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A REVISED ESTIMATE OF
SMS SOUNDER PERFORMANCE

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1. INTRODUCTION

A temperature sounding of the atmosphere based on radiance measurements in the 15 μm CO_2 band can be made only if the clear column radiances over the spatial region of interest are determined within a standard error of 0.25 erg/(sec - ster - cm^2 - cm^{-1}). Instrumentation we have proposed for sounding from SMS can achieve this only by averaging a large number of individual measurements, each with a standard error much larger than that required for sounding. In evaluating the performance of this instrumentation we have been assuming that the standard error of the mean of N measurements, each of standard error σ, is just $\sigma/\sqrt{N}$. However, this assumption is not correct and leads to a considerable overestimate of the instrument performance. The actual amount of noise reduction through averaging depends on characteristics of the detector noise and on the frequency of measurements as well as the number of measurements.

The precise nature of this dependence and how it effects SMS sounder performance estimates is the subject of the following discussion.

2. DETECTOR NOISE CHARACTERISTICS

Let x(t) denote the time dependent signal output from a detector observing a time independent scene. Under these conditions the time dependent variations are due to detector noise fluctuations. This noise is usually described in terms of the noise power spectral density which is given by:

$$P(f) = \lim_{T\to\infty} \frac{2}{T} \left| \int_{-T/2}^{T/2} x(t) e^{-i2\pi ft} dt \right| , \qquad (1)$$

where f is a frequency. P(f)df represents the contribution to the signal variance from frequencies between f and f+df. The total variance is thus:

$$\sigma^2 = \int_0^\infty P(f)df = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt . \qquad (2)$$

Another useful function which characterizes the detector noise is the autocovariance function, defined by:

$$C(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)\, dt , \qquad (3)$$

where we have assumed a zero mean for x(t) in order to simplify notation. The autocovariance function can be related to the power spectrum through a cosine transform, i.e.

$$C(\tau) = \int_0^\infty P(f) \cos(2\pi f \tau)\, df . \qquad (4)$$

Equations (1) through (4) are the positive frequency versions of those given by Blackman and Tukey, 1958.

In addition to the general relations stated in equations (1) through (4), we must finally deal with the specific form of P(f) for the detectors to be used on SMS. The approximate form for P(f) in this case contains two components each arising from independent processes. One part is independent of frequency and arises from generation-recombination noise (g-r noise). The second component is proportional to 1/f and arises from a number of sources. These are usually lumped together in "1/f noise".

The frequency at which these two components are equal is called the crossover frequency, denoted by f_c. In practice, the detector signal will be filtered to eliminate all frequencies below f_{min} and above f_{max}. The effective noise power spectrum is thus given by:

$$P(f) = \sigma^2 \frac{1 + f_c/f}{f_{max} - f_{min} + f_c \log \frac{f_{max}}{f_{min}}} , \quad f_{min} \leq f \leq f_{max} \qquad (5)$$

$$P(f) = 0 \quad , \quad f < f_{min} \quad \text{or} \quad f > f_{max} \quad ,$$

where, as before, σ^2 is the total signal variance. We are now in a position to calculate the expected noise in the mean of a particular set of measurements.

3. SINGLE SCAN NOISE REDUCTION

The expected variance of the mean of a sample of size N (containing N measurements) is defined as:

$$\sigma_M^2 = E[(M-\mu)^2] = E(M^2) - \mu^2 \quad , \qquad (6)$$

where E denotes expectation value and μ is the population mean, which is assumed to be zero for simplicity. The sample mean ~~N~~ M is just:

$$M = \frac{1}{N} \sum_{i=1}^{N} X(t_i) \quad , \qquad (7)$$

where $X(t_i)$ is the detector signal at time t_i. Substituting (7) in (6) we find:

$$\sigma_M^2 = E\left[\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} X(t_i) X(t_j)\right] . \qquad (8)$$

Since the expectation value of a sum of random variables is equal to the sum of the expectation values of the random variables (even if they are not independent), equation (8) reduces to

$$\sigma_M^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} E[x(t_i)x(t_j)] \quad . \tag{9}$$

For $i = j$ the expectation of the product is just the population variance, i.e.

$$E(X(t_i)^2) = \sigma^2 \quad , \tag{10}$$

Thus equation (9) can be rewritten as

$$\sigma_M^2 = \frac{1}{N}\sigma^2 + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j\neq i} E(x(t_i)x(t_j)) \quad . \tag{11}$$

