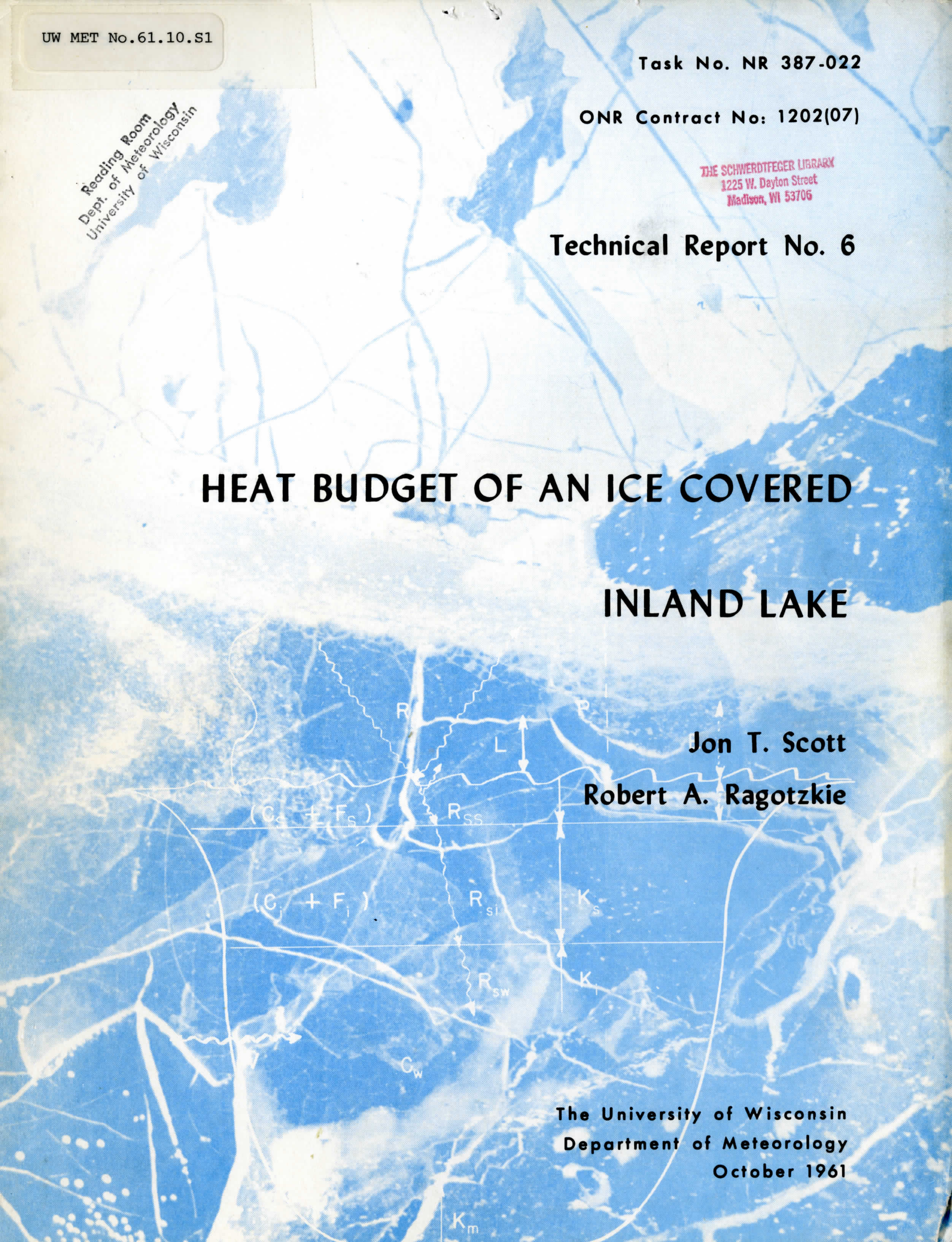
Task No. NR 387-022

ONR Contract No: 1202(07)

Technical Report No. 6

HEAT BUDGET OF AN ICE COVERED INLAND LAKE



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Department of Meteorology
October 1961

HEAT BUDGET OF AN ICE COVERED INLAND LAKE

by

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COVER: Aerial photo of ice on Green Lake, Wisc. taken from 1000 feet on Jan. 22, 1961.

ACKNOWLEDGEMENTS

The authors wish to express their thanks to Charles R. Stearns for his help in designing and installing the instruments used on the Lake Mendota tower, and to Bernhard Lettau for his assistance in gathering much of the data. In addition, we are indebted to Professor Reid A. Bryson for his invaluable advice and counsel during the course of the work.

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ABSTRACT

The heat budget for Lake Mendota is evaluated for two winter seasons. Winter is defined as extending from the time just before freezing of the lake surface until all ice has melted in the spring. Change in heat storage, radiation flux, conduction (molecular and/or eddy) and advection are determined separately for snow, ice, and water where applicable. Sensible and latent heat flux at the lake surface are not directly measured, but the heat budget method appears to be promising for determining at least their sum.

Aerial and lake surface measurements of albedo are compared with each other and to visual estimates of albedo based on ice or snow conditions and age of the snow surface. Results indicate that visual estimates by experienced observers are reliable to within 10 percent.

Heat conduction in the snow-ice-complex is measured using a thermocouple and flux-plate assembly.

Seven distinct periods of the winter season are described using the heat budget data for the two years. Net radiation, air temperature, and amount and type of snow cover are the most important factors affecting the heat budget of the winter lake.

I. INTRODUCTION

A. Purpose

From the date of freezing of its surface in winter to the spectacular breakup in spring, an inland lake follows a pattern of events governed by the nature of the physical environment and by its morphology. The reaction of the lake ice to certain rigors of its environment is visible from the appearance of many fascinating, often beautiful, and sometimes puzzling, phenomena. A more important, though less obvious response of the winter lake, is the change in its thermal energy budget. This report is an attempt to describe and evaluate the heat budget of a lake in winter, including that of its ice and snow cover.

The present study on Lake Mendota is part of a general investigation of natural climatic indicators in a region from Wisconsin to Northern Canada in which it is hypothesized that a lake can be used as a measure or indicator of climate by acting as an "integrator" of various meteorological parameters. The heat budget study was chosen because it is itself basic to climatic studies and because the larger terms of the heat budget may be directly related to other meteorological parameters.

B. Literature

The pioneer and early proponent of lake heat budget studies was Birge (1915), who, with his associates, accumulated a vast amount of data on temperature distribution in lakes, some of which remained unpublished until Neess and Bunge (1957) summarized Birge's original notes and manuscripts. A winter heat budget for Lake Mendota was computed by Juday (1940), which included some estimates of the heat exchange mechanisms. But even with the vast amount of effort expended by Birge, Juday, and their co-workers, there remained many unmeasured heat budget terms.

Interest in the winter heat budget of Lake Mendota was renewed when Bunge and Bryson (1956) reviewed past work in lake ice research and attempted to measure some of the heat budget terms. Dutton and Bryson (1960) calculated a mean annual heat budget for Lake Mendota considering the larger parameters. In this present study, all of the main heat budget processes for a lake in winter will be considered for relatively short time periods. Particular emphasis will be given to simple methods of estimating the heat budget parameters and to the variation of these parameters throughout two winter seasons.

C. Winter Heat Budget Equations

The general heat budget equation which applies to a surface of land or water is

$$R_n = C + E + L \tag{1}$$

where R_n is net radiation, C is change in heat storage, E is energy used for latent heat exchange, and L is the exchange of sensible heat between the surface and the air.

For a lake with an ice cover and perhaps also a snow cover, it is most convenient to consider separate budgets for the water, ice, and snow. In Figure 1 a schematic diagram is given indicating the various heat budget parameters for these three media.

Heat energy directed toward a medium is positive and that directed away is negative.

In this report the cgs system will be used ($1 \text{ cal cm}^{-2} = 1$ langley).

Using Figure 1 (in which the symbols are defined), the heat budget equation for the water is, neglecting signs

$$C_w = K_i + K_m + V + R_{sw}. \qquad (2)$$

Smaller terms involving biological activity and friction are negligible and are not included.

When there is a snow cover, and a complete ice cover, the descriptive equation for the heat budget of the ice is, again neglecting signs

$$C_i + F_i = K_i + K_s + R_{si}. \qquad (3)$$

If the snow is deep enough, the terms L, E, and R_l are negligible or zero.

When there is no snow cover but a complete ice cover, similarly the heat budget of the ice becomes

$$C_i + F_i = R_{si} + R_l + K_i + K_s + L + E + P \qquad (4)$$

In this case, precipitation (P) represents only sensible heat addition which is present in rainfall since snowfall would change conditions so that equation (3) would apply.

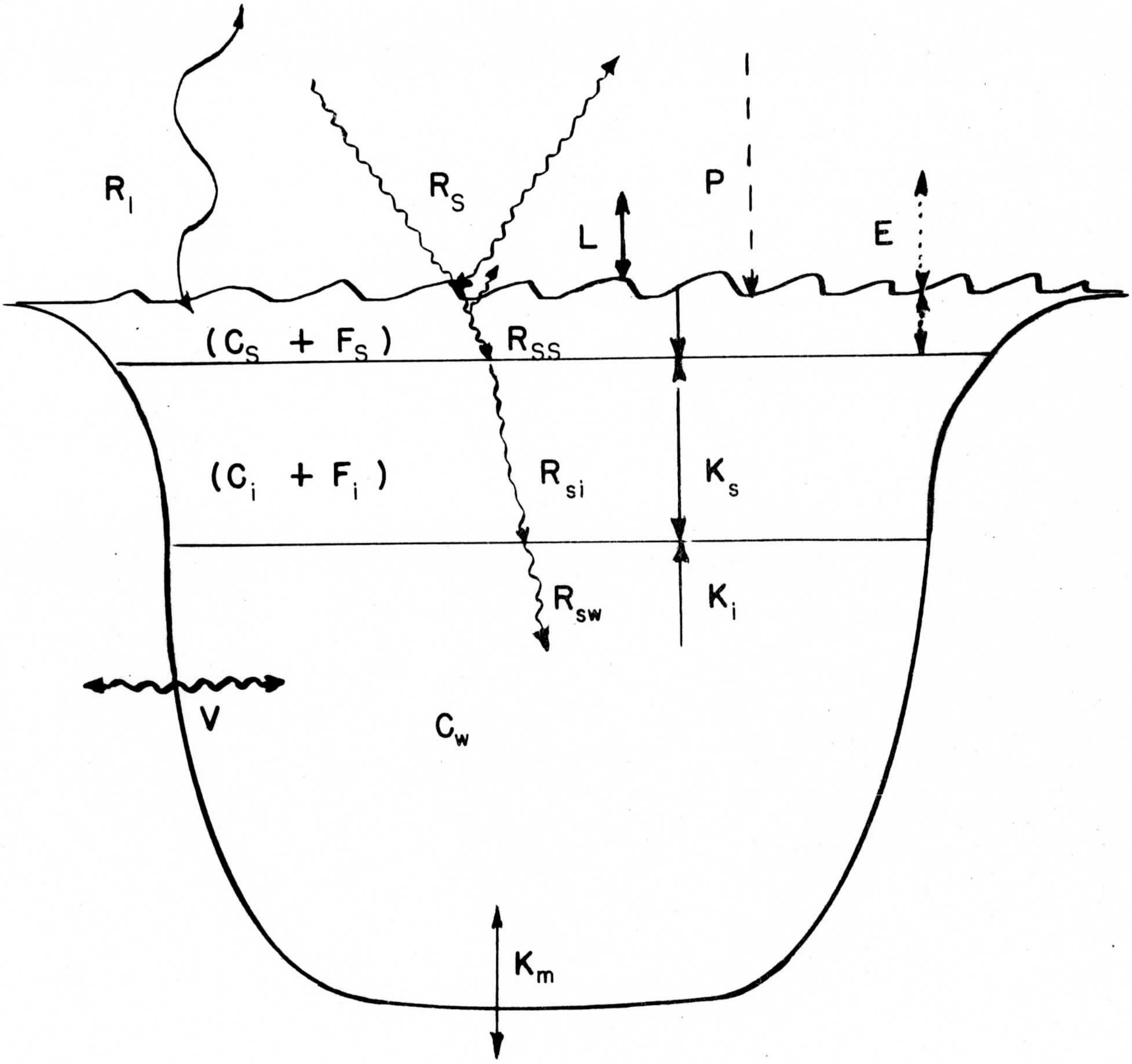


Figure 1: Schematic representation of the heat budget of a lake with a winter cover of ice and snow.

R_s - net incoming solar radiation
R_{ss} - incoming solar radiation absorbed by snow
R_{si} - incoming solar radiation absorbed by ice
R_{sw} - incoming solar radiation transmitted to water
R_l - net long wave radiation
L - sensible (turbulent) heat exchange with the air

E - latent heat exchange with the air

K_s - sensible heat conducted from ice to snow

K_i - sensible (turbulent or molecular) heat exchange through the water to the ice

K_m - sensible (turbulent or molecular) heat exchange between the water and bottom sediments

C_s - change in sensible heat stored in snow below a reference temperature of 0° C

F_s - change in amount of latent heat of freezing stored in the snow

C_i - change in sensible heat stored in ice below a reference temperature of 0° C

F_i - change in amount of latent heat of freezing stored in the ice

C_w - change in amount of sensible heat stored in the water above a reference temperature of 0° C

V - advection of heat in and out of lake in water

P - sensible or latent heat added in rain, snow etc. (above or below a reference 0° C so that latent heat of snow is negative in sign).

When there is a snow cover, an additional equation applies to the heat budget of the snow,

$$C_s + F_s = P + R_{ss} + R_1 + K_s + L + E \qquad (5)$$

II. METHODS OF ESTIMATING OR MEASURING THE HEAT BUDGET TERMS

A. The Storage Change Terms

The heat storage of a temperate lake is continuously changing. In order to estimate these changes (as heat flux per day = ly/day), the total heat storage of each medium must be measured at different times and the differences obtained.

1. Change in Heat Storage of the Water

The procedure used by Dutton and Bryson (1960) to determine the total sensible heat stored in the water at any time can be briefly described as follows. The amount of heat depends on temperature, heat capacity, and total mass of the water. Since the mass of water in a lake varies with depth, the hypsometric curve of the lake can be used to obtain the amount of heat stored in a unit layer at any depth relative to the amount in the surface layer. The total amount of heat in the entire water column weighted for the hypsometric curve can then be obtained and expressed as cal cm^{-2} or langleys (ly).

The Whitney Underwater Electric Thermometer was used to measure temperature at one meter intervals of depth. An ice-water mixture in the hole in the ice sheet afforded a good field check each time the instrument was used and temperature measurement was considered accurate to within $\pm 0.1^\circ$ C.

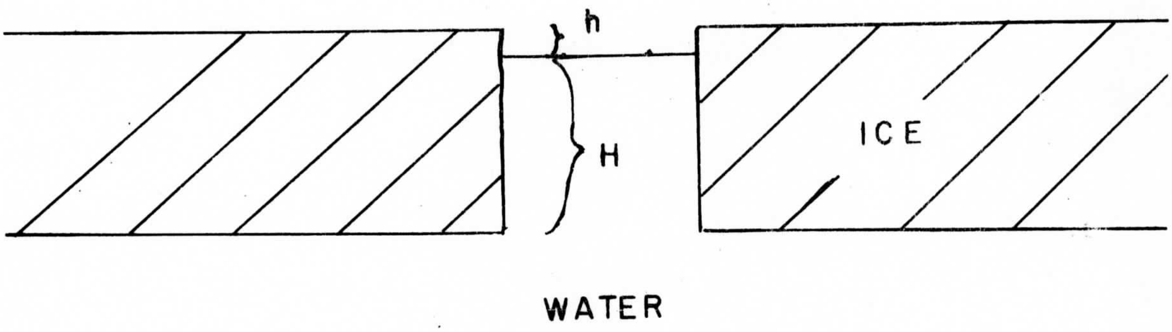
Series of temperature soundings were made eight times in the 1958-59 winter and twenty times for the 1959-60 winter at from 5-30 locations on Lake Mendota. Storage change in the water is summarized in Appendices 1 and 2.

2. Storage Change in the Ice

There is both latent and sensible heat storage in the ice. If we take the convenient reference temperature of 0° C, sensible heat in the ice must either be zero or "negative". At any time the total "negative" sensible heat stored in the ice is small compared to the total latent heat of fusion represented by a certain thickness of ice. Since the heat storage change is obtained for a period of several days to a few weeks, the sensible heat change (C_i) can usually be neglected. The normal diurnal change in sensible heat storage is as large or larger than the change over the various measuring periods. For lakes with thicker ice than that usually observed in Lake Mendota and in colder regions, the sensible heat change may be more important if the ice temperature, which depends on air temperature, is significantly different from one measuring time to the next.

Latent heat in the ice, at any one time, is given by the product of ice thickness, its density, and the latent heat of freezing. The heat of freezing for fresh water of 80 cal/g and an ice density of 0.92 g/cm^3 were used. The density value was checked by measuring the displacement of the ice cover above the water level when there was no snow on the ice (see Figure 2).

Fig. 2: Schematic, showing method of determining ice density over a large area of lake surface.



From Figure 2, the ice density (ρ_i) is given by

$\rho_i = \frac{H}{H + h} \rho_w$, where ρ_w, the water density, is assumed to be 1.0 g/cm^3. For fifteen locations on the lake on March 31, 1960, the ice density was 0.92 g/cm^3. Several measurements taken in the 1960-61 winter gave ρ_i values not significantly different from this value. Ice thickness was measured nearly every day at one site on the lake for both the 1958-59 and 1959-60 winter seasons, and at the times when temperatures were taken, the thickness was measured at from 10-30 different locations.

