	15.9	5.7	12.0	16.0	8.7	27.4	-4.6	29.5	17.3
	46.8	24.4	41.6	26.7	9.1	17.8	12.4	1.5	13.4	8.4	-3.0	18.1
6	19.5	3.8	-2.7	16.4	16.1	1.7	1.0	32.2	9.3	8.3	14.6	10.9
	36.0	31.0	28.7	8.4	9.6	15.5	7.3	15.6	3.3	0.4	10.4	15.1