If $X(t_i)$ and $X(t_j)$ were independent random variables then we would have $E(X(t_i) \cdot X(t_j))_{i\neq j} = E(X(t_i)) \cdot E(X(t_j)) = \mu^2 = 0$, and the result $\sigma_M = \sigma/\sqrt{N}$ would follow. Since $X(t_i)$ and $X(t_j)$ are not independent a considerably different result obtains. Recognizing that, for measurements made on the same scan line,

$$E(X(t_i) \cdot X(t_j)) = C(t_j - t_i) \quad , \tag{12}$$

we may use equation (4) in (11) to display the dependence on the power spectrum, i.e.

$$\sigma_M^2 = \frac{1}{N}\sigma^2 + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j\neq i}\int_0^\infty P(f)\cos(2\pi f(t_j - t_i))\,df \quad . \tag{13}$$

We may further simplify these results by specifying the measurement intervals $t_j - t_i$. If we assume that the N measurements are equally spaced over the time interval T, then there are N-1 different time lags

$$\tau_k = \frac{(k-1)}{N-1}T \quad , \quad k = 2,3,\ldots N. \tag{14}$$

The number of times a given time lag τ_k appears in the double sum over i and j is given by:

$$M(\tau_k) = 2(N + 1 - k) \quad , \quad k = 2, 3, \ldots N \quad . \tag{15}$$

Note that:

$$\sum_{k=2}^{N} M(\tau_k) = \sum_{k=2}^{N} 2(N+1-k) = N(N-1) \; , \tag{16}$$

as expected.

Employing (14) and (15) in equation (13) we find

$$\sigma_M^2 = \frac{\sigma^2}{N} + \frac{2}{N^2} \sum_{k=2}^{N} (N+1-k) \int_0^{\infty} P(f) \cos(2\pi f \tau_k) df . \tag{17}$$

Inserting the specific form for P(f) stated in equation (5), (17) becomes

$$\sigma_M^2 = \frac{\sigma^2}{N} \Bigg\{ 1 + \frac{2/N}{f_{max} - f_{min} + f_c \ln f_{max}/f_{min}} \sum_{k=2}^{N} (N+1-k) \int_{f_{min}}^{f_{max}} \left(1 + \frac{f_c}{f}\right) \cos(2\pi f \tau_k) df \Bigg\} \tag{18}$$

where τ_k is given by equation (14).

In order to display the significance of this result for the sounding performance of SMS let us consider a specific, and highly relevant, example. A detector of .4 mr IGFOV scanning across the earth at 100 satellite revolutions per minute will scan across a GARP grid square ($\sim$12 mr on a side) in a time $T \approx 1.2 \times 10^{-3}$ sec. There are 30 adjacent IFOV's in one GARP grid in one scan. The frequencies f_{max} and f_{min} are chosen to be 12,500 Hz and .1 Hz respectively. The crossover frequency f_c is 2,000 Hz. We may now determine the expected variance of the mean of these 30 measurements taken in one scan (assuming a uniform scene). Evaluation of (18) yields $\sigma_M^2 = .46\sigma^2$ (actual SMS example), whereas our previous assumptions (incorrect) predicted the result $\sigma_M^2 = .033\sigma^2$ (random independent measurements). The ratio of the actual one scan error to that estimated on the basis of random independent measurements is thus $(.46/.0333)^{\frac{1}{2}} = 13.8^{\frac{1}{2}} = 3.7$. Thus the actual noise reduction in the mean is only about one fourth of what we had previously assumed.

In most practical situations not all 30 measurements would be useable, because of cloud contamination. The variance as a function of the actual number of samples used in the mean is shown in figure 1, together with results for somewhat different choices for f_{min} and T. It is interesting to note that we may crudely approximate the expected variance of the mean as

$$\sigma_M^2(N) \approx \sigma^2 \left(a + \frac{b}{N}\right) , \tag{19}$$

where $a \approx .44$ and $b \approx .56$ for the example we have just discussed. Roughly speaking a represents the fractional contribution to the variance from frequencies below the GARP grid spatial frequency, and b the contribution from frequencies higher than the GARP grid spatial frequency. (Note that $a + b = 1$.) Obviously, the N samples taken within a GARP grid on one scan are only effective in averaging variations occurring within this region. Since it is only the variance due to frequencies greater than the GARP grid spatial frequency which can be reduced in one scan, the form displayed in equation (19) is to be expected.