The value of heat storage change obtained was well within the 10% accuracy desired.

Results of computation of storage change in the ice (F_1) are given in Appendices 1 and 2.

3. Storage Change in the Snow

As with the ice, the change in sensible heat storage (C_s) in the snow varied more diurnally than seasonally and was neglected.

Latent heat "stored" in the snow crystals was more important and in late winter was often as high as 1/4 the amount of latent heat in the ice. The total latent heat in the snow is given by the product of the weight of the snow per unit area of lake surface, and the latent heat of freezing (80 cal/g).

Two methods were used to obtain the total weight of snow per area of lake. In the 1958-59 winter, density and depth measurements were made near the Hydrobiology Laboratory near the south shore of the lake (see Figure 4). The density "instrument" was a metal cylinder of known volume which was slid into the snow pack at various depths. From the weight of the snow in the cylinder, density was computed. Depths were taken at 10-20 random sites and were averaged.

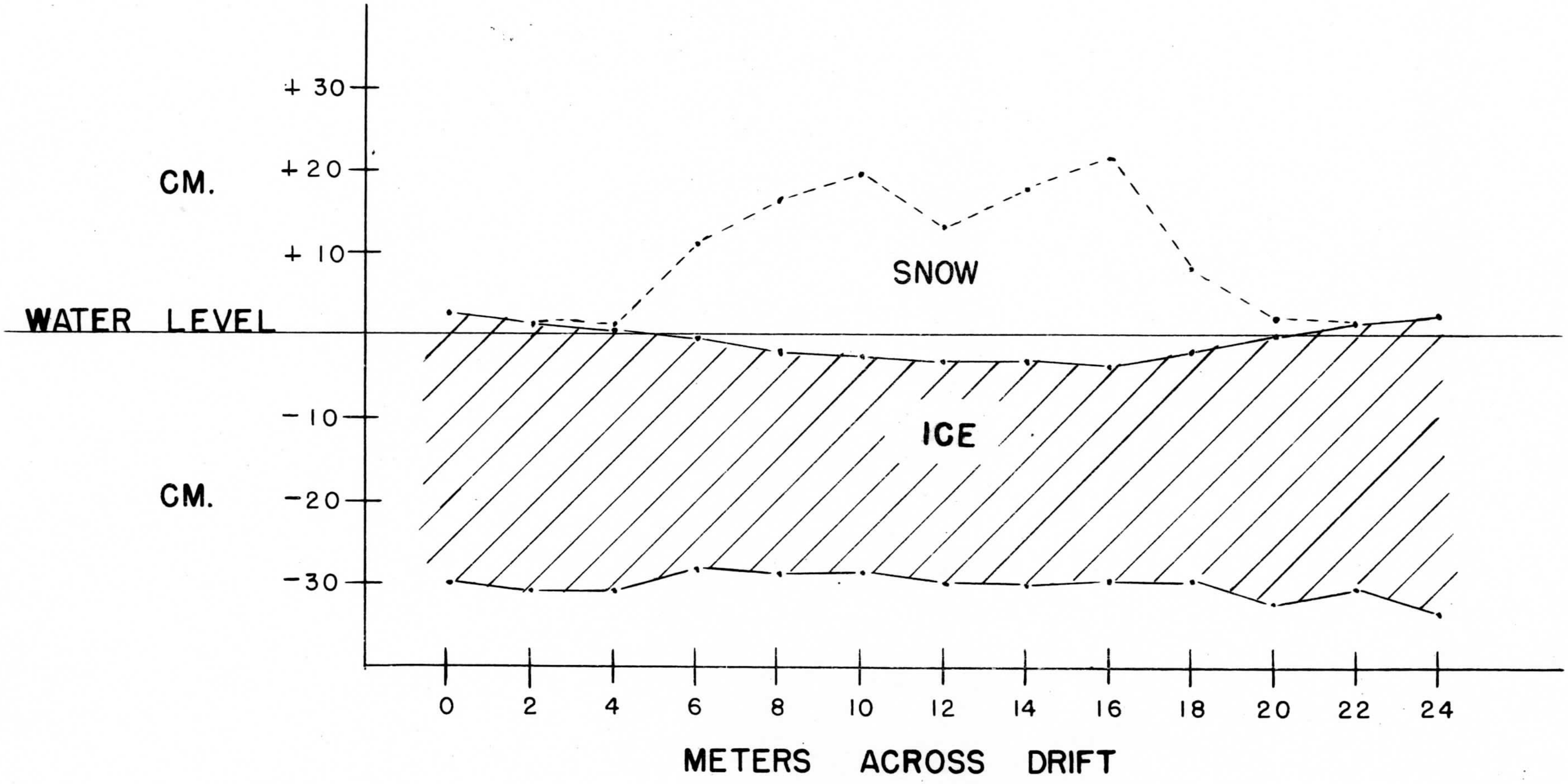
The second method of obtaining snow weight per area was tested in the 1959-60 season and involved the same technique employed in finding ice density (Figure 2).

When snow falls on the ice, the ice sinks because it acts as if it were freely floating on the water. The weight of snow is equal to the weight of water displaced by the ice. Referring to Figure 2, we find the weight of snow per cm^2 to be $(1-\rho_i)(Z_i) - h$, where ρ_i is assumed to be 0.92 g/cm^3, Z_i is ice thickness and h is the distance from the top of the ice to the water. Thus, a lake with an ice cover can be used as a "snow gauge" which may be more accurate than conventional gauges because it integrates over a relatively large area.

When snow drifting occurred, this method was more difficult to use, because the ice "sank" more under deep drifts. Figure 3 shows this effect rather strikingly. Measurements of snow depth, ice thickness, and level of the top of the ice are shown, taken across a deep drift. As the figure illustrates, water level was 3 cm above the ice level in the center of the drift. Some of the error in snow weight determination caused by drifting was reduced by sampling once in a deep snow area and once in a shallow area (between drifts) when an area of large drifting was encountered.

Generally, more consistant results were obtained by the second method than by the depth-density technique.

FIGURE 3: DEPRESSION OF ICE SHEET UNDER A SNOW DRIFT



Results are discussed in section IIIA and were applied to the calculation of the best estimate of latent heat storage change in the snow pack (F_s). These latter values are summarized for various periods of the two winter seasons in Appendices 1 and 2.

B. Radiation Terms

1. Incoming and Reflected Solar and Sky Radiation

Incoming solar radiation is regularly measured with Eppley pyrheliometers at the U. S. Weather Bureau Station located about a mile northeast of Lake Mendota and at the Solar Energy Laboratory about 2/3 of a mile south of the Lake (see Figure 4).

Data from the two stations were averaged for the 87 day period from February 1 to April 25, 1960. The mean values were 383.7 ly/day and 374.6 ly/day for Weather Bureau and Solar Energy Laboratory data respectively. Except in a few cases the daily values were within 5-10% of each other.

Data for incoming solar radiation from these two stations were considered to be sufficiently accurate for this study. The problem in determining net-solar and sky radiation was then reduced to an estimation of the albedo of the lake surface. In general, it was thought than an estimate of the albedo could be made by noting the ice or snow conditions daily. Such an estimate was normally accurate to within 10% of the true value. This method of estimation

FIGURE 4: OUTLINE MAP OF LAKE MENDOTA

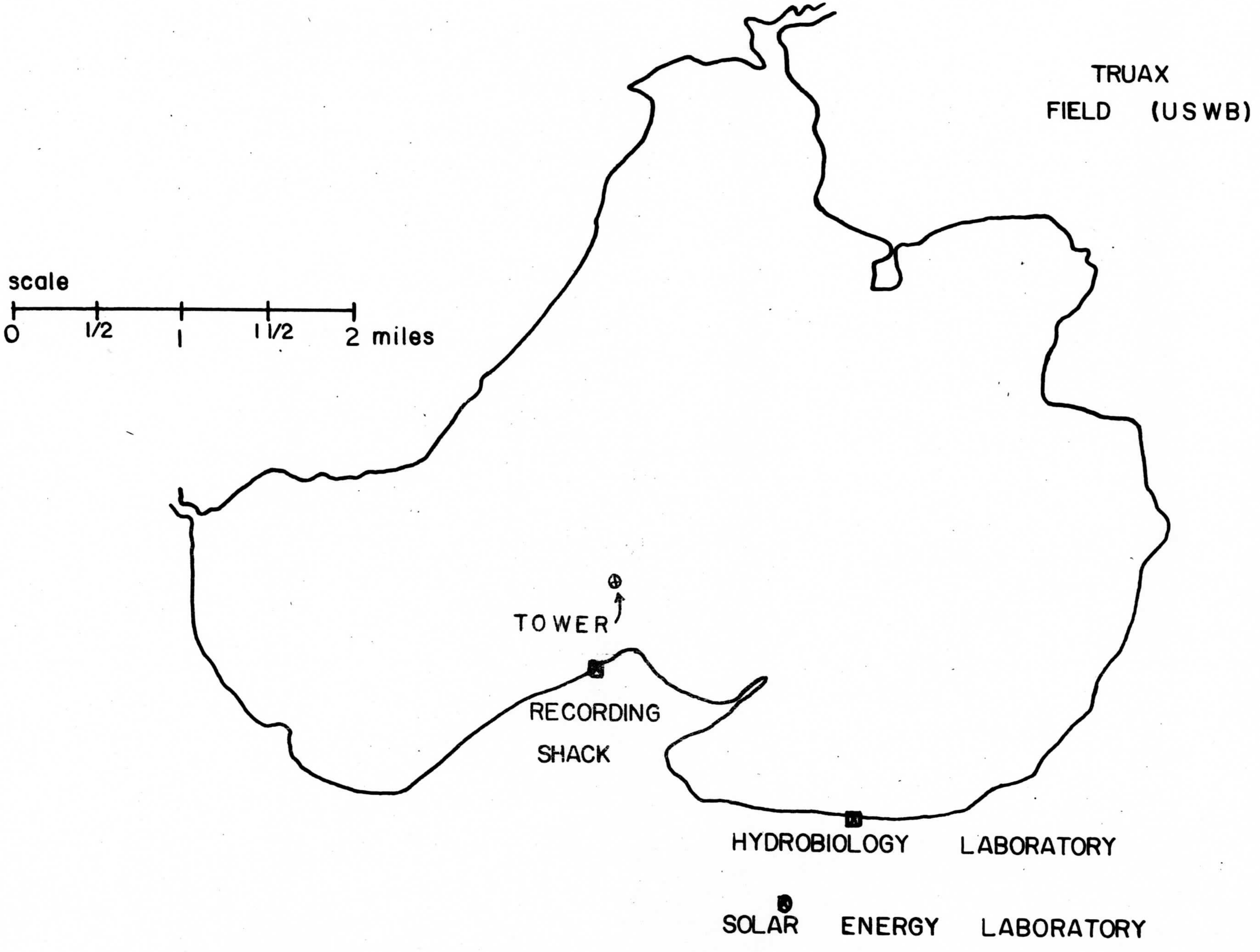
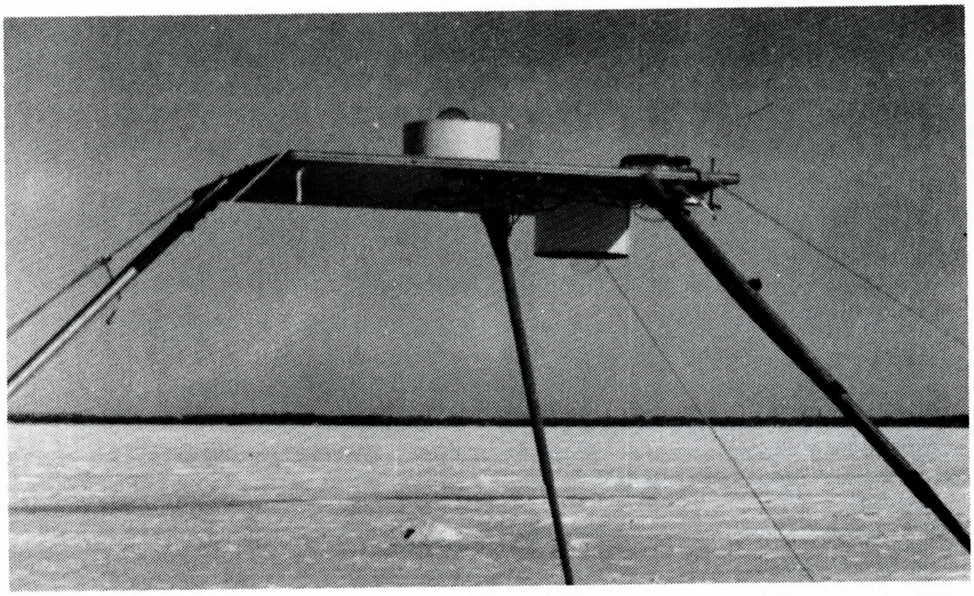
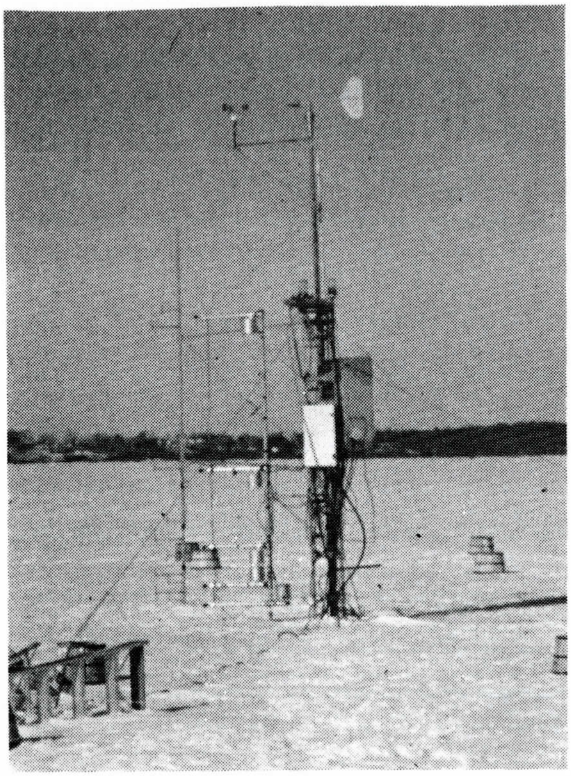


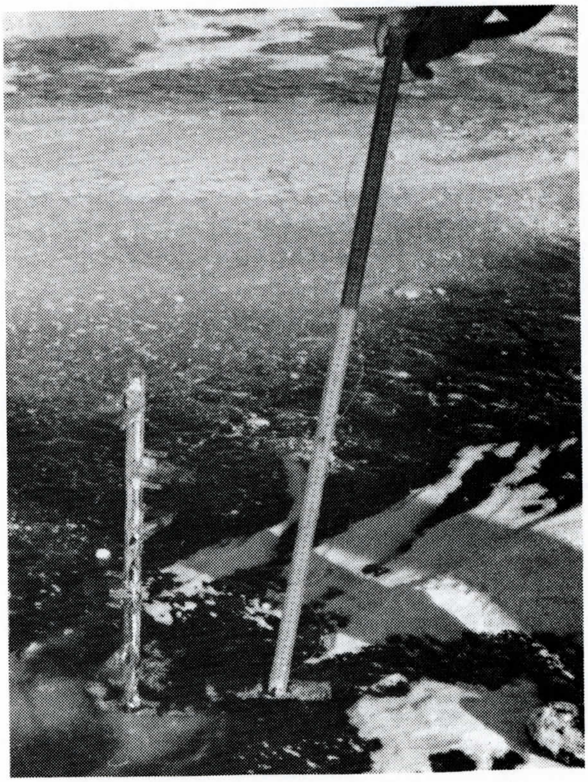
Figure 4A: Installations at the Lake Mendota tower



a. Radiation instruments



b. Main tower



c. Temperature profile instrument frozen in ice was checked by measurements taken at a special instrument tower located on Lake Mendota, 450 meters from the south shore near "second point". Eppley instruments were set up in January 1960, in upright and inverted positions (see Figure 4). Data were recorded on Brown (Minneapolis-Honeywell) Recorders at an instrument shack on the nearby shore.

Errors due to signal transmission, recording and data reduction, amounted to ±1.5 ly/day considering a twelve hour sunshine period.

The largest error was caused by frosting on the glass bulb which occurred mostly in the early morning. On a few occasions when there were high radiation intensities, the pen on the Brown Recorders went off the chart. Davis (1957) discusses other errors involved with use of the Eppley instrument. In particular, he has found an albedo estimate 4% higher than the true value when using an inverted Eppley.

Albedo measurements of Lake Mendota by Bauer and Dutton (1960), using an instrumented airplane, were available for the 1959-60 heat budget study. These albedoes were obtained every few days.

When no direct albedo measurements were available, estimated values were used. The accuracy of these was improved by reference to the rather large literature on the subject [see SIPRE (1951), Miller (1955), Wallen (1948), Anderson (1954) and Bauer (1960)] . There is further discussion in section III B of this report.

The daily albedo, measured or estimated, was applied to the daily value of incoming solar radiation to obtain net solar and sky radiation at the lake surface. The daily values were then summed for various periods of the two seasons (Appendices 3a and 3b).

2. Solar Radiation Absorption

The solar radiation terms of importance to the winter heat budget are the amounts absorbed by the snow, ice, and water. Some technique must be used to apportion the net-solar radiation in the correct amount to each medium. According to Birge, Neess and Bunge (1957), and Kalitin (SIPRE 1951), 10-15 cm. of snow absorbs more than 99 per cent of the solar radiation. The problem is not as simple when there is a snow cover thinner than this. Generally, three techniques are avialable. These are: (1) Direct measurement below each media, (2) Estimation by differences if all of the other heat budget parameters are known, and (3) by calculation from an assumed absorption (extinction) coefficient for ice and/or snow.

