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1. A study of the correlation between cloud motion and wind field has been initiated. Cloud heights and displacements are being obtained from a ceilometer and movie pictures, while winds are being measured from pilot balloon observations on a near-simultaneous basis.
2. Cloud motion vectors obtained from ATS-III time-lapse cloud pictures, using the WINDCO program, are being processed for 27, 28 July, 1969, in the Atlantic. The purpose is to investigate the relationship between observed features of cloud clusters (e.g., growth, intensification, decay, etc.) and the ambient wind field derived from cloud trajectories on a wide range of space and time scales.

TECHNICAL STATUS REPORT ON

THE POINTING ERRORS OF GEOSYNCHRONOUS SATELLITES

by Aniruddha Das

This report presents the first phase of the work completed on the pointing error analysis. This phase consisted of chosing a configuration of the spacecraft and obtaining a mathematical model of the dynamic problem. The assumptions made and the derivation of the guiding equations are briefly described in the following pages.

The simulation includes elastic shells, plates, beams, rigid bodies and point masses. The effects of thermal stresses, large angular velocities and the effect of the motion of the centre of mass due to vehicle deformation, are included in the analysis. In this formulation, the spatial dependences are maintained linear. But the time-dependences are nonlinear. It is in these respects that this formulation claims to be more exact than any previous one.

The current phase of the research is centered on the solution of the homogeneous part of the problem. The governing equations are a complex set of coupled integro-differential equations. Attempts are now being made to obtain the uncoupled eigenfunction expansions for each of the variables.

Contents

ASSUMED CONFIGURATION OF THE FLEXIBLE SATELLITE