4. MULTIPLE SCAN NOISE REDUCTION

The rotational frequency of SMS is 100 rpm or 1.67 Hz, and the corresponding period is T = .6 sec. For f_{min} of the order of 0.1 Hz there will be some correlation between measurements in the same GARP grid even

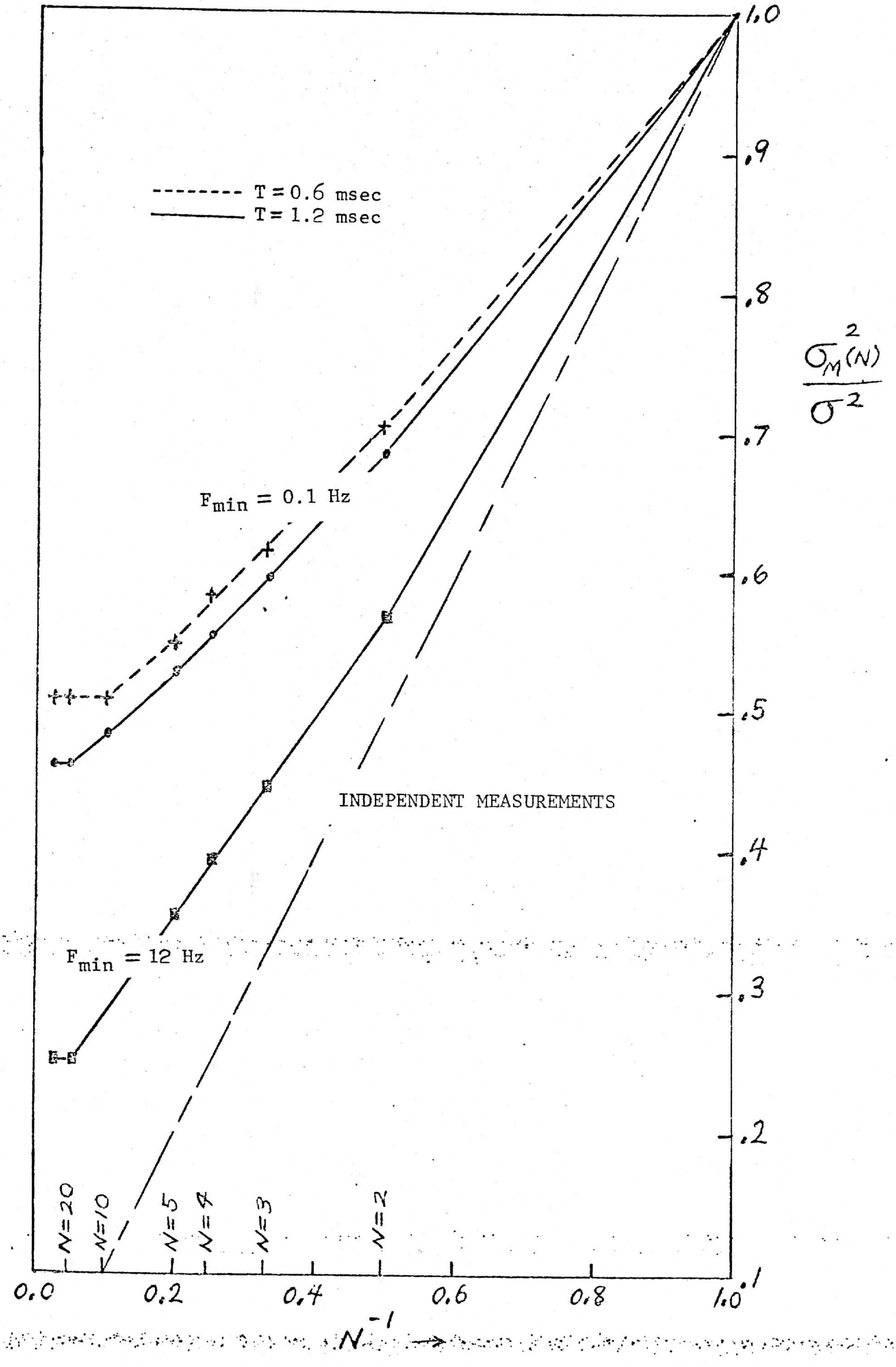

Figure 1. Fractional reduction of variance through averaging.

on different scan lines. This cross scan correllation should be a small effect generally, and completely negligible if pulsed biasing is used on the detectors. Thus we shall assume that the means over GARP grid lines are random independent variables although, as we have seen, the individual measurements on each line are not. Under these assumptions we find that the expected variance of the mean of N_L lines of data is given by

$$\sigma_M^2 = \frac{1}{N_L} \sigma^2 F(N) , \tag{20}$$

where F(N) is the fractional reduction of variance per scan line when N sample measurements are averaged for each line. The function F(N) which is determined by equation (18) implicitly depends on all the additional parameters displayed in that equation. Note that F(N) takes the value .46 at N = 30 for the specific example discussed in the previous section.