Measurements were not made in either of the two seasons discussed here, and the heat budget method will be discussed in section IV. Because the solar radiation term was one of the largest parameters in the heat budget, the third method was used to obtain an approximation of this term. There is extensive literature concerning the value of ice and snow absorptivity coefficients [see SIPRE (1951), Lyons and Stoiber (1959)]. Absorptivity coefficients for snow range from 0.10 cm^{-1} to about 0.40 cm^{-1} depending on the density. Gerdel (in SIPRE 1951) obtained a density variation which is in agreement with most investigators. He found:

snow density (g cm^{-3})	0.261	0.232	0.397	0.448
k (cm^{-1})	0.280	0.184	0.106	0.106

The recent literature concerning ice absorptivity coefficients was reviewed by Lyons and Stoiber (1957) who found absorptivity coefficient to be dependent upon whether or not there were bubbles or other imperfections in the ice.

Computation of solar radiation absorbed by the ice and snow and transmitted to the water is shown in Appendix 4. Absorptivity coefficients were chosen according to the best available values in the literature for given conditions of the snow or ice. For instance in mid-winter an absorptivity coefficient of .25 cm^{-1} was used for snow and .05 cm^{-1} was taken for the "bubbly" ice.

When snow melted from the ice in the spring of 1960, the ice was more uniform, and so daily values of absorption of solar radiation were computed. For this computation (March 25 - April 12, 1960), it was assumed that 50 per cent of the net solar radiation was in the range of wave lengths from .45$\mu$to .70$\mu$where an absorptivity coefficient of 0.028 cm^{-1} was assumed to apply; 25% was in the range .70$\mu$to 1.20$\mu$where a coefficient of 0.10 cm^{-1} applied; and 25% was greater than

1.20μ where 100.0 cm^{-1} applied [see Lyons and Stoiber (1959), and Johnson (1954)]. The heat budget method afforded a relatively good check for the above computations and much of the discussion in section IV pertains to the estimation of this important term.

3. Long Wave Radiation

Long wave radiation was not measured and had to be estimated by empirical formulae. Two such formulas were tested. The first given by Budyko (1958) is derived from theoretical considerations of the effect of atmospheric moisture on the long wave radiation flux. The radiation flux in ly/min from a surface with clear sky is given by Budyko as

$$R_o = \epsilon \sigma T^4 (0.39 - 0.058 \sqrt{e}) \qquad (6)$$

where ϵ is surface emmisivity, σ is the Stefan-Boltzmann constant, T is surface temperature (K^o) and e is vapor pressure (mm Hg) of the air above the surface. Budyko determined the effect of cloud cover and arrived at the following empirical equation for net-long-wave radiation (R_1)

$$R_1 = R_o(1 - CN)$$

where N is cloud cover in tenths, and C is a constant depending on latitude (0.70 for Madison, Wisconsin).

The second formula tested here was used by Dutton and Bryson (1960) for computing long-wave radiation from Lake Mendota. It was originally derived by Ångstrom (1915) and modified by Sverdrup (1946). Wallen (1948) discusses the application of Ångstrom's formula to the heat balance of the Kårsa glacier for the case of short time periods Ångstrom's formula is

$$R_1 = \sigma T^4 \left[.194 + .236(10)^{-0.069e} \right] (1 - CN) \quad (8)$$

Where C = .83 and the other parameters are the same as in equation (7).

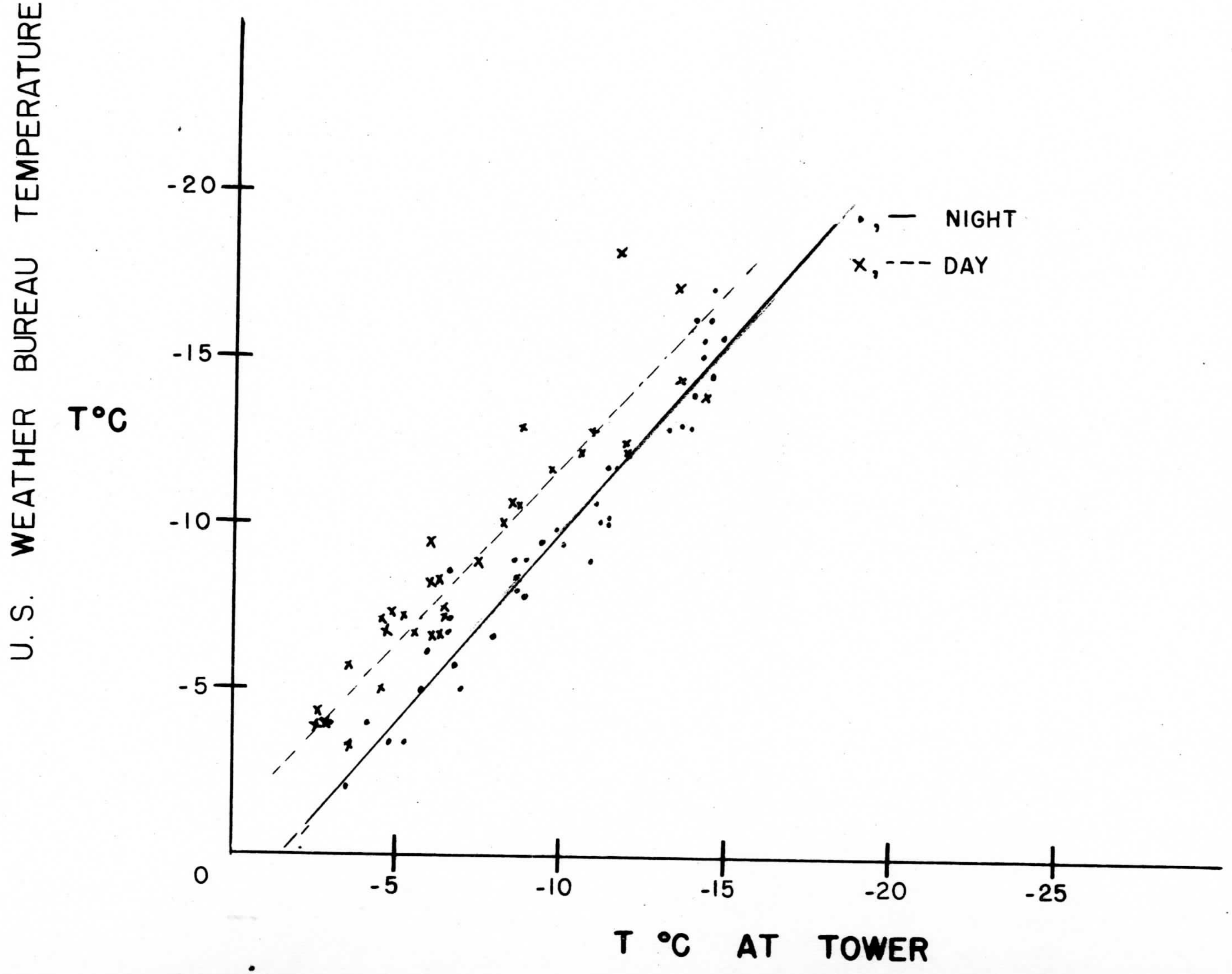
Both of these formulae apply to the mean conditions, but Anderson (1954) has concluded from the Lake Hefner studies that empirical equations are reliable for short periods, if 10% accuracy is acceptable.

For the computation of net long-wave radiation using the above formulae, it was assumed that the air temperature at the nearby Weather Bureau Station was the same as the Lake Mendota surface temperature, if the latter was not measured. Justification for this assumption is given by the scatter diagram in Figure 5, where hourly values of the Weather Bureau air temperature are plotted against Lake Mendota surface temperature (from a thermocouple - Section II-C-1). In addition, day and night means of these hourly temperatures were not greater than 3°C different for any one day or night, and the mean of eight day or night periods were different by only 1.55°C and 0.31°C, respectively.

Dew point and cloud cover data were also taken from the U. S. Weather Bureau records.

For the 1959-60 ice season, both formulae (7 and 8) were checked and were normally within 5% of agreement. In

FIG. 5 HOURLY LAKE SURFACE TEMPERATURE AT LAKE MENDOTA TOWER COMPARED TO AIR TEMPERATURE AT THE U. S. WEATHER BUREAU, TRUAX FIELD, FEB. 18 - 20, 1960.



only 4 cases out of 34 computations for various periods did the differences exceed 10%. For the 1958-59 winter, only the Budyko formula was used. Results are summarized in Appendices 3a and 3b.

4. Net Radiation

Net radiation was measured on Lake Mendota for part of both seasons, using the Economical Net Radiometer (1958). Brown Recorders were used for recording the thermocouple sensor data. There were no known serious errors involved in the recording and processing of the data, but there were errors caused by frosting, snow, or rain. Usually, the instrument was removed during rain or snow.

Results are compared to the computed or "estimated" net radiation in section III B.

C. Conduction Terms

Sensible (turbulent) heat transfer between the lake surface and the atmosphere will be considered in section IV. The other terms involving molecular or eddy conduction are: (1) molecular conduction in the ice to the snow or free ice surface (K_s), (2) molecular or eddy conduction through water to the ice (K_i), and (3) net conduction between the bottom sediments and the lake water (K_m).

1. Conduction in the Ice and Snow

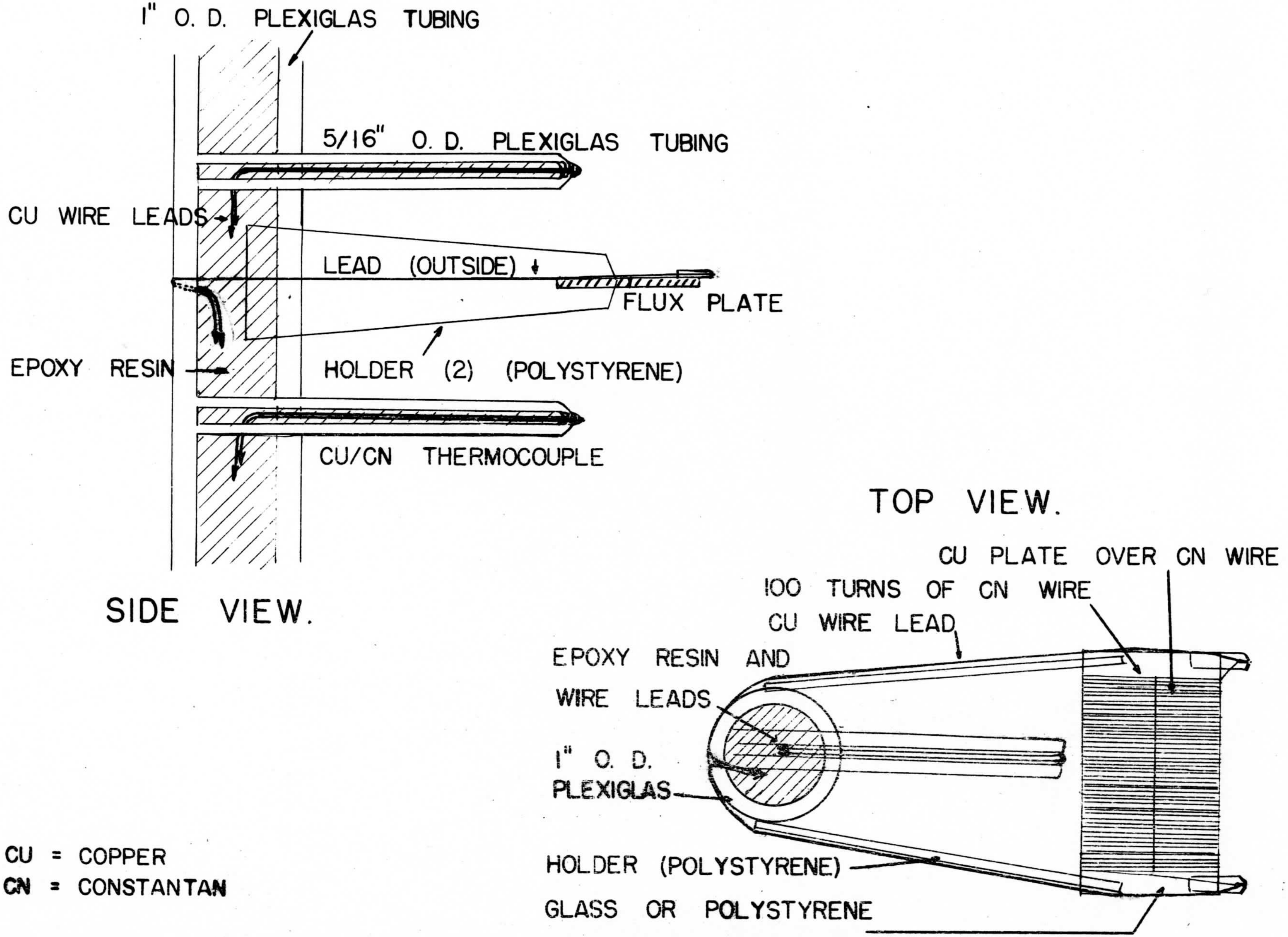
Net conduction in the ice is upward to the snow or free ice surface and because it is large may be measured by difference using the heat budget equation (see section IV).

An attempt was made to obtain continuous measurements of conduction in the ice-snow-water system for the 1959-60 winter. An ice "conduction instrument" was installed at the Lake Mendota instrument tower. This instrument contained thermocouples at 5 cm intervals over a vertical range from 50 cm above the ice surface to 150 cm below the lake surface, and flux plates at 10 cm intervals between every other thermocouple. Each successive thermocouple pair with fluxplate were on alternate sides of a one inch O. D. plexiglas tube holder, so that disturbance of the thermal field would be less serious (see Figure 6). The plexiglas tubing was filled with an epoxy-resin to give the instrument a thermal conductivity similar to ice, and the whole instrument (except the flux plates) was covered with aluminized mylar to reduce radiation effects. Conductivity along the copper and constantan wires was reduced by using thin (no. 30) wire.

Flux plates were constructed from 1mm thick glass microscope slides (for ice placement) or 1.88 mm thick polystyrene (for snow placement) wound with one hundred turns of constantan wire and half-plated with copper (see Figure 6).

Readings from selected thermocouples and flux plates were taken at ten or fifteen minute intervals through February and March of 1960. Errors due to data reduction from Brown Recorder charts and signal transmission amounted to $\pm$ 0.1°C. Possible errors due to radiation on

FIG. 6 SCHEMATIC OF "CONDUCTION INSTRUMENT"



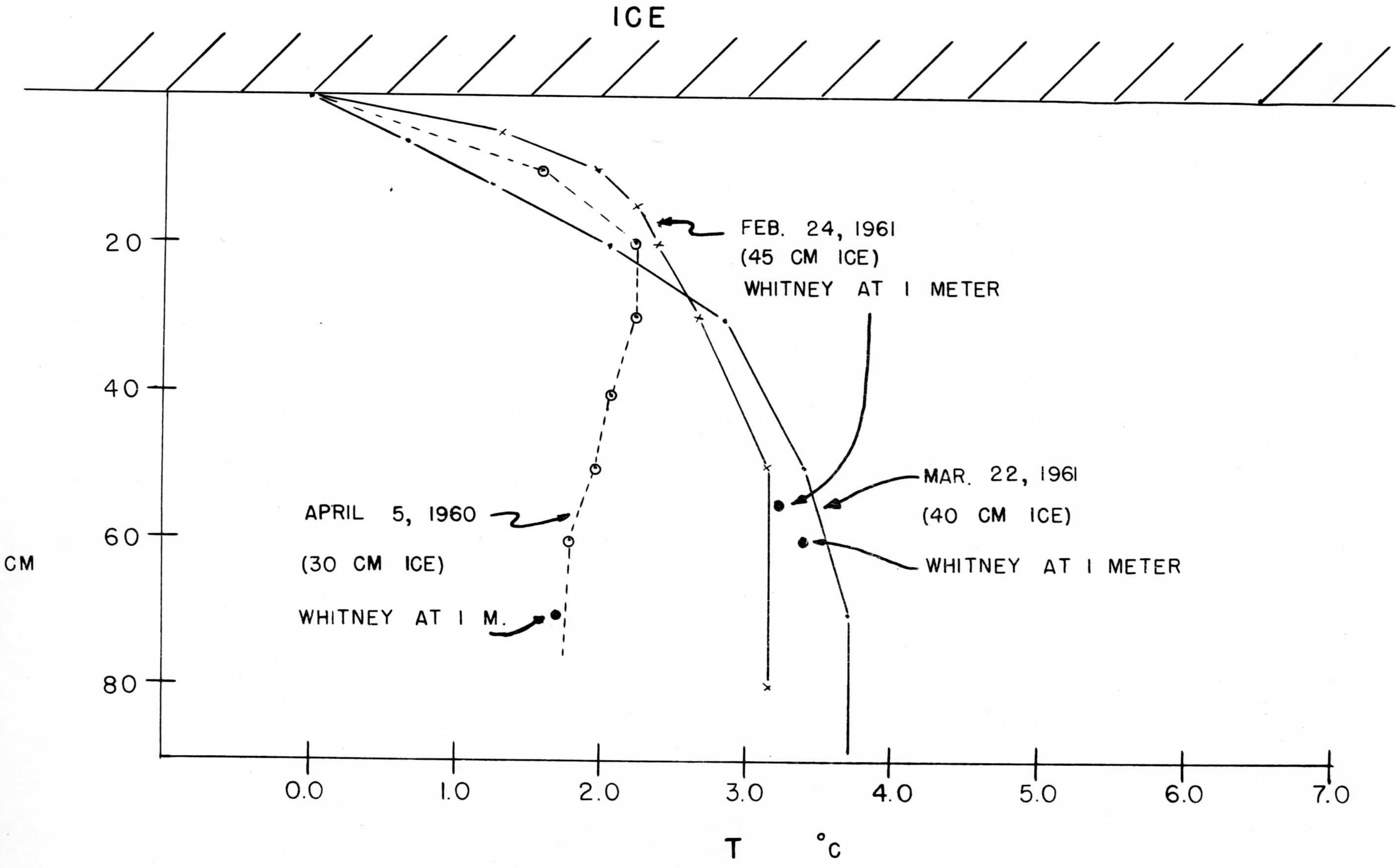
the sensors and conduction along the wire leads were not determined, but several checks of the instrument temperatures (for air exposed sensor) proved these errors to be relatively small. Some results are given in section IIIC.