The results of this section and of the previous one show that the effect of correllation in the detector noise is large enough by itself to necessitate sounding for 13.8 times as long to achieve the same clear conditions performance that we claimed in the preliminary SMS report. However, a somewhat compensating error was made in the modulation transfer function. Changing W_α from .55 to its correct value of 1.0 amounts to factor of 3.3 reduction in the required sounding time. The net result of correcting both of these errors is a requirement for a factor of 4.2 increase in sounding time, for clear conditions. The required relative increase in sounding time under cloudy conditions is not this great for reasons which will be discussed in a later section.

5. DETECTOR NOISE SIMULATION

In order to simulate the SMS sounding performance for varying meteorological conditions and to evaluate the effectiveness of different data handling algorithms it is necessary to have available a simulated detector signal $s(t)$ such that

$$s(t) = r + x(t) \tag{21}$$

where r is the true signal and $x(t)$ is the time dependent error which satisfies equation (1) and behaves as if generated by a stationary Gaussian random process. According to Blackman and Tukey (1958), a stationary Gaussian random process may be regarded as the result of passing white Gaussian noise through a filter with a prescribed transmission function, while white Gaussian noise may be regarded as the superposition of the outputs of a set of simple harmonic oscillators with (a) a continuous distribution in frequency, (b) uniform amplitudes, and (c) independent and random phases.

We shall approximate white Gaussian noise by a finite sum (instead of a continuous integral) over the frequency range of the filter, i.e.

$$w(t) = \sum_{n=1}^{N} a_n \cos(2\pi f_n t + \phi_n) \quad , \tag{22}$$

where ϕ_n is a random phase, $f_1 \leq f_{min}$ and $f_n \geq f_{max}$ (f_{min} and f_{max} are the bounds of the frequency range of the transmission system), N is a number large enough such that MAX $(f_n - f_{n-1})$ is significantly smaller than the sampling frequencies which will eventually be used (we assume $f_{n-1} < f_n$ for all $n > 2$), and the amplitude A_n is proportional to the square root of the bandwidth $\frac{1}{2}(f_{n+1} - f_{n-1})$ which is appropriate to the nth frequency. If we choose $a_n = \left(\frac{f_{n+1} - f_{n-1}}{f_n - f_1}\right)^{1/2}$ the variance of $w(t)$ is determined to be unity, and the fractional contribution to the variance from frequencies between f_ℓ and f_n is just $(f_\ell - f_n)/(f_n - f_1)$.

Passing the white noise $w(t)$ through the appropriate transmission system (which we will subsequently determine) we may regard only the amplitudes

a_n as affected. Thus we may express x(t) as

$$x(t) = \sum_{n=1}^{N} b_n \cos(2\pi f_n t + \phi_n) , \tag{23}$$

where, if T(f) is the transmission of the filter at frequency f, we have

$$b_n = T(f_n)a_n = T(f_n)\left[\frac{f_{n+1} - f_{n-1}}{f_N - f_1}\right]^{1/2} . \tag{24}$$

The transmission function, or b_n, is determined from the power spectrum P(f) through equation (1). According to equation (1) the fractional contribution to the signal variance from frequencies between $f_a = f_n - \frac{1}{2}(f_{n+1} - f_{n-1})$ and $f_b = f_n + \frac{1}{2}(f_{n+1} - f_n)$ is given by

$$F.\ VAR. = \int_{f_a}^{f_b} P(f)df \Big/ \int_0^{\infty} P(f)df , \tag{25}$$

whereas, according to (23) the fractional variance is just

$$F.\ VAR. = b_n^2 \Big/ \sum_{n=1}^{N} b_n^2 \tag{26}$$

Equating these two expressions we have

$$b_n^2 = \frac{\sum_{n=1}^{N} b_n^2}{\int_0^{\infty} P(f)df} \cdot \int_{f_a}^{f_b} P(f)df \tag{27}$$

or

$$b_n^2 \approx \left[\frac{\sum_{n=1}^{N} b_n^2}{\int_0^{\infty} P(f)df}\right] \frac{1}{2} P(f_n)\left[f_{n+1} - f_{n-1}\right] . \tag{28}$$

Requiring the total variance to satisfy

$$\sigma^2 = \sum_{n=1}^{N} \frac{1}{2} b_n^2 = \int_0^{\infty} P(f)df , \tag{29}$$

we find the following expression for b_n, i.e.

$$b_n = \left[P(f_n)\left\{f_{n+1} - f_{n-1}\right\}\right]^{\frac{1}{2}} \tag{30}$$

This result can be used with equation (24) to determine the transmission function, although we will have no further use for it in this discussion.

Equations (30) and (23) form the basis for the numerical simulation of x(t). In practice approximately 300 frequencies with variable logarithmic spacing between f_{min} and f_{max} are chosen randomly. Once these are chosen the amplitudes b_a are calculated from equation (30). At the beginning of each scan line the phases $\emptyset_n$ are chosen randomly and the simulated signal x(t) is calculated by summing the terms in (23) at each time desired in that scan.

The simulated signal was used to determine the one scan variance reduction for the same conditions specified in the example discussed in section 3. The mean reduction of variance per scan line was found, from the simulation results, to be

$$\sigma_M^2/\sigma^2 = 0.451 \pm 0.013 . \tag{31}$$

This compares favorably with the expectation value of .46 determined in section 3.

6. Compensating for Very Low Frequency Noise

With an SMS rotation rate of 100 RPM the spatial frequency of the earth's disc is 33 Hz, while the electronic bandpass starts at f_{min} = .1 Hz. Thus there is a considerable amount of noise variance σ_{VL}^2, where

$$\sigma_{VL}^2 = \sigma^2 \frac{\int_{.1}^{33} \left(1+\frac{f_c}{f}\right) df}{f_{max} - f_{min} + f_c \ln \frac{f_{max}}{f_{min}}} , \tag{32}$$

which arises from a component of x(t) which is roughly constant over the earth's disc. For the example situation discussed in sections 3,4, and 5, equation (32) takes the specific form

$$\sigma_{VL}^2 = \sigma^2 \frac{11,633}{36,554} \approx .32\,\sigma^2, \tag{33}$$

which shows that the very low frequency variance is almost one third of the total. More important it is approximately two thirds of the low frequency variance which cannot be reduced by measurements made on a single scan of a GARP grid.

Because of the substantial benefits possible in reducing the effective contribution from σ_{VL}^2 to the total variance we have examined the possibility of using, for each scan, the average signals x(t) measured just before the earth comes into view and just after the earth leaves the field of view to subtract out the very low frequency components from x(t) measured within a GARP grid. The compensating equation which we tested is

$$x(t)_{CORR} = x(t) - \left[x_1 + \frac{t - t_1}{t_2 - t_1}(x_2 - x_1)\right], \tag{34}$$

where $x(t)_{CORR}$ denotes the corrected signal, and x_1 and x_2 are averages taken before and after the earth scan at times t_1 and t_2. More precisely, we have

$$x_1 = \frac{1}{\Delta}\int_{t_1-\frac{\Delta}{2}}^{t_1+\frac{\Delta}{2}} x(t)\,dt, \tag{35}$$

$$x_2 = \frac{1}{\Delta}\int_{t_2-\frac{\Delta}{2}}^{t_2+\Delta/2} x(t)\,dt,$$

$$t_2 - t_1 = T_E + \Delta,$$

where T_E is the time taken to scan the earth's disc and Δ is the time interval over which the averages are taken.

The compensation equation was tested with a simulated signal x(t) as described in section 5. The results are presented in the following table:

Δ (sec)	No of IFOV's in Time Δ	$(\sigma_M^2/\sigma^2)_{CORR}$
.0012	30	.65 $\pm$.02
.0048	120	.34 $\pm$.02
.015	375	.29
.060	1500	.28

The improvement in noise reduction at Δ of .015sec or .060sec is comparable to cutting off the power spectrum at $f_{min} \approx 12$ Hz. Comparing the best compensated results with the random independent results (see section 3) we find that the ratio of standard errors for our standard example is $(.28/.0333)^{\frac{1}{2}} = 8.4^{\frac{1}{2}} = 2.7$, while the uncompensated error ratio was found to be 3.7. With compensation it is necessary to sound for only 60% of the time required without compensation, a considerable improvement.

7. NOISE REDUCTION UNDER CLOUDY CONDITIONS

In this section we shall compare the performance of a detecting system with random uncorrellated noise, for which

$$\sigma_{MU}^2 = \frac{1}{N_{SC}\,N_{SA/SC}}\,\sigma^2 , \qquad (36)$$

with the performance of the SMS detection system with correllated noise, for which the variance of the mean is approximately given by

$$\sigma_{MC}^2 = \frac{1}{N_{SC}}\left(X + \frac{1-X}{N_{SA/SC}}\right)\sigma^2 . \qquad (37)$$

In equations (36) and (37) N_{SC} is the number scans over which means are taken, $N_{SA/SC}$ is the number of samples obtained per scan, and X is roughly equivalent to the fraction of variance due to low frequency noise.