2. Conduction through Water to the Ice

This term, (K_1), can be determined by measuring the temperature gradient in the layer just below the ice, if the effective thermal conductivity of the water in this layer is known.

Temperature gradients just below the ice are shown in Figure 7. These temperatures were carefully measured with an open bead thermistor and are accurate to $\pm$ 0.1°C. Unfortunately, such temperatures were not taken throughout both winters. Therefore, the temperature gradients for most of the two years were approximated by using the less detailed Whitney thermometer readings at one meter below the ice. This temperature was assumed to apply at a level of either 10 or 20 cm. below the ice, depending on the type of snow cover for the period before the measurement. The temperature gradient was then determined by assuming that the ice-water interface was 0°C and the gradient was linear. Figure 7 indicates that the above assumptions were good. For instance when there was a long period with no snow cover, such as was the case for the April 5, 1960 and February 24, 1961 readings in Figure 7, then the Whitney reading, if applied to the point at 10 cm below the ice, gave a good

FIG. 7 DETAILED TEMPERATURE TAKEN WITH A BEAD THERMISTOR SHOWING THE GRADIENTS JUST BENEATH THE ICE BOTTOM.



approximation of the actual temperature gradient. By the same procedure for a time after a long period with deep snow cover (exemplified by the March 22, 1961 temperatures in Figure 7) the Whitney temperature, if applied at 25 cm below the ice, can be used to approximate the gradient just under the ice.

The effective thermal conductivity (λ_{eff}) of the water just beneath the ice is very nearly the same as its molecular conductivity. A value of λ_{eff} was obtained in the spring of 1960. When there was an isothermal condition in the ice, then any heat conducted from the water resulted in melting at the bottom of the ice. Effective conductivity of the water was determined by

$$\lambda_{eff} = - \frac{Q}{\frac{\Delta T_w}{\Delta Z}} \qquad (9)$$

Where Q was heat flux determined from bottom ice melt, and $\frac{\Delta T_w}{\Delta Z}$ was vertical temperature gradient in the water.

Bottom melt was measured in a reference stake experiment [see Figure 8 and Bryson and Bunge (1956)] .

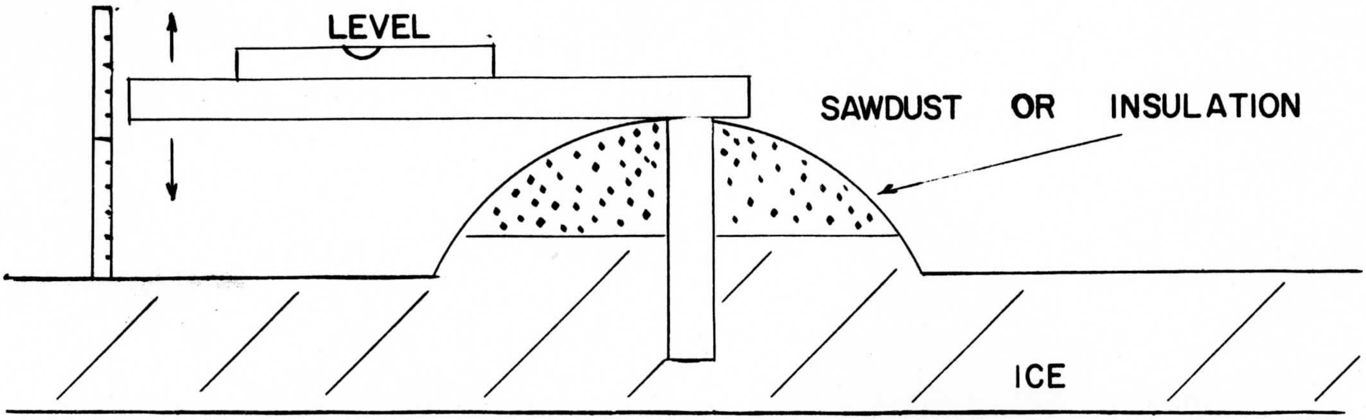


Figure 8. Reference stake experiment

If top melt and ice thickness decrease were measured at different times (e.g. daily), then rate of bottom melt could be determined. Results from the reference stake experiment are given in Figure 9 showing amount of top melt, bottom melt, and thickness decrease for the period. March 28 to April 8, 1960. Using Figure 9 to determine Q and from temperatures taken six times over this period, equation 9 gave a mean ratio $\frac{\lambda_{eff}}{\lambda_{mol}}$ of 2.25. For the April 5 sounding (Figure 7) this ratio was 2.20. Measurements in mid-winter from the ablation stake experiment indicated that the ratio was slightly greater than 1.0. For this a value for $\frac{\lambda_{eff}}{\lambda_{mol}}$ of 1.5 was chosen in order to obtain an order of magnitude of the term K_i. Because K_i in mid-winter is small compared to the large heat budget terms, the overall error introducted by this procedure is small.

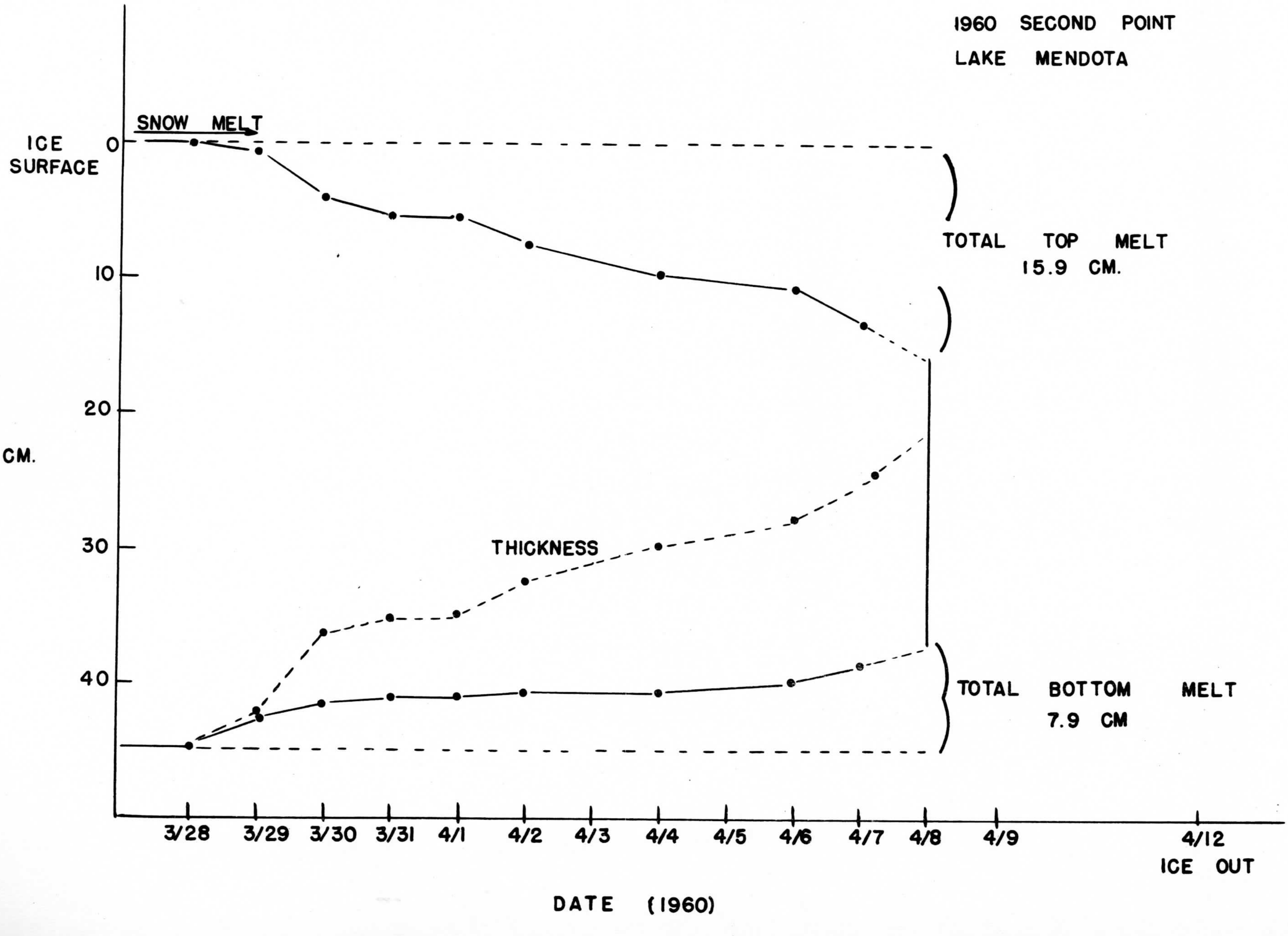
3. Conduction from Lake Bottom

As Lake Mendota water cools in fall and early winter, its temperature drops below that of the mud sediments, and heat is conducted from the bottom to the water. Birge and Juday (1927) have estimated the average conduction to the water for the ice covered season to be 650 ly.

Temperature difference between the water and mud decreases from December to April, and therefore, conduction from the bottom (K_m) decreases.

The following two assumptions were used for esti-

FIG 9 RESULTS OF REFERENCE STAKE ABLATION EXPERIMENT.



mating the term K_m: (1) that Birge and Juday's value of 650 ly/winter applied, and (2) that heat flux from the bottom was proportional to the temperature difference between the mean mud temperature and the mean of the water temperatures nearest the bottom.

The mean temperature difference was determined from 20-100 temperature soundings using the last temperature before the Whitney thermometer reached the bottom. Results are given in Appendix 6.

D. Advection Terms

Advection terms which affect the water heat budget are: (1) underwater springs, (2) outlet advection, (3) inlet advection and runoff.

1. Underwater Springs

McCaskey (1955) computed a water budget of Lake Mendota for the year, 1951-54 and found by difference, a mean underground inflow of forty cubic feet per second. This value is relatively constant throughout the year. Ground water in the Lake Mendota area has a temperature which does not vary significantly from 10°C (unpublished data 1957-1960 U. S. Geological Survey). From these data, advection from underground was computed to be 2.4 ly/day or about 250 ly for an ice covered season (Appendix 7).

2. Outlet Advection

Volume outflow at the outlet of Lake Mendota (Yahara River) was not measured, but a flow gauge is located downstream at the Lake Waubesa outlet. For each of the winter months, the ratio of Lake Mendota flow to the Lake Waubesa outflow was obtained from data given by McCaskey (1955) for the years 1951-1954. These ratios were applied to the Lake Waubesa flow for the 1958-60 winter months to obtain an approximation of Lake Mendota outflow.

Several checks of the outflow water temperature showed it to be the same as the water temperature at one meter below lake surface (about 1/2 m below the ice). Computation of outlet advection is shown in Appendix 7 in which outflow data was converted to the term ly/day for each 1°C of water temperature. The product of this value and the water temperature yields the total heat loss in ly/day.

3. Inlet Advection

Inlet advection was not measured. Ratio of inflow to outflow obtained from McCaskey's data (1955) varied from 0.5 to 2.0 for winter months with a mean value of 0.75.

Inlet water temperatures taken at scattered intervals varied from 0°C to 4°C. Inflow advection was thus hard to determine, but the few temperatures indicate that the inflow heat advection is somewhat less or equal to the outlet advection. The net advection was estimated by assuming that the inflow was 75% of the outflow except during periods when the inflowing water was known to be as large as the outflow. For the latter periods, inflow advection was assumed to be 100% of outflow advection.

There can be no important error involved by the use of the rough assumptions for estimating these small terms for Lake Mendota, since they are always small compared to the larger heat budget parameters.

E. Heat Exchange at the Lake Surface

Heat exchange at the surface (other than radiation) involves sensible heat exchange (L), latent heat exchange (E), and precipitation (P). The first two are parameters of large importance to the heat budget but difficult to measure directly. They could be approximated by difference in some cases. This method is fully discussed in section IV.

Precipitation data were obtained from the nearby U. S. Weather Bureau Station records.

III. RESULTS

In this section, specific results of more than usual interest to heat budget studies will be presented.

A. Storage Terms

1. Snow Storage Estimates- The Lake as a "Snow Gauge"

The method of determining snow load by measuring the amount of lake ice sink (II-A 3) is of interest, because it integrates over a relatively large area, and because snow gauges in present use are often inaccurate when drifting occurs in a snowfall. Also, if careful measurements are made on a lake for a period in which new snowfall is known (or zero), it may be possible to measure latent heat exchange (evaporation, condensation, etc.) by this method.

Data for the period from February 26 to March 24, 1960 are given in Table 1 showing the snow load on Lake Mendota as estimated by a mean of five to fifteen measurements, compared to the weight of snow accumulation taken from the Weather Bureau records.

Table 1: Estimates of the snow load on Lake Mendota in g cm^{-2} using the "ice sink" method, compared with the accumulated values of water equivalent in the snowfall at Truax Field Weather Bureau Station (1960).

Date	No. of Locations	Lake Mendota Snow g cm^{-2}	Water Equiv. g cm^{-2}
Feb. 25	9	3.6	4.8
Mar. 3	5	4.3	5.3
Mar. 10	15	5.4	5.5
Mar. 17	7	6.1	6.1
Mar. 24	15	5.1	6.1

Data from the "ice sink" method show that Lake Mendota lost some of its snow load by melting or evaporative losses. For instance the lake lost 1.0 g cm^{-2} of snow load from Mar. 17-24. In cases where discrepancies occurred between the values

FIG. 10 HEAT STORAGE IN WATER, ICE, AND SNOW OF LAKE MENDOTA FOR THE 1959-1960 WINTERS

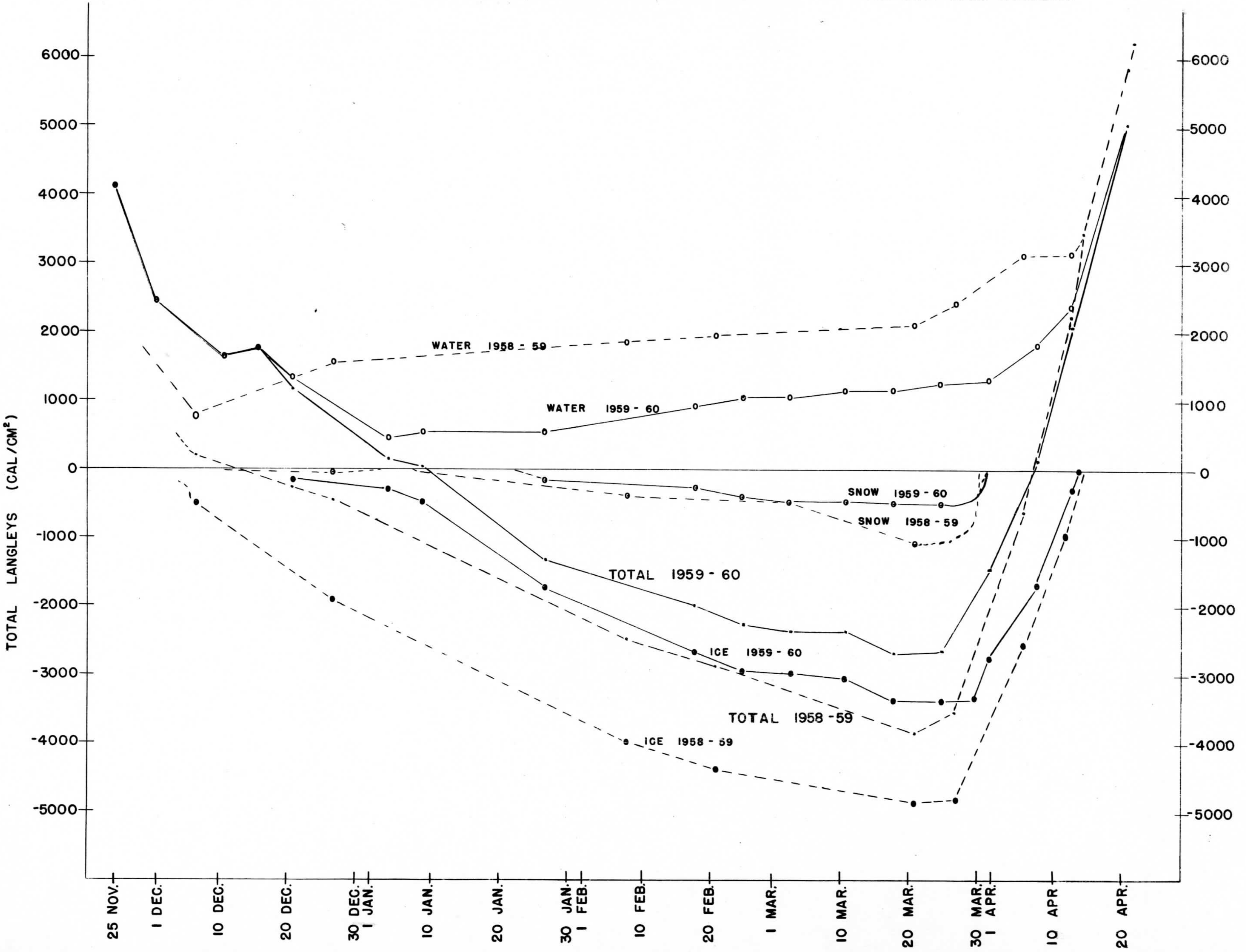
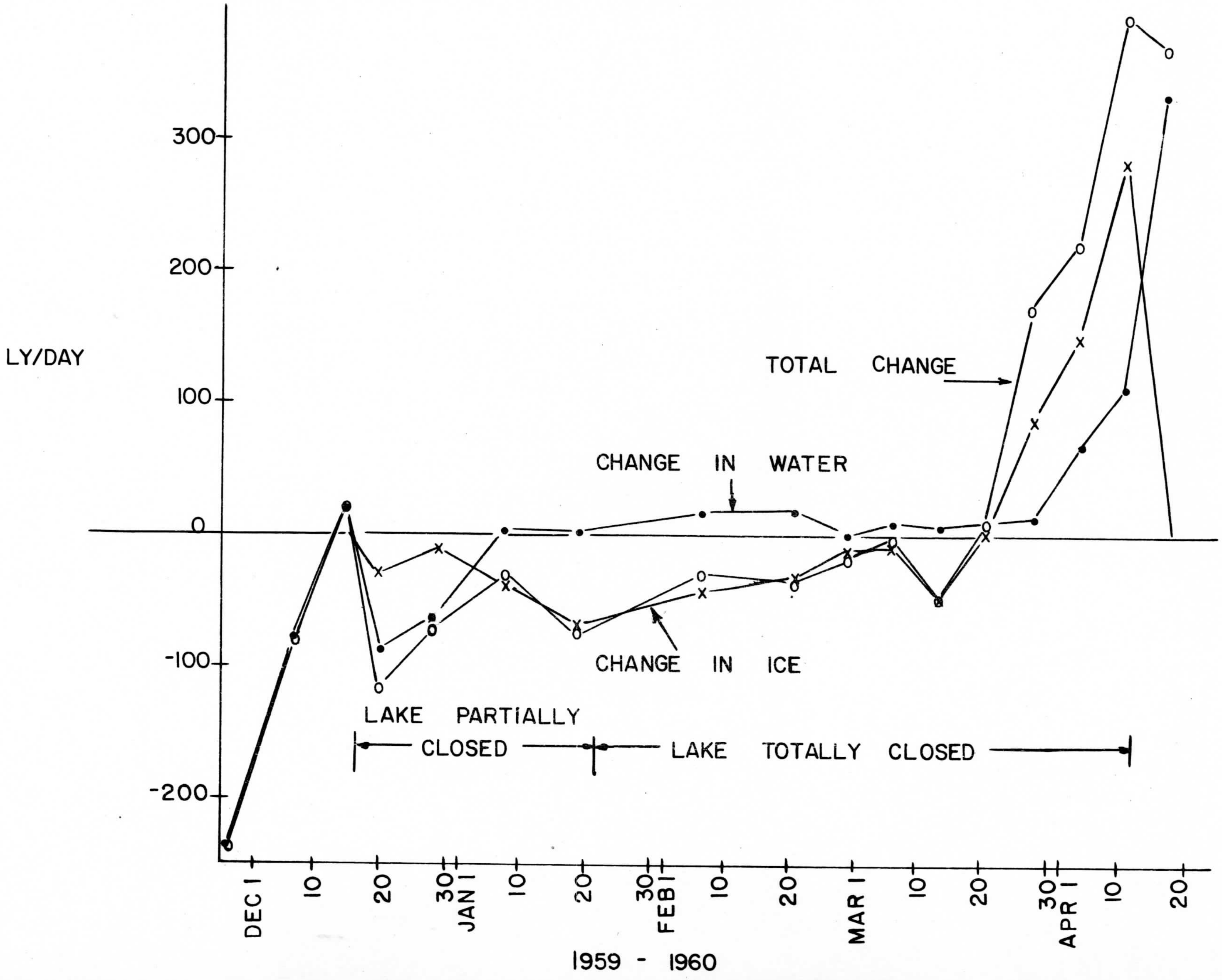


FIG 11 CHANGE IN HEAT STORAGE IN THE WATER AND ICE AND TOTAL CHANGE IN LAKE MENDOTA FOR 1959 - 1960 WINTER.



obtained by the "ice sink" method and those from the U. S. Weather Bureau records, the "ice sink" data were considered more reliable.

2. Heat Storage in 1958-59 and 1959-60

Total heat storage value for water, ice, and snow are plotted in Figure 10 for the two winters. The typical course of the lake heat storage is demonstrated for both years, but in the 1958-59 winter, all of the terms were larger. As the figure illustrates, there is a rapid loss of heat in late fall or early winter followed by a gradual loss in winter with a tapering off of this loss in late winter. In spring, there is abrupt change about two weeks before the lake opens with gains in all media resulting in rapid melting of ice and snow and warming of the water.

In Figure 11 storage change is plotted in ly/day illustrating even more strikingly that the mid-winter is a period of small changes preceded by early winter and followed by spring melting, both periods of exceedingly rapid change.

B. Radiation Terms

1. Net Solar and Sky Radiation - Albedo

Measurements made at the Lake Mendota tower with upright and inverted Eppley pyrheliometers are given in Table 2. In this table a comparison is made of two methods of estimating net solar and sky radiation (col. 10 and 11), three sources of incoming radiation (cols. 2,3, and 4), and three sources of Lake Mendota albedo (cols. 6, 7, and 9).

Table 2: Incoming solar radiation data from various sources for the period February 26, to March 9, 1960.

(1) Date	(2) R_s Truax	(3) R_s Sol. En. Lab	(4) R_s Tower	(5) R_s Refl. Tower	(6) Albedo	(7) Aerial Measurement	(8) Time in (7)	(9) Est. Albedo	(10) $R_s$net Tower	(11) $R_s$net Est.
Feb 26	339	288	264	219	.83	-	-	.80	45	62
Feb 27	477	470	406*	317	.78*	.71	1015	.80	89*	94
Feb 28	396	315	376*	285	.76*	-	-	.75	91*	89
Feb 29	486	468	471*	341	.73*	-	-	.75	131*	120
March 1	492	493	463	339	.73	.64	0943	.75	124	123
March 2	366	372	345	273	.79	-	-	.75	72	92
March 3	409	368	374	314	.84	-	-	.80	60	72
March 4	508	470	395**	344	.87**	-	-	.80	51**	98
March 5	500	527	478**	406	.85**	.70	1025	.80	72**	102
March 6	460	476	449**	351**	.78**	-	-	.80	98**	94
March 7	532	542	462**	398**	.86**	-	-	.80	64**	80
March 8	494	481	432**	362	.86**	-	-	.80	61**	98
March 9	293	230	259	225	.87	-	-	.80	34	39

Note: On Feb 25, at 1029 the albedo determined by Bauer (Col. 7) was .77, and on March 10, at 1032, it was .71.

Explanation of columns: (1) date, (2) Incoming solar and sky radiation measured at Truax Field Weather Bureau Station in ly/day, (3) Incoming solar and sky radiation measured at the Solar Energy Laboratory ly/day, (4) Incoming solar and sky radiation measured at the Meteorology Department Instrument tower on Lake Mendota ly/day, (5) Reflected solar and sky radiation in ly/day measured at the instrument tower, (6) "albedo" computed from columns (4) and (5), (7) albedo determined by aerial measurements, (8) time of day of aerial albedo determination, (9) estimated albedo determined by ovservations of surface conditions (section II-B), (10) net solar and sky radiation in ly/day the instrument tower on Lake Mendota (from col. 5 minus 4), (11) net solar and sky radiation in ly/day determined from the mean of the Truax and Solar Energy Lab value of incoming solar and sky radiation by applying the albedo estimate in col. 9.

*partly estimated because recorder pen went off chart paper near noon.

**partly estimated because there was snow or frost on the instrument which was not wiped off in time. This usually occurred in early morning time, in most cases, in the form of frost on the Eppley instrument and appeared to be a serious error on March 4, 5, 7, and 8.

For the period in Table 2, the estimated net solar radiation, using the estimated albedo values (col. 11), was 15% higher than the measured value (col. 10). This may have been partly due to frost errors on the upfacing Eppley instrument. While the measured net solar radiation was considered to be more reliable (February 26 - March 3 and March 9), the "estimated" net solar radiation was only 6% higher than the measured value (col. 10).

Measurements of incoming and reflected solar radiation (col. 4 and 5) were obtained from values every fifteen minutes, integrated over the day period. Incoming solar radiation measured on the lake was somewhat lower than the average of the Weather Bureau and Solar Energy Laboratory data (col. 2 and 3) probably due to the frost error.

The estimated albedo value compared favorably with the Lake Mendota tower measurement, but was lower than the aerial measurement (Bauer 1960) by about 10%.

Lake Mendota tower albedo measurements were compared to those found by Bauer (Table 3). In general, the tower values were 5 to 10% higher than the aerial measurements. Three possible reasons for this were: (1) the lake tower value was about 4% too high, because the inverted Eppley was not calibrated in that position (Davis 1957), (2) the lake surface, "seen" by the instrument at the one tower site,

Table 3: Albedo measurements at the Lake Mendota instrument tower compared with measurements for various times of the day. The snowfall at Truax Weather Bureau Station is included.

Date	Weather Bureau	Time (1)	albedo (1)	Time (2)	albedo (2)	Time (3)	albedo (3)	Mean of (1-3)	Albedo measurements by Bauer (albedo)	Albedo measurements by Bauer (time)
Feb 25	-	-	-	-	-	-	-	-	.77	1029
Feb 26	trace	1000	.83	1200	.84	1400	.82	.83	-	-
Feb 27	none	0900	.74	-	-	1400	.75	.74	.71	1015
Feb 28	trace	0900	.84	-	-	1400	.84	.84	-	-
Feb 29	none	0900	.75	-	-	1500	.74	.74	-	-
Mar 1	none	1000	.73	1200	.74	1400	.74	.74	.64	0943
Mar 2	0.4	1000	.80	1200	.78	1400	.77	.78	-	-
Mar 3	1.5	1000	.82	1200	.82	1400	.80	.81	-	-
Mar 4	0.2	1100	.77	1200	.80	1400	.76	.78	-	-
Mar 5	none	1000	.77	1200	.76	1300	.77	.77	.70	1025
Mar 6	0.3	1000	.74	1200	.79	1400	.79	.78	-	-
Mar 7	trace	1000	(.93*)	1200	.78	1400	.77	.78	-	-
Mar 8	trace	1000	(.87*)	-	-	1400	.80	.80	-	-
Mar 9	0.2	1000	.86	1200	.87	1400	.85	.86	-	-
Mar 10	-	-	-	-	-	-	-	-	.71	1032

*Value probably high, because snow or frost accumulated on the upward looking instrument. Value in parenthesis not included in the mean column.

actually had a higher albedo than the larger surface, "seen" by the aircraft instrument, and (3) the aircraft value was low, due to radiation absorption by the atmosphere between the airplane and the surface (see Bauer 1960).

One further point is illustrated in Table 3, in which snowfall measured at the Weather Bureau Station is also shown Even a "trace" of snow maintained a high albedo on the lake Giddings and LaChapelle (1961) have shown that a base of 2 - 3 cm of snow is needed for such a "trace" to produce a high albedo. Miller (1955) measured the decrease of albedo on a snow surface with time after a new snowfall His results for winter were:

Days since last snowstorm	0	2	4	6	8	10
Albedo	0.83	0.76	0.71	0.67	0.65	0.64

Results in Table 3 for the period February 28 to March 1 were in agreement with Miller's results.

2. Net Radiation Measurements

Most of the net radiation measurements made on Lake Mendota for the two winters were discontinuous and did not afford a comparison with all of the calculated results reported in Appendix 3. Measurements made with the Economical Net Radiometer (Suomi and Kuhn 1959) are compared to some of the calculated values in Table 4.

Table 4: Estimated values of net radiation compared with values measured on Lake Mendota using The Economical Net Radiometer for the period, February 11 to March 27, 1959.

		Calc. R_s (1)	Calc. R_1 (2)	Calc. R_n (3)	Meas. R_n (4)	Tot. Est. R_n (5)	Tot. Meas. R_n (6)	(7)	(8)
Feb. 11-21	Days*	+ 80	-51.3	+ 28.7	--	- 33.4	+ 27.5	11	5
	Nights*	--	62.1	- 62.1	--	--	--	--	5
Feb 22-March 21	Days	+139.8	-48.4	+ 91.4	+ 71.2	+ 36.0	+ 39.3	28	22
	Nights	--	-55.4	- 55.5	- 31.9	--	--	--	19
Mar. 22-27	Days	+287.1	-56.5	+231.6	+176.3	+179.7	+141.6	7	5
	Nights	--	-50.9	- 50.9	- 34.7	--	--	--	3

(1) Incoming net solar radiation from Appendix 3a ly/day (calculated)
(2) Net long wave radiation from Appendix 3a in ly/day or night (calculated)
(3) Net radiation estimated from Appendix 3a (calculated)
(4) Net radiation measured (days and nights) on Lake Mendota in ly/day or ly/night
(5) Total daily net radiation estimated (col. 1 and 2) ly/day
(6) Total daily net radiation measured ly/day

* All figures in ly/day for daytime, nighttime or total daily period.

(7) Total days and nights
(8) Total days and nights of measured net radiation (col. 4 and 6) usable

The radiometer gave poor results for the period February 11 - 21, shown in Table 4, because less than half of the eleven days of data were reliable.

For the second period in Table 4 (February 22 - March 21), most of the measured radiation data were reliable, and there was agreement with the calculated value (cols. 5 and 6); but the measured radiation for the day and night periods (col.4) were lower than the estimated values (col. 3). The difference at night may have been due to frost accumulation on the instrument, which may have caused some of the nighttime measurements to be low. The lower day measurement may have been due to a reflection error correction of the instrument which was not considered in the computation, or to a low estimate of the lake albedo.

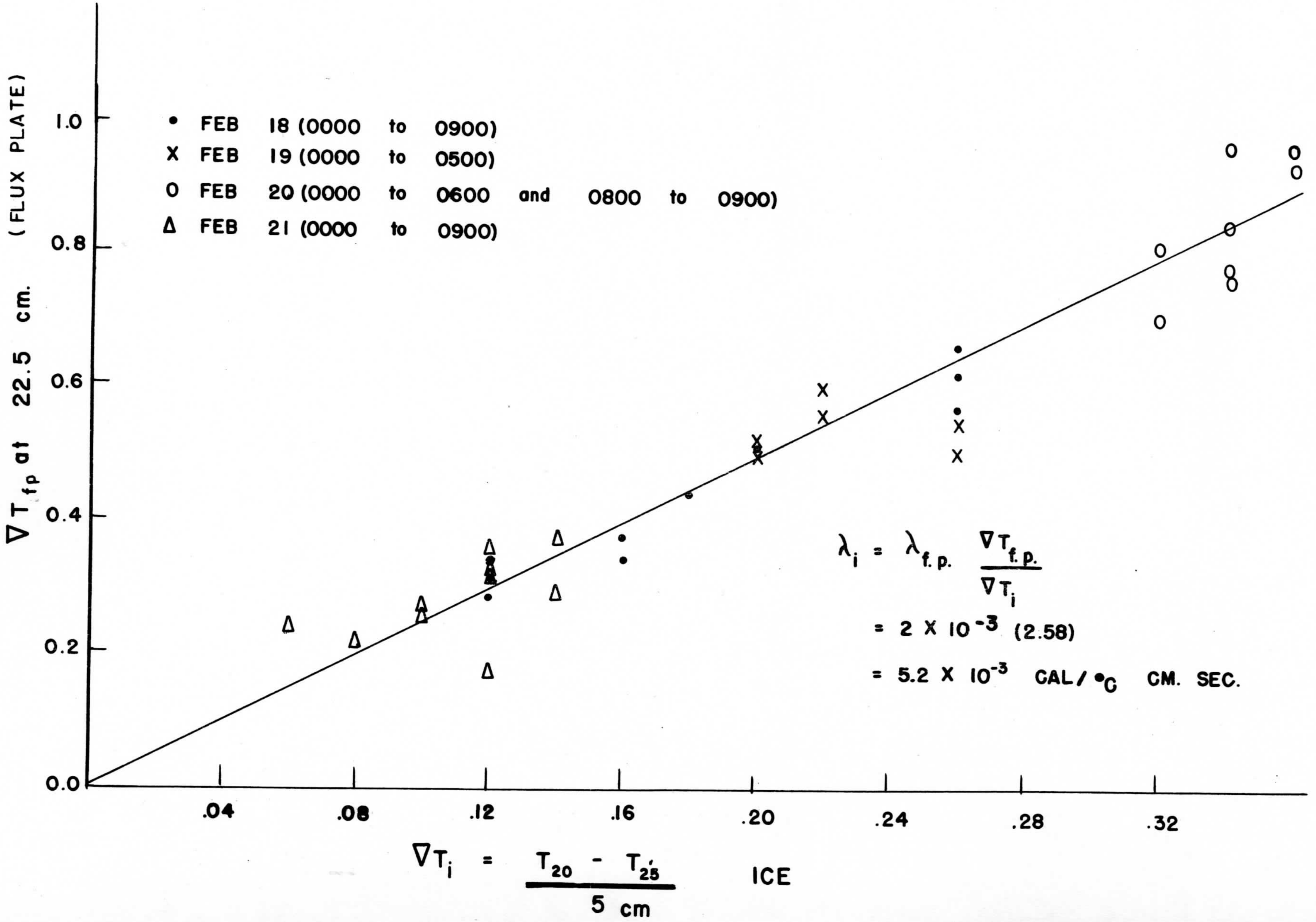
For the period, March 22 - 27, the difference of about forty ly/day (col. 5 and 6) in Table 4 was probably due to the unfortunate placement of the radiometer. Snow melted from most of the lake, but not from under the instrument. The lower measured net radiation for the days was probably caused by a higher albedo under the radiometer.