For a square grid of data consisting of N scan lines, a maximum of N clear samples per scan line, and a fractional cloud cover C, equations (36) and (37) take the forms

$$\sigma_{MU}^{2}(C) = \frac{1}{(1-C)N^{2}}\sigma^{2} \qquad (38)$$

$$\sigma_{MC}^{2}(C) = \frac{1}{N}\left(X + \frac{1-X}{(1-C)N}\right)\sigma^{2}, \qquad (39)$$

where we have assumed the relations

$$N_{SC} \approx N \qquad (40)$$

$$N_{SA/SC} \approx (1-C)N \qquad (41)$$

We shall next justify this approximation.

If cloudy FOV's are randomly distributed in the square matrix of N^2 elements, then the probability that a particular FOV is cloudy is just C, since the mean cloudiness is just $N^2C/N^2 = C$. The mean number of samples per scan line which are cloudy is NC/N which also reduces to C. Thus, the number of clear samples per scan is given by (41). The probability that a given scan line is cloudy (i.e. completely cloudy) is C^N. Thus the fractional number of scans which are at least partially clear is given by

$$N_{SC} = N(1-C^{N}) \qquad (42)$$

which reduces to (40) for $C^N \ll 1$. If one takes a somewhat different point of view and requires that C is not the mean cloud cover over a statistically large number of grids, but in fact is the precise fraction of cloud cover for each grid in the set, then the probability that K lines out of N are completely cloudy is different from the Kth power of the probability that one line is completely cloudy, i.e.

$$\text{Prob}\left(\begin{smallmatrix}\text{k lines}\\ \text{completely}\\ \text{cloudy}\end{smallmatrix}\right) = \begin{cases} \prod_{i=0}^{KN} \frac{(CN^2-i)}{(N^2-i)} & \forall\ CN^2 \geq KN \\ 0 & \forall\ CN^2 \leq KN \end{cases} \tag{43}$$

Equation (43) also leads to (40) for N = 30 and C ≤ .9. Even for C = .9 we find $N_{SC} \approx .95$ N. Although clouds are not distributed randomly, the approximations we have developed are probably sufficient for making crude performance estimates of the SMS system in cloudy regions.

Dividing equation (39) by equation (38) we find

$$\frac{\sigma_{MC}^2(C)}{\sigma_{MU}^2(C)} \approx NX(1-C) + (1-X), \tag{44}$$

which displays the ratio of variance reduction with correllated noise to variance reduction with uncorrellated noise when taking the mean over a grid of N^2 samples with a fraction C of them cloud contaminated. This equation is useful in estimating the increase in sounding time (as a function of C) which is required to make up for the erroneous assumption of uncorrellated noise which was made in the preliminary SMS sounder report. For example, for x ≈ .46 and N = 30 we find the following sounding time increase factors:

C	$\sigma_{MC}^2/\sigma_{MU}^2$	NET INCREASE	NET INCREASE WITH VLF C
0.0	13.8	4.2	2.7
0.25	10.5	3.2	2.0
0.50	7.2	2.2	1.4
0.75	3.8	1.2	.76
0.85	2.6	0.8	.50
0.90	1.8	0.6	.38

The net increase includes the effect of correcting the modulation transfer function and the net increase with VLFC includes the additional benefits of Very Low Frequency Noise Compensation as discussed in section 6.

An improved calculation of the effect of cloudiness, based on the results of section 3, is discussed in the appendix.

8. CONCLUSIONS

A revised estimate of SMS sounder performance with instrumentation proposed in the preliminary report was made on the basis of three significant changes in analysis:

1) a corrected estimate of noise reduction by statistical averaging

2) a change in W_α from .55 to 1.0

3) inclusion of VLF noise compensation

The revised estimate, which is currently approximate, shows that:

1) under clear conditions previously estimated performance can be duplicated with an increase in sounding time by a factor of 2.7.

2) under moderately cloudy conditions (∼ 60% cloudy) no increase in time is required.

3) under very cloudy conditions (∼ 90% cloudy) sounding time may be reduced by a factor of 2.

For most channels adequate accuracy can be obtained within one hour, while the Q-branch will require somewhat less than three hours. Since these time intervals are still reasonably short the possible benefits of introducing chopping are probably not worth the accompanying complexities.

It should be noted that the performance estimates made here are based on clear looks only and do not include the effects of the changed conditions on cloud discrimination algorithms. This is being investigated and will be reported on as results become available.

Please note the results of improved calculations, which are discussed in the appendix.

REFERENCES

Blackman, R.B. and J.W. Tukey, 1958, "The Measurement of Power Spectra from the Point of View of Communications Engineering"- Part I, The Bell System Technical Journal, January, pp. 185-282.