C. Conduction in the Ice and Snow

1. Conductivity of Ice and Snow

Thermal conductivity of Lake Mendota ice was determined from the "ice conduction instrument" (Figure 12). Temperature difference from the thermocouples at 20 and 25 cm

FIG 12 DETERMINATION OF ICE CONDUCTIVITY AT LAKE MENDOTA TOWER (1960).



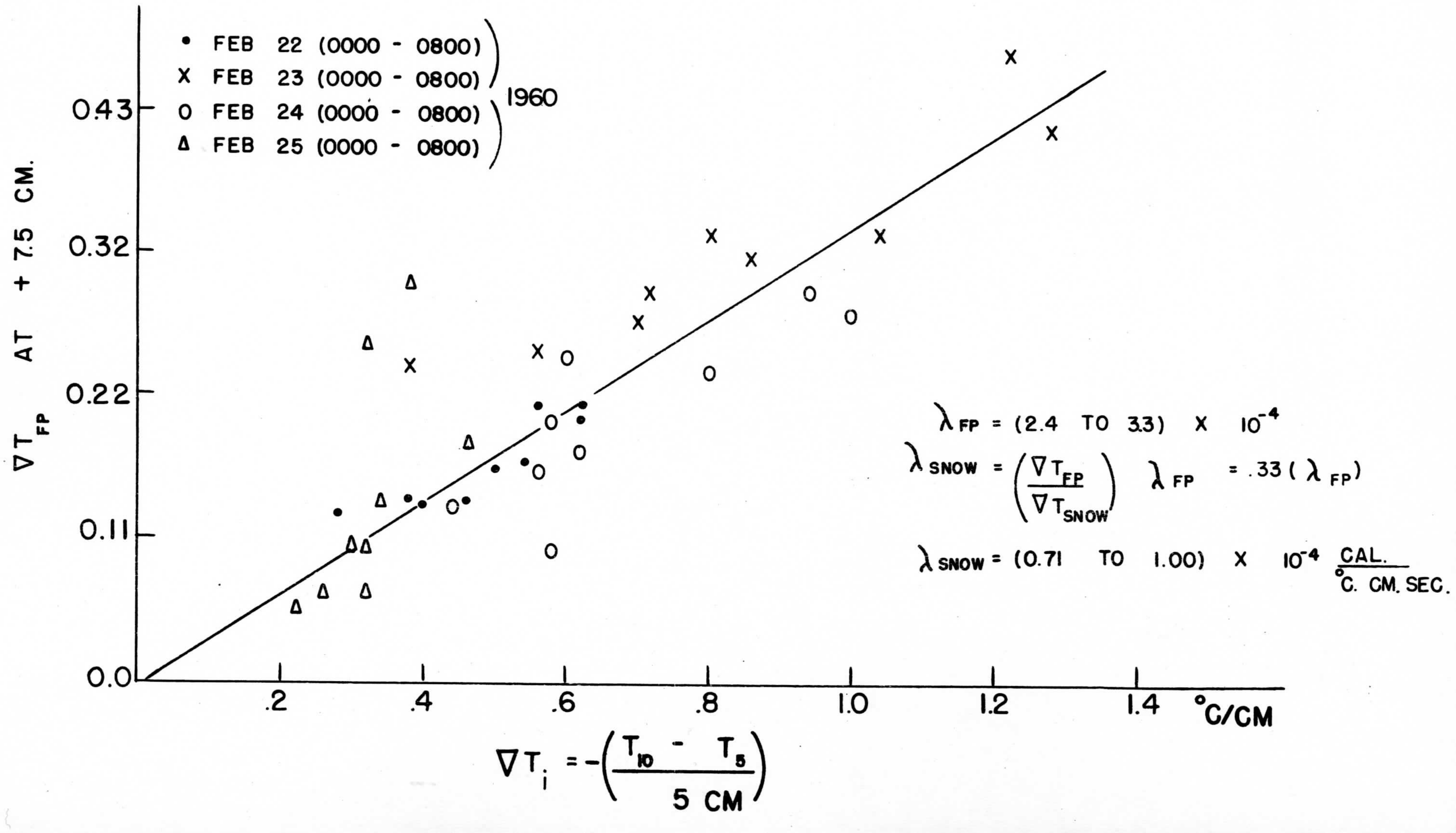
below the top of the ice is plotted against temperature difference across the flux plate 22.5 cm below the ice surface for four nights of hourly values. Flux plate conductivity was taken to be the value of the glass used or 2.0×10^{-3} cal/°C cm sec. Conductivity of the ice was then given by the slope of the line in Figure 12 (fitted by eye) multiplied by λ_{fp}. The value obtained was 5.2×10^{-3} cal/°C cm sec which is in good agreement with past determinations (SIPRE 1951). Since ice conductivity is a function of temperature, the conductivity determined in Figure 12 is an average value.

The same procedure as above was used to determine conductivity of snow for a four day period beginning on February 22 just after a snowfall. Thermocouples at 5 and 10 cm and the flux plate at 7.5 cm above the ice were used (Figure 13). Conductivity of the polystyrene used in constructing the flux-plate was from 2.4 to 3.0×10^{-4} cal/°C cm sec. Snow conductivity was determined to be 0.7 to 1.0×10^{-4} cal/°C cm sec for the snow density of 0.17 g/cm^3. For comparable densities, the conductivities of snow listed in SIPRE (1951) are somewhat higher and vary from 0.5 to 5.0×10^{-4} Cal/°C cm sec.

2. Conduction in Ice

Conduction in the ice was calculated for three layers in the ice over a five day period, shown in Table 5.

FIG 13 DETERMINATION OF SNOW CONDUCTIVITY AT LAKE MENDOTA TOWER (1960).



Hourly values of conduction were added over day and night periods assuming a constant conductivity of 5.0×10^{-3} cal/ ^{o}C cm sec During this same period, the ice near the instrument increased in thickness by 3.5 cm, representing 257 ly released at the bottom of the ice, which had to be conducted upward to the surface. From Table 5 we note that 303 ly were conducted to the surface in the bottom layers if the computation was correct. The difference of 45 ly was too large to be explained totally as heat conducted from the water, since Appendix 5 gives a more realistic value of 3.7 ly/day for this period or about 15 ly for the four days. The discrepancy may have been due to errors in temperature measurement, use of the wrong conductivity, or error in ice thickness measurement. A more likely source was solar radiation absorbed in these layers which was later conducted upwards. If this last explanation is correct, then a mean of about 8 ly/day was absorbed in these lower layers.

Table 5 also illustrates that there was preferential absorption of solar radiation in the surface layers because there was greater conduction there. About 50% of the sun's radiation reaching the surface is composed of wave lengths greater than $.7\mu$ (Johnson 1954). These are strongly absorbed near the top surface of the ice (Lyons and Stoiber 1959). From Table 5, we find about $\frac{(800-300)}{4} \frac{(\text{ly})}{(\text{days})}$ or 125 ly/day preferentially absorbed in the surface layer. Net solar radiation for the period was 240 ly/day, and thus

Table 5: Computed conduction at various levels in the ice using temperature gradients from the temperature profile instrument at the Lake Mendota instrument tower. Hourly values are determined and integrated over the periods of time shown. Figures are given in langleys. Ice conductivity was taken to be 5 x 10^{-3} cal/°C cm sec.

Date	Time	Surface to 5 cm below (ly)	-20cm to -25cm (ly)	-25cm to -30cm (ly)
Feb. 17-18	1800-0600	102	30	24
Feb. 18	0700-1700	95	26	29
Feb. 18-19	1800-0600	122	44	49
Feb. 19	0700-1700	121	46	39
Feb. 19-20	1800-0600	220	70	72
Feb. 20	0700-1700	73	42	49
Feb. 20-21	1800-0600	65	38	33
Feb. 21	0700-1000	7	7	9
Totals		805	303	304

Table 6: Ice Phenology for Lake Mendota

Year	Closing Date	Opening Date	Days of ice cover	Date of complete ice cover	Days of complete ice cover
100 year mean	Dec. 19	Apr. 6	109	--	--
1958-59	Dec. 7	Apr. 15	131	Dec. 7	131
1959-60	Dec. 29	Apr. 12	98	Jan. 22	86

a total of about 105 ly/day are absorbed in the remaining layer (5-20 cm) or transmitted to the water. (considering 10 ly/day absorbed in the lower layers).

IV. DISCUSSION

The data in Appendices 1-7 were broken into various time periods, depending on the date when water temperatures were measured, giving a broken rather than a continuous description of the heat budget. This was not a serious problem because the nature of the heat budget of Lake Mendota changed markedly as winter progressed. Bryson and Bunge (1956) defined three periods of the ice season. They were termed the "ice growth period", "the equilibrium period", and the "wastage period". Because early winter and spring data were available, four other "periods" could be delineated here. These were termed the "cooling period", the "ice build-up period", the "break-up period", and the "initial warm-up period". In a few cases, the date of change from one period to the next was somewhat arbitrary, but in general, each "period" was markedly different from the others.

The "ice build-up" period was defined, because in the 1959-60 winter, the entire lake surface didn't freeze in the normal manner, that is, in one or a few days. Ice phenology for the mean and for the two winters is shown in Table 6.

Actually, ice began to form Dec. 16, 1959, so that in the 1959-60 winter there were 111 days of at least partial ice cover.

In the following discussion of the heat budget for the various "periods", the remainder needed to balance the budget equation (2-5) will be determined for each case. These remainders will be apportioned either quantitatively or qualitatively among the unmeasured terms. If more than one term is unknown, a qualitative statement will be made as follows:

"negligible" - probably $<$ 1 ly/day
"small" - probably $<$ 5 ly/day
"significant"- probably $>$ 5 ly/day

Significant terms can often be roughly estimated if they are known to be much larger than the other unmeasured terms.

A. The Cooling Period

A few weeks before ice forms on Lake Mendota, there is a period of rapid heat loss. Birge (Neess and Bunge 1957) found that Lake Mendota does not freeze until the mean water temperature decreases to about 1.0°C. The mean water temperature at closing in 1959 was 0.6°C. Heat budget data for the cooling period for 22 days, from Nov. 25 to Dec. 16, 1959 is as follows:

$$\begin{array}{lcccccccccccccccl} C_w & = & R_{sw} & + & R_l & + & K_m & + & V & + & P & + & E & + & L & & \text{Remainder} \\ -108 & = & +126 & - & 107 & + & 5.3 & + & 2.0 & - & 3.7 & - & 66^* & - & 66^* & & -132.9 \end{array}$$

*based on Bowen ratio = 1.0 (Dutton and Bryson, 1960)

(a) P determined from snowfall data

The water lost heat at the rate of 108 ly/day which is about half the mean rate, according to Dutton and Bryson (1960). Net radiation was +19 ly/day. and all but the remaining terms (-L and -E) were small.

B. Ice Build-up Period

Late November and early December of 1958 were abnormally cold. Lake Mendota cooled rapidly and froze during the night of December 7th. But the corresponding period in 1959 was mild, and although ice began to form on December 16. there was not a complete ice cover until January 27 which, therefore, was defined as the end of this period (Table 7).

1. Water Budget

Lake Mendota water lost heat during this period, because it was exposed to surface losses ($-R_1$. -L and -E) and to losses by melting of floating ice (K_i). The latter term was probably the largest of the terms not estimated.

At one period (January 4-5), the mean water temperature reached 0.25 $^{\circ}$C, but, despite an air temperature as low as -1°F, the lake did not freeze because of strong winds. At other times, winds were calm but air temperatures were too high. Thus, the lake is greatly affected by extremes of weather, and one peculiar condition may produce a great difference in the characteristics of the heat budget for an entire winter.

C. Ice Growth Period

After Lake Mendota was completely covered with

Table 7: Heat budget data for the "ice build-up" period, lasting from December 17, 1959, to January 27, 1960 or 42 days.

Water Budget:

C_w	=	R_{sw}	+	R_l	+	K_m	+	K_i	+	V	+	P	+	E	+	L	Remainder
-28.7	=	+66	-	52	+	4.9	-	sign.	+	2.2	-	18.6	-	sign.	-	sign.	-31.3

Ice Budget:

F_i	=	R_{si}	+	R_l	+	K_i	+	K_s	+	P	+	E	+	L	
-40.6	=	+13	-	46	+	sign.	-	negl.	+	negl.	-	small	-	sign.	- 7.6

Snow Budget:

F_s	=	R_{ss}	+	R_l	+	K_s	+	P	+	E	+	L
-2.4	=	+65	+	64.5	+	sign.	-	8.8	-	small	-	sign.

Note:

(a) Long wave radiation was weighted according to the area and time of ice, water, or snow cover.

(b) Snowfall before January 10 fell into the water and formed ice or melted. "Snow Budget" based on period January 10-27.

(c) precipitation term from snowfall data ice, there followed a relatively rapid increase of ice thickness and a slow warming of the water.

1. Water Budget

The remainder of 9.9 ly/day in the 1960 budget was thought to be due to the large drifting of snow which reduced the reliability of the solar radiation term (R_s). Some areas on the ice were clear of snow, and so the mean snow depth used in Appendix 4 probably did not apply. By difference, R_{sw} becomes 11.9 ly/day (see Table 8).

Advection and conduction terms were relatively more important to the water budget in mid-winter, because all other terms were small.

2. Ice Budget

Ice thickness increased at a rate of 0.50 and 0.75 cm/day for the 1960 and 1958-59 winters, respectively, through upward conduction (K_s) in the ice to the snow.

3. Snow Budget

A snow cover slows the rate of ice growth by acting as an insulator. The effect of the low snow conductivity is demonstrated in Figures 14 and 15 in which temperatures in the ice, snow, and air are plotted for one day periods. High temperature gradients (Figure 14) with no snow cover resulted in rapid heat loss. Figure 15 illustrates that the new snow cover reduced the temperature gradients and hence the heat loss by conduction in the ice

Table 8: Heat budget data for the ice growth period which lasted 29 days, from January 28 - February 25 in 1960 and 64 days, from Dec. 7 to Feb. 8 in 1958-59.

Water Budget:

	C_w	=	R_{sw}	+	K_m	+	K_i	+	V	Remainder
1959-60	+17.1	=	+2.0*	+	5.8	-	2.9	+	2.3	+9.9*
1958-59	+17.9	=	+14.0	+	6.5	-	5.5	+	2.4	+0.5

Ice Budget:

	F_i	=	R_{si}	+	R_l	+	K_i	+	K_s	+	E	+	L	Remainder
1959-60	-41.3	=	+8.0	-	4.0	+	2.9	-	sign.*	±	negl.	-	small	-48.2*
1958-59	-61.8	=	+23.0	-	23.0	+	5.5	-	sign.*	-	small	-	small	-67.3*

Snow Budget:

	F_s	=	R_{ss}	+	K_s	+	P	+	E	+	L	Remainder
1959-60	-6.5	=	+56.0	-	80.2	+	(45.0)*	-	8.0	- small	- sign.	-6.3
1958-59	-4.3	=	+39.0	-	83.0	+	(60.0)*	-	6.1	- sign.	- sign.	-4.4

Note:

(a) long wave radiation weighted for the number of days and approximate area of snow sover.

(b) remainder term probably belongs in the marked term. For the snow budget, the term is estimated from the remainder in the ice budget and is placed in parentheses.

FIG 14 ICE AND AIR TEMPERATURE AT THE LAKE MENDOTA TOWER FEB 18 - 19, 1960.