Appendix: NOISE EXPECTATION PROGRAM

In order to evaluate the effect of resolution, cloud cover, and low frequency cutoff on the statistical noise reduction performance of the SMS sensing system, we have created a relatively simple program which incorporates the results of sections 3, 4, and 7, specifically equations (13) and (20). The program evaluates performance in two ways: first, the effective noise equivalent radiance (NER) obtained from a complete set of non-overlapping scans of one GARP grid is determined; second, the number of additional detectors required to improve the performance to an effective NER of .25 erg/(cm^2 - sec - cm^{-1} - sterad) is determined (as well as the resulting mean on-time dissipation).

The input parameters of this program are:

FMIN = low frequency cutoff (KHz)

RC = fractional cloud cover

The output quantities and the equations from which they are calculated are:

ALPHA = length (in milliradians) of a side of a square IFOV

NER (1 sample) = $6.839\ (\mathrm{DFN}/50.94)^{\frac{1}{2}}\ (.2/\mathrm{ALPHA})$

The constants here have been adjusted so that this result agrees with the 1 sample NER determined in our preliminary report after changing the modulation transfer to 1.0 from 0.55. The noise equivalent bandwidth (DFN) in KHz is determined from

$$\mathrm{DFN} = f_{min} - f_{max} + f_c \ln \frac{f_{max}}{f_{min}} ,$$

where f_{max} (KHz) = 5.2/ALPHA.

SAMPLE/LINE = 12.0 * (1-RC)/ALPHA

For convenience we have assumed that the angular dimensions of a 400km x 400km GARP grid are 12.0 mr x 12.0 mr. The actual dimensions are closer to 11 mr x 11 mr at the subsatellite point.

F, 1 LINE (1 detector) = σ_M^2/σ^2, the fractional reduction of variance per scan line. This is determined from equation (13), section 3.

NO OF LINES = 12.0/ALPHA

F (N Lines) = $\frac{1}{N}$ F(1 line)

EFF NER = NER (1 Sample) $*(\sigma_M^2/N\sigma^2)^{\frac{1}{2}}$,

the effective noise equivalent radiance of the mean of measurements made on N lines at the rate of (1-RC)*N per line. Units are ergs/(cm^2 - sec - cm^{-1} - sterad).

DTCTRS/LINE = $(\text{EFF NER}/.25)^2$

LINES OF DETECTORS = 2.4/ALPHA, the number of lines which must be scanned simultaneously in order to permit stepping 2.4 mr each rotation.

DISSIPATION = .5 $(\text{ALPHA}/.2)^2$ * (Lines of Detectors) * (DTCTRS/LINE)

This is the on-time dissipation in milliwatts for a detector array which steps 2.4 mr each rotation and achieves through averaging clear columns, the effective NER required for sounding in one frame. This does not include any form of dissipation except I^2R losses in the detectors.

The results displayed in the following two tables show that instrument performance, judged on the basis of dissipation, is almost independent of resolution when lead conduction and cloud discrimination are not included. Thus, the tradeoff in resolution size essentially amounts to only a balance between lead conduction and cloud discrimination capabilities, not between the 1-sample NER and the number of clear FOV's. This suggests that a new search for the optimum sounding resolution should be made at once.

These results also demonstrate the weak dependence of performance on cloudiness (approximately determined in section 7) and the substatial improvement in performance obtained by increasing the lower cutoff frequency (as discussed in section 6).

FMIN= .00010 KHZ, FC= 2.00 KHZ, FRACTIONAL CLOUD COVER= .00
F=SIG M**2/SIG**2

ALPHA	NER 1 SAMPL	SAMPLS /LINE	F,1 LINE 1DETCTR	NO.OF LINES	F N LINES	EFF NER	DETCTRS /LINE	LINES OF DETCTRS	DISSI- PATION
.2	6.839	60	.326	60	.005	.504	4.07	12.00	24.39
.3	4.113	40	.400	40	.010	.411	2.71	8.00	24.35
.4	2.897	30	.453	30	.015	.356	2.03	6.00	24.33
.5	2.219	24	.494	24	.021	.318	1.62	4.80	24.31
.6	1.790	20	.526	20	.026	.290	1.35	4.00	24.28
.8	1.283	15	.575	15	.038	.251	1.01	3.00	24.22
1.0	.994	12	.611	12	.051	.224	.81	2.40	24.16
1.2	.809	10	.639	10	.064	.205	.67	2.00	24.11
1.5	.631	8	.671	8	.084	.183	.53	1.60	24.02
2.0	.459	6	.709	6	.118	.158	.40	1.20	23.88
2.4	.376	5	.731	5	.146	.144	.33	1.00	23.76