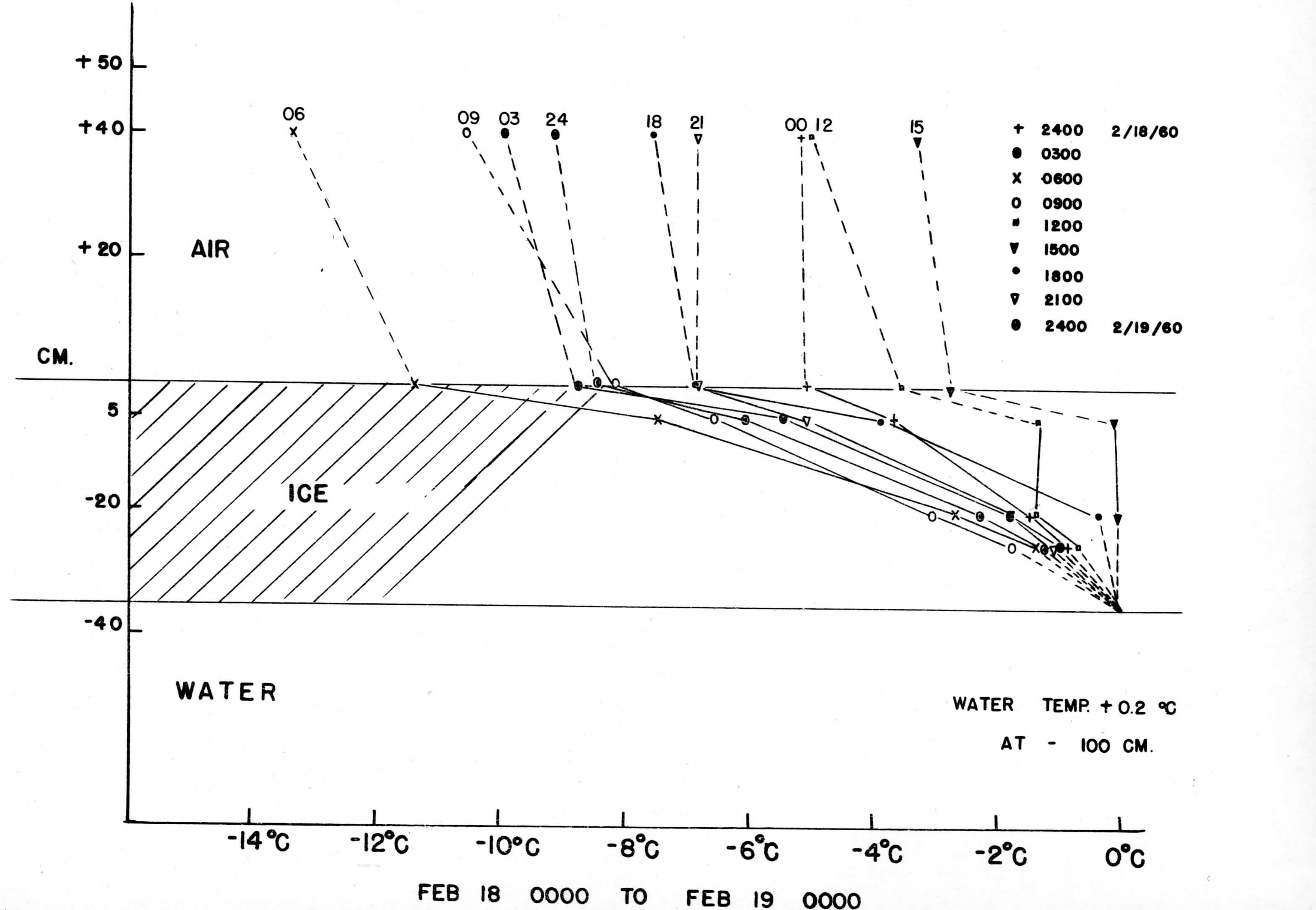
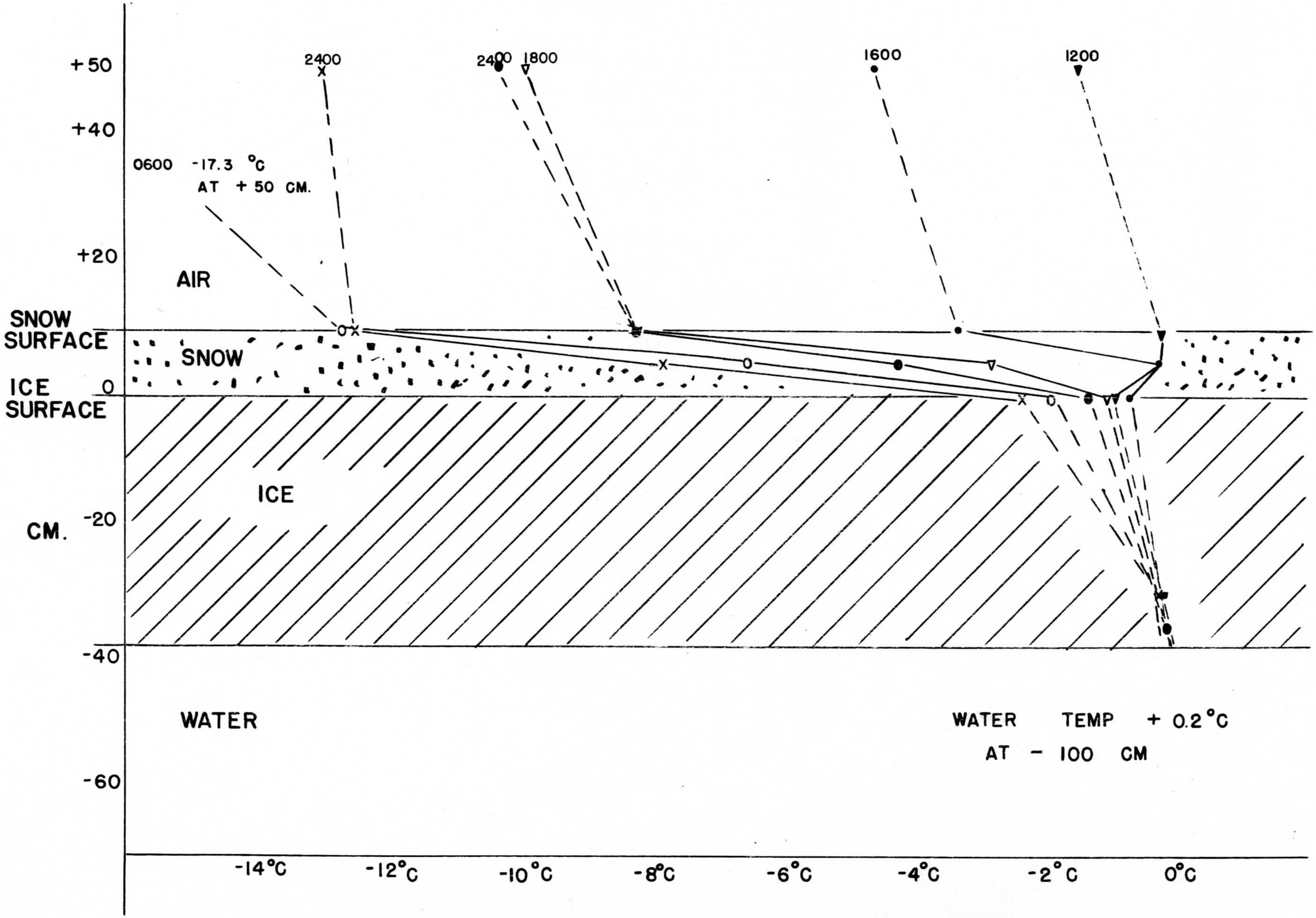


FIG 15 ICE SNOW AND AIR TEMPERATURE AT LAKE MENDOTA TOWER

FEB 23 - 24, 1960.



FEB 23 0000 TO FEB 24 0000, 1960
(CLEAR, SUNNY DAY)

by a factor of about five for similar air temperatures.

For the snow heat budget in Table 8, the conduction term (K_s) was estimated from the ice heat budget by the reasonable assumption that for the ice the terms -E and -L were small. There was probably some error in the estimation of the other large terms in the snow budget, (e.g. R_{ss} and R_l) because air temperature indicated that the sensible heat loss was "large", but the negative remainders needed to balance the budget were small.

D. Equilibrium Period

In late winter, ice increase slows and then stops because of a combination of deeper snow, warmer air temperatures, higher positive net radiation, or thicker ice. The beginning of this period has been set arbitrarily as the date when the ice increase has slowed to -20 ly/day of heat loss, due to ice accretion at the bottom of the ice sheet.

1. Water Budget

The most probable errors in the water budget, indicated by the small remainder terms, were in the determination of the solar radiation (R_{sw}) and conduction (K_i) terms. Estimation of these terms can be improved easily, but exact determination of bottom conduction (K_m) and advection (V) would be more difficult.(see Table 9).

Table 9: Heat Budgets for the equilibrium period which lasted for 28 days from February 26 to March 24 in 1960, and for 42 days from February 9 to March 27 in 1959.

Water Budget:

	C_w	=	R_{sw}	+	K_m	+	K_i	+	V	Remainder
1959-60	+6.6	=	0.0	+	4.7	-	3.4	+	2.2	+3.1
1958-59	+11.6	=	3.0	+	3.9	-	5.3	+	2.2	+7.8

Ice Budget:

	F_i	=	R_{si}	+	R_1	+	K_i	+	K_s	+	E	+	L	Remainder
1959-60	-17.0	=	+2.0	-	0.0	+	3.4	-	(22.4)*	+	0.0	+	0.0	-22.4*
1958-59	-17.4	=	+18.0	-	0.0	+	5.3	-	(30.7)*	+	0.0	+	0.0	30.7*

Snow Budget:

	F_s	=	R_{ss}	+	R_1	+	K_s	+	P	+	E	+	L	Remainder
1959-60	-4.2	=	+99.0	-	96.4	+	(22.4)*	-	3.5	±	small-		sign.	-12.5
1958-59	+2.4	=	+123.0	-	106.7	+	(30.7)*	-	13.6	±	small-		sign.	-31.0

Table 10: Ice and snow budgets for the week March 11-17, 1960

Ice Budget:

F_i	=	R_{si}	+	K_i	+	K_s	Remainder
-47.2	=	+1.0	-	5.3	-	(40.9)*	-40.9 *

Snow Budget:

F_s	=	R_{ss}	+	R_1	+	K_s	+	P	+	E	+	L	Remainder
-8.0	=	+99.0	-	93.9	+	(40.9)	-	6.8	-	small	-	sign.**	-47.2 **

Accurate determination of the solar radiation term (R_{sw}) may be important for biological studies especially on shallow lakes where snow may cut off light for photosynthesis.

2. Ice Budget

Conduction upwards in the ice was estimated by differences (Table 9) because for the ice the long wave radiation (R_1), latent heat (E), and sensible heat (L) terms were zero.

From Table 9, it was shown that the ice thickened at about 0.25 cm/day or from less than 1/2 to 1/3 of the increase for the "ice-growth" period. Data for the 1959 period were somewhat misleading, because ice increased on the top surface rather than in the normal manner at the bottom of the ice. The large amount of snowfall depressed the ice, forcing water to the surface which formed slush and froze.

In 1960 there was a week of cold weather from March 11-17 leading to a second brief "ice-growth" period (Table 10).

From the budget data in Tables 9 and 10, it can be concluded that with a thick ice cover, if a snow cover is present, air temperature regulates the amount of ice increase. From experience, it was noted that a period of several cold days was needed to produce an appreciable amount of ice.

3. Snow Budget

One of the largest differences between the heat budgets of the two years was caused by the heavier snowfall in the later part of the 1959 winter. Before the ice begins to melt, this snow must melt first. In both seasons the snow melted rapidly (Table 11).

The large remainders needed to balance the budgets were attributed to the sensible heat (+L) and condensation (+E) gains. Because the net radiation is small for this period due to the high albedo of the snow cover, the air temperature was the deciding factor in determining the date of snow melt. If the arrival of a warm "spring-like" period was delayed for several days, the ice melting was likewise delayed.

E. Wastage Period

After the snow melts, decreasing albedo results in higher net radiation, which, combined with higher air temperature, causes rapid ice melt. Normally, Lake Mendota is ice free about twenty days after the snow has melted.

1. Water Budget

The main problem in the water budget for this interesting period is in the determination of the two large terms, solar radiation transmitted through the ice (R_{sw}) and conduction or convection through the water to the ice (K_i). In Table 12, K_i was determined by difference because the radiation absorbed by the ice was computed daily. The

Table 11: Snow heat budget for the period of rapid snow melt. In 1960, there were three days, from March 26-28 and in 1959, seven days, from March 22-27.

	F_s	=	R_{ss}	+	R_l	+	K_s	+	P	+	E	+	L	Remainder
1959-60	+135.8	=	+94.0	-	68.9	+	negl.	+	0.0	+	sign.	+	sign.	+110.8
1958-59	+110.2	=	+152.0	-	107.4	+	negl.	+	15.2	+	sign.	+	sign.	+ 80.8

Table 12: Heat budget of the wastage period. In 1960 there were fourteen days shown from March 25-April 7 and in 1959, seventeen days from March 28 to April 13.

Water Budget:

	C_w	=	R_{sw}	+	K_m	+	K_i	+	V	Remainder
1959-60	+41.2	=	+49.5	+	1.9	-	(-11.2)*	+	2.0	-11.2*
1958-59	+42.8	=	+56.0	+	2.6	-	(-18.2)*	+	2.4	-18.2*

Ice Budget:

	F_i	=	R_{si}	+	R_l	+	K_i	+	P	+	E	+	L	Remainder
1959-60	+120.4	=	+163.0	-	81.4	+	sign.	+	negl.	±	small	+	sign.	+48.8
	+228.3	=	+248.0	-	101.1	+	sign.	+	negl.	±	small	+	sign.	+81.4

direct determination of K_i (Appendix 5) was unreliable for the last week in March, but for one week of the wastage period (April 1-7, 1960), measurement of K_i (see Figure 9 and Appendix 5) was sufficiently accurate so that the term, R_{sw}, could be determined by difference. For this one week, the water budget was,

$$\begin{array}{cccccccccc} C_w & = & R_{sw} & + & K_m & + & K_i & + & V & \text{Remainder} \\ +68.9 & = & +(108.1)^{*} & + & 1.9 & + & 43.1 & + & 2.0 & +108.1^{*} \end{array}$$

If the R_{sw} value of 108 ly/day is applied to the Bouguer-Lambert law with the assumption that the ice was homogeneous and diffuse, then an "effective" absorptivity coefficient for the ice of 0.031 cm^{-1} results. This applies to total solar radiation. Assuming the ice absorbs all of the radiation of wave lengths greater than .7 microns (50% of total), a coefficient of 0.0115 cm^{-1} if found. This is lower than Lyon's and Stoiber's (1959) value for "bubbly" ice (0.028 cm^{-1}), but higher than published values (0.0004 to 0.007 cm^{-1}) for clear ice (Lyons and Stoiber 1959 and SIPRE).

From the above it is concluded that the heat budget affords a good check on instrumental techniques for measuring an "effective absorptivity" of ice.

2. <u>Ice Budget</u>

Ice melted at an average rate of about 1.7 cm/day in 1960 and 3.3 cm/day in 1959. The difference was due largely to a greater amount of incoming solar radiation in 1959, which also indirectly affected the air temperature.

The greatest amount of melt occurred when the three terms, R_{net}, L and E were all positive. This was experienced when the Madison area was under the influence of maritime tropical air. At these times, ice melt rates as high as 8 cm/day were measured. From Table 12, it is seen that melt at the bottom of the ice, (K_i), is large, but not as important as the other large terms [see Neess and Bunge (1957) and Bryson and Bunge (1956)].

F. Break-up Period

In both seasons studied in this report, the ice left the lake in a spectacular fashion (two days). When the ice thickness decreases to 20 cm or less with sizeable leads present, a wind will cause the ice sheet to pile up on a shore, often causing considerable property damage. Some effects of ice shove are shown in Figure 15.

In both seasons, the ice cover decreased from over 95% cover to about 30% in one daylight period. When such a large area of water surface was exposed, the heat budget changed drastically. In 1959, water temperatures were taken on April 13, 14, and 15, because a boat could be forced through the "rotten ice", but on April 8, 1960, the ice was too firm to push a boat through, yet not strong enough to support a person. Thus, data for the 1960 break-up contained the last three days of the "wastage period" (April 8-10).

1. Water and Ice Budgets

For the 1959 break-up, the two predominant terms

Figure 15: Wind induced ice damage in spring (March 27, 1961).

a. Ice pushed 100 feet in "tongues" on a low shore. Boat house in background was severely damaged

b. Large boulder lifted by ice push

c. Boat wrecked by ice push

Table 13: Heat budget for the break-up period. In 1960, five days were included, from April 8-12 and in 1959 there were two days, April 14-15.

Water Budget:

	C_w	=	R_{sw}	+	R_l	+	K_m	+	K_i	+	V	+	E	+	L	Remainder
1959-60	+114.8	=	+218.0	-	33.2	+	negl.	-	sign.	+	2.4	+	sign.	+	sign.	-72.4
1958-59	+139.5	=	+501.0	-	125.0	+	1.6	-	sign.	+	2.4	-	small	+	sign.	-240.5

Ice Budget:

	F_i	=	R_{si}	+	R_l	+	K_i	+	E	+	L	Remainder
1959-60	+280.4	=	+258.0	-	99.5	+	sign.	+	sign.	+	sign.	+121.9
1958-59	+457.0	=	+ 53.0	-	60.0	+	sign.	-	small	+	sign.	+464.0

Notes:

(a) long wave radiation weighted for relative amount of area of water and ice.
(b) solar radiation also apportioned by area (Appendix 4).

Table 14: Heat budget of the period just after ice break-up. In 1960, data is from April 13-20 and in 1959, from April 16-22.

	C_w	=	R_{sw}	+	R_l	+	K_i	+	K_m	+	V	+	E	+	L	Remainder
1959-60	+334.0	=	+351.8	-	79.6	-	36.6*	-	negl.	±	negl.	+	sign.	+	sign.	98.4
1958-59	+401.0	=	+406.4	-	103.5	+	0.0	-	negl.	±	negl.	-	negl or -sign.	+	sign.	98.1

not estimated were conduction to the ice (K_i) and sensible heat gain by the water and ice (L). Latent heat exchange was small since the mean dew point temperature was -1.0°C. If the assumption was made that sensible heat was gained by the ice at the same rate as by the area of exposed water, these terms could be determined for the 1959 budget. Since open water covered 2/3 of the lake area over the two days, we have from Table 13,

$$-K_i \quad + \quad 2/3L \quad = \quad -240.5 \text{ (water budget)}$$

$$K_i \quad + \quad 1/3L \quad = \quad 464.0 \text{ (ice budget)}$$

and L = 223.5 ly/day, K_i = 390 ly/day, L to ice = 74.5 ly/day, L to water = 149 ly/day. Thus, ice melted at a rate of 6 cm/day due to mixing with water (K_i) while sensible heat from the air caused 1 cm/day melt. An interesting feature of this period is that despite the large heat loss for melting ice (-390 ly/day) the water gained heat at a rate of 139.5 ly/day.

G. Initial Warm-up Period

As soon as the ice was gone, Lake Mendota warmed rapidly. In 1960 the mean water temperature increased from 2.0°C at break-up to 4.1°C eight days later; and in 1959, it increased from 2.8°C at break-up to 5.2°C seven days later.

The large remainders were attributed to gains by the surface flux terms (L and E). Weather Bureau data for this period are shown below:

	Mean water temp. °C (from lake soundings)	Mean dew pt. at noon °C	Mean dew at midnight °C	Mean air temp. °C	Mean wind speed mph
April 13-20, 1960	2.52	+6.1	+4.8	+10.7	12.3
April 16-22, 1959	4.04	-0.7	+1.7	+ 8.8	12.0

From the temperature, dew point and wind speed data, it would be expected that L plus E was larger for the 1960 warm up period, although the data in Table 14 showed them to be about equal. A possible error was in determining the conduction term, (K_i), for the 1960 warm up, because the ice thickness could not be determined accurately on April 12th of that year (See *Table 14).

V. CONCLUSIONS

1. The three most important factors in the winter heat budget of Lake Mendota are, (1) radiation, (2) air temperature, and (3) snow cover. Radiation affects the heat budget directly, while air temperature governs the loss or gain of sensible heat. Snow cover indirectly affects the heat budget because of its high reflectivity in the solar energy wave lengths and because it acts as an insulator against conduction losses.

2. With some improvement in the methods, the heat budget can be used as a check against instrumental techniques for determining sensible heat flux (L) because

during winter, evaporation is often negligible for long cold periods.

3. Empirical methods for determining long wave radiation flux are good if 10% accuracy is sufficient. However, the heat budget study is vastly improved if continuous measurements of net-radiation can be obtained.

4. Heat conducted through the water to the ice can be determined if accurate water temperatures are taken just beneath the ice. An exposed bead thermistor is suitable for this purpose.

5. A method of obtaining snow load by measuring amount of ice "sink" was found to be extremely useful for the heat budget study. Because the measurement integrates over a large area, this method should be tested for obtaining latent heat exchange (E) at the surface as well as for measuring snowfall amount and total snow load.

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Appendix 1: Rate of change of the storage terms in ly/day for the years 1959-60 and 1958-59, based on water temperatures, ice thickness and water equivalent of snow.

1959-60 Period	# Days	C_w	F_i	F_s	Tot.
Nov. 25-Dec. 1	7	-235.7	0	0	-235.7
Dec. 2-Dec.11	10	- 82.8	0	0	- 82.8
Dec. 12-Dec.16	5	+ 21.0	0	0	+ 21.0
Dec. 17-Dec.21	5	- 87.2	- 29.2	0	-116.4
Dec. 22-Jan. 4	14	- 62.9	- 10.4	0	- 32.6
Jan. 5-Jan. 9	5	+ 4.0	- 36.6	0	- 32.6
Jan. 10-Jan.27	18	+ 1.5	- 68.5	- 5.5	- 72.5
Jan. 28-Feb.18	22	+ 16.9	- 43.5	- 1.2	- 27.8
Feb. 19-Feb.25	7	+ 18.0	- 34.5	-23.1	- 39.6
Feb. 26-Mar. 3	7	+ 0.4	- 12.6	-16.8	- 29.0
Mar. 4-Mar.10	7	+ 9.1	- 8.3	-12.4	- 11.6
Mar. 11-Mar.17	7	+ 7.0	- 47.2	- 8.0	- 48.2
Mar. 18-Mar.24	7	+ 10.0	+ 0.3	+11.4	+ 22.7
Mar. 25-Mar.31	7	+ 13.8	+ 89.9	+58.1	+161.8
Apr. 1-Apr. 7	7	+ 68.9	+151.1	0	+220.0
Apr. 8-Apr.12	5	+114.8	+280.4	0	+395.2
Apr. 13-Apr.20	8	+334.4	+ 36.6	0	+370.0

1958-59	# Days	C_w	F_i	F_s	Tot.
Dec. 7-Dec.27	21	+ 37.8	- 90.8	- 2.8	- 55.6
Dec. 28-Feb. 8	43	+ 8.2	- 47.7	- 5.1	- 44.6
Feb. 9-Feb.21	13	+ 8.4	- 31.0	- 6.7	- 29.3
Feb. 22-Mar.21	28	+ 4.9	- 18.0	-20.3	- 33.4
Mar. 22-Mar.27	7	+ 44.7	+ 1.3	+110.2	+155.2
Mar. 28-Apr. 6	10	+ 74.2	+223.2	+ 48.0	+354.4
Apr. 7-Apr.13	7	- 3.6	+235.6	0	+232.0
Apr. 14-Apr.15	2	+139.5	+457.0	0	+596.5
Apr. 16-Apr.22	7	+401.0	0	0	+401.0

Appendix 2: Summary of storage by change by "period" of the winter season for 1959-60 and 1958-59. Values in ly/day.

Year	Period	No. of Days	C_w	F_i	F_s	Tot.
COOLING PERIOD						
1959-60	Nov. 25-Dec. 16	22	-107.8	--	--	-107.8
ICE BUILD-UP PERIOD						
1959-60	Dec. 17-Jan. 27	42	- 28.7	- 40.6	- 2.4	- 71.7
ICE GROWTH PERIOD						
1959-60	Jan. 28-Feb. 25	29	+ 17.1	- 41.3	- 6.5	- 30.7
1958-59	Dec. 7-Feb. 8	64	+ 17.9	- 61.8	- 4.3	- 48.2
EQUILBRIUM PERIOD						
1959-60	Feb. 26-Mar. 24	28	+ 6.6	- 17.0	- 4.2	- 15.6
1958-59	Feb. 9-Mar. 27	48	+ 11.6	- 17.4	+ 2.4	- 3.4
WASTAGE PERIOD						
1959-60	Mar. 25-Apr. 7	14	+ 41.2	+120.4	+29.0	+190.6
1958-59	Mar. 28-Apr. 13	17	+ 42.8	+228.3	+28.2	+299.3
BREAK-UP PERIOD						
1959-60	Apr. 8-Apr. 12	5	+114.8	+280.4	0	+395.2
1958-59	Apr. 14-Apr. 15	2	+139.5	+457.0	0	+596.5
INITIAL WARM UP						
1959-60	Apr. 13-Apr. 20	8	+334.0	+ 36.6	0	+370.6
1958-59	Apr. 16-Apr. 22	7	+401.0	0	0	+401.0

Appendix 3a: Summary of albedo, net solar radiation, net long wave radiation, and net radiation for the years 1959-60 and 1958-59. Radiation values in ly/day.

1959-60 Period	No. of Days	Mean Albedo* (1)	(2)	Net Solar and Sky Radiation	Net Long Wave Radiation	Net Radiation
Nov. 25-Dec. 1	7	.03	.03	158.7	-110.5	48.2
Dec. 2-Dec. 11	10	.03	.05	129.7	-116.1	13.6
Dec. 12-Dec. 16	5	.03	.02	73.0	- 94.6	- 11.6
Dec. 17-Dec. 21	5	.03	--	152.6	-129.6	23.0
Dec. 22-Jan. 4	14	.08	--	79.8	- 77.0	2.8
Jan. 5-Jan. 9	5	.12	.12	139.4	-131.5	7.9
Jan. 10-Jan. 27	18	.37	a	104.9	- 86.4	18.5
Jan. 23-Feb. 18	22	.71	.65	67.5	- 85.5	- 18.0
Feb. 19-Feb. 25	7	.81	.71	60.3	- 80.0	- 19.7
Feb. 26-Mar. 3	7	.77	.70	94.0	-105.4	- 11.4
Mar. 4-Mar. 10	7	.81	.72	87.8	-127.4	- 39.6
Mar. 11-Mar. 17	7	.77	.71	99.8	- 93.9	- 5.9
Mar. 18-Mar. 24	7	.76	.65	122.1	-114.9	+ 7.2
Mar. 25-Mar. 31	7	.41	.35	143.0	- 68.9	+ 74.1
Apr. 1-Apr. 7	7	.17	.13	323.1	- 87.3	+235.8
Apr. 8-Apr. 12	5	.09	.08	476.0	-132.7	+343.3
Apr. 13-Apr. 20	8	.05	.06	351.8	- 79.6	+282.1

*Column (1) is the mean of daily values determined by estimation, (see text, section II B 1), and column (2) is the mean of the albedo values measured by Bauer(1960), using his "hemispheric" values.

a)Two measurements were taken during this time, but the lake was partly open.

Period	No. of Days	Mean Albedo	Net Solar and Sky Radiation	Net Long Wave Radiation	Net Radiation
Dec. 7-Dec. 27	21	.65	56.9	-105.2	- 48.3
Dec. 28-Feb. 8	43	.69	57.4	-111.8	- 54.4
Feb. 9-Feb. 21	13	.73	79.5	-113.4	- 33.9
Feb. 22-Mar. 21	28	.61	139.8	-103.8	+ 36.0
Mar. 22-Mar. 27	7	.31	287.1	-107.4	+179.7
Mar. 28-Apr. 6	10	.23	300.3	-100.7	+199.6
Apr. 7-Apr. 13	7	.15	310.9	-101.6	+209.3
Apr. 14-Apr. 15	2	.10	554.3	-185.0	+369.3
Apr. 16-Apr. 22	7	.06	406.4	-103.5	+302.9

Appendix 3b: Summary of the radiation terms by period of the winter season.

Year	Period	No. of Days	Mean Albedo (1)	Net Solar and Sky Radiation	Net Long Wave Radiation	Net Radiation
COOLING PERIOD						
1959-60	Nov. 25-Dec. 16	22	.03	126.0	-107.2	+ 18.8
ICE BUILD UP PERIOD						
1959-60	Dec. 17-Jan. 27	42	.15	107.4	-103.1	+ 4.1
ICE GROWTH PERIOD						
1959-60	Jan. 28-Feb. 25	29	.76	66.0	- 84.2	- 18.2
1958-59	Dec. 7-Feb. 8	64	.68	76.0	-109.0	- 33.0
EQUILIBRIUM PERIOD						
1959-60	Feb. 26-Mar. 24	28	.78	101.0	- 96.4	- 4.6
1958-59	Feb. 9-Mar. 27	48	.60	144.1	-106.7	+ 37.4
WASTAGE PERIOD						
1959-60	Mar. 25-Apr. 7	14	.29	233.1	- 78.1	+155.0
1958-59	Mar. 28-Apr. 13	17	.18	304.6	-101.1	+203.5
BREAKUP PERIOD						
1959-60	Apr. 8-Apr. 12	5	.09	476.0	-132.7	+343.3
1958-59	Apr. 14-Apr. 15	2	.10	554.3	-185.0	+369.3
INITIAL WARM UP						
1959-60	Apr. 13-20	8	.06	351.8	- 79.6	+272.2
1958-59	Apr. 16-22	7	.06	406.4	-103.5	+302.9

(1) Estimation method, see section II-B-1.

Appendix 4: Solar radiation absorption in ly/day for the years 1959-60 and 1958-59 based on the Bouguer-Lambert Law $R_s = R_o e^{-kz}$.

1959-60 Period	# Days	Snow Depth cm	Ice Thickness cm	k for snow cm^{-1}	k for ice cm^{-1}	R_{ss} (snow) ly/day	R_{si} (ice) ly/day	R_{sw} (water) ly/day
Dec. 17-Dec. 21	5	0	a	-	.03	0	6	147
Dec. 27-Jan. 4	14	0	b	-	.03	0	7	73
Jan. 5-Jan. 9	5	0	c	-	.05	0	37	102
Jan. 10-Jan. 27	18	6	d	.25	.05	65	12	28
Jan. 28-Feb. 18	22	7	29	.25	.05	56	9	3
Feb. 19-Feb. 25	7	10	38	.25	.05	55	4	1
Feb. 26-Mar. 3	7	14	42	.25	.05	91	3	-
Mar. 4-Mar. 10	7	17	44	.25	.05	87	1	-
Mar. 11-Mar. 17	7	19	46	.25	.05	99	1	-
Mar. 18-Mar. 24	7	18	46	.25	.05	121	1	-
Mar. 25-Mar. 31	7	5-8*	35-46*	.20	*	41	82	20
Apr. 1-Apr. 7	7	0	24-35*	-	*	0	244	79
Apr. 8-Apr. 12	5	0	0-21*	-		0	258	218

a) 50% of lake surface water, 50% of surface with ice with a mean of 3 cm. b) 50% water, 50% 6 cm. ice. c) 30% water, 70% 10 cm. ice. d) 20% water and 80% 20 cm. ice.

* special calculation was made for daily values of net solar radiation. (see section II-B-2)

1958-59

1958-59 Period	# Days	Snow Depth cm	Ice Thickness cm	k for snow cm^{-1}	k for ice cm^{-1}	R_{ss} (snow) ly/day	R_{si} (ice) ly/day	R_{sw} (water) ly/day
Dec. 7-Dec. 25	19	2	18	.25	.05	25	16	22
Dec. 26-Dec. 30	5	0	25	-	.05	0	187	64
Dec. 31-Jan. 17	18	5	33	.25	.05	45	12	7
Jan. 18-Feb. 8	22	12	50	.25	.05	56	2	1
Feb. 9-Mar. 21	41	20	56	.20	.05	118.5	2	0.5
Mar. 22-Mar. 27	7	5	64	.20	.05	152	115	20
Mar. 28-Apr. 6	10	0	48	-	.05	0	264	36
Apr. 7-Apr. 13	7	0	23	-	.05	0	311	86
Apr. 14-Apr. 15	2	0	0-15e	-	.05	0	53	501

e) ice going out on April 14; 20% ice cover on morning of April 15.

Appendix 5: Computation of conduction to the ice from the water (in ly/day).

1959-60 Period	No. of Days	Assumed ΔZcm	$\frac{\Delta T}{\Delta Z}$ °C/cm	Molecular Conduction	$\frac{\lambda eff}{\lambda mol} = 1.5$ ly/day	$\frac{\lambda eff}{\lambda mol} = 2.5$	Measured
Dec. 17-Dec. 21	5	10	.116	13.5			
Dec. 22-Jan. 4	14	10	.045	5.2			
Jan. 5-Jan. 9	5	10	.032	3.7			
Jan. 10-Jan. 27	18	10	.021	2.4			
Jan. 28-Feb. 18	22	20	.016	1.8	2.7		
Feb. 19-Feb. 25	7	20	.021	2.4	3.7		
Feb. 26-Mar. 3	7	20	.020	2.3	3.5		
Mar. 4-Mar. 10	7	20	.019	2.2	3.3		
Mar. 11-Mar. 17	7	20	.019	2.2	3.3		
Mar. 18-Mar. 24	7	20	.020	2.3	3.5		
Mar. 25-Mar. 31	7	10	.062	7.2	10.8	18.0	40.3*
Apr. 1-Apr. 7	7	10	.139	16.2	24.3	40.5	43.1
Apr. 8-Apr. 12	5	10	.181	21.1	31.6	52.7	
1958-59							
Dec. 7-Dec. 27	21	20	.035	4.1	6.1		
Dec. 28-Feb. 8	43	20	.030	3.5	5.2		
Feb. 9-Feb. 21	13	20	.025	2.9	4.4		
Feb. 22-Mar. 21	28	20	.022	2.6	3.9		
Mar. 22-Mar. 27	7	10	.065	7.6	11.4	18.9	
Mar. 28-Apr. 6	10	10	.270	31.4	47.1	78.6	
Apr. 7-Apr. 13	7	10	.457	53.2	79.8	133.0	

*High melt water runoff in this area for this period in measuring area.

Appendix 6: Estimation of conduction from the bottom of Lake Mendota to the water. Assumption is made that Birge and Juday's (1927) value of 650 ly per winter applies and that heat flux is proportional to the mean difference between the last water temperature and the mud temperature ($\overline{\Delta T}$).

Period	No. of Days	$\overline{\Delta T}$	Conduction (K_m) in ly/day	Total conduction (K_m) for period (ly)
1959-60				
Nov. 25-Dec. 16	22	.79	5.3	116
Dec. 17-Jan. 27	42	.74	4.9	207
Jan. 28-Feb. 25	29	.87	5.7	168
Feb. 26-Mar. 24	28	.70	4.7	131
Mar. 25-Apr. 7	14	.30	1.9	27
Apr. 8-Apr. 12	5	.19	negl.	1
Apr. 13-Apr. 20	8	.02	negl.	--
TOTAL	148			650
1958-59				
Dec. 7-Feb. 8	64	.94	6.5	415
Feb. 9-Mar. 27	48	.57	3.9	188
Mar. 28-Apr. 13	17	.38	2.6	44
Apr. 14-Apr. 15	2	.23	1.6	3
Apr. 16-Apr. 20	7	--	--	--
TOTAL	138			650

Appendix 7: Approximation of advection term (V) in Lake Mendota for the 1959-60 and 1958-59 winters. Inflow is assumed to be 75 per cent of out flow (see text section II D).

Period	No. of Days	Outflow in ly/day/°C	Outflow Temp. °C (mean of sounding at 1 m.)	Outflow in ly/day	Assumed Inflow* ly/day as % outflow	Inflow in springs ly/day	V in ly/day
1959-60							
Nov. 25-Dec. 16	22	0.93	1.85	-1.73	75%	+2.4	2.0
Dec. 17-Jan. 27	42	1.12	0.54	-0.60	75	+2.4	2.2
Jan. 28-Feb. 25	29	1.24	0.47	-0.58	75	+2.4	2.3
Feb. 26-Mar. 24	28	1.24	0.58	-0.72	75	+2.4	2.2
Mar. 25-Apr. 7	14	1.44	1.13	-1.63	75	+2.4	2.0
Apr. 8-Apr. 12	5	1.64	2.00	-3.28	100	+2.4	2.4
Apr. 13-Apr. 20	8	1.64	3.06	-5.02	100	+2.4	2.4
1958-59							
Dec. 7-Feb. 8	64	0.22	0.80	-0.19	100	+2.4	2.4
Feb. 9-Mar. 27	48	0.54	1.06	-0.57	75	+2.4	2.2
Mar. 28-Apr. 13	17	2.54	3.52	-8.94	100*	+2.4	2.4
Apr. 14-Apr. 15	2	2.98	2.75	-8.20	100	+2.4	2.4
Apr. 16-Apr. 22	7	2.98	4.08	-12.40	100	+2.4	2.4

* Inflow greater than outflow.