FRACTIONAL CLOUD COVER= .50

.2	6.839	30	.342	60	.006	.516	4.26	12.00	25.58
.3	4.113	20	.420	40	.010	.421	2.84	8.00	25.55
.4	2.897	15	.475	30	.016	.365	2.13	6.00	25.52
.5	2.219	12	.518	24	.022	.326	1.70	4.80	25.49
.6	1.790	10	.552	20	.028	.297	1.41	4.00	25.46
.8	1.283	7	.603	15	.040	.257	1.06	3.00	25.39
1.0	.994	6	.641	12	.053	.230	.84	2.40	25.33
1.2	.809	5	.670	10	.067	.209	.70	2.00	25.28
1.5	.631	4	.703	8	.088	.187	.56	1.60	25.19
2.0	.459	3	.743	6	.124	.162	.42	1.20	25.04
2.4	.376	2	.796	5	.159	.150	.36	1.00	25.87

FRACTIONAL CLOUD COVER= .75

.2	6.839	15	.358	60	.006	.528	4.46	12.00	26.77
.3	4.113	10	.439	40	.011	.431	2.97	8.00	26.73
.4	2.897	7	.497	30	.017	.373	2.22	6.00	26.68
.5	2.219	6	.541	24	.023	.333	1.78	4.80	26.64
.6	1.790	5	.576	20	.029	.304	1.48	4.00	26.59
.8	1.283	3	.658	15	.044	.269	1.15	3.00	27.71
1.0	.994	3	.689	12	.057	.238	.91	2.40	27.26
1.2	.809	2	.761	10	.076	.223	.80	2.00	28.72
1.5	.631	2	.775	8	.097	.196	.62	1.60	27.75

FMIN= .01200 KHZ, FC= 2.00 KHZ, FRACTIONAL CLOUD COVER= .00
F=SIG M**2/SIG**2

ALPHA	NER 1 SAMPL	SAMPLS /LINE	F,1 LINE 1DETCTR	NO.OF LINES	F N LINES	EFF NER	DETCTRS /LINE	LINES OF DETCTRS	DISSI- PATIO
.2	6.162	60	.170	60	.003	.328	1.72	12.00	10.31
.3	3.607	40	.220	40	.005	.267	1.14	8.00	10.28
.4	2.488	30	.259	30	.009	.231	.85	6.00	10.25
.5	1.875	24	.291	24	.012	.206	.68	4.80	10.22
.6	1.492	20	.318	20	.016	.188	.57	4.00	10.19
.8	1.046	15	.362	15	.024	.162	.42	3.00	10.13
1.0	.798	12	.396	12	.033	.145	.34	2.40	10.08
1.2	.641	10	.424	10	.042	.132	.28	2.00	10.02
1.5	.491	8	.457	8	.057	.117	.22	1.60	9.94
2.0	.350	6	.500	6	.083	.101	.16	1.20	9.80
2.4	.283	5	.525	5	.105	.092	.13	1.00	9.67

FRACTIONAL CLOUD COVER= .50

.2	6.162	30	.189	60	.003	.346	1.92	12.00	11.50
.3	3.607	20	.245	40	.006	.282	1.27	8.00	11.47
.4	2.488	15	.289	30	.010	.244	.95	6.00	11.44
.5	1.875	12	.325	24	.014	.218	.76	4.80	11.41
.6	1.492	10	.355	20	.018	.199	.63	4.00	11.37
.8	1.046	7	.404	15	.027	.172	.47	3.00	11.31
1.0	.798	6	.442	12	.037	.153	.38	2.40	11.25
1.2	.641	5	.473	10	.047	.139	.31	2.00	11.19
1.5	.491	4	.511	8	.064	.124	.25	1.60	11.10
2.0	.350	3	.559	6	.093	.107	.18	1.20	10.96
2.4	.283	2	.639	5	.128	.101	.16	1.00	11.79

FRACTIONAL CLOUD COVER= .75

.2	6.162	15	.209	60	.003	.364	2.11	12.00	12.68
.3	3.607	10	.270	40	.007	.296	1.41	8.00	12.65
.4	2.488	7	.318	30	.011	.256	1.05	6.00	12.60
.5	1.875	6	.357	24	.015	.229	.84	4.80	12.56
.6	1.492	5	.390	20	.020	.208	.78	4.00	12.51
.8	1.046	3	.486	15	.032	.188	.57	3.00	13.63
1.0	.798	3	.518	12	.043	.166	.44	2.40	13.18
1.2	.641	2	.619	10	.062	.159	.41	2.00	14.64
1.5	.491	2	.629	8	.079	.138	.38	1.60	13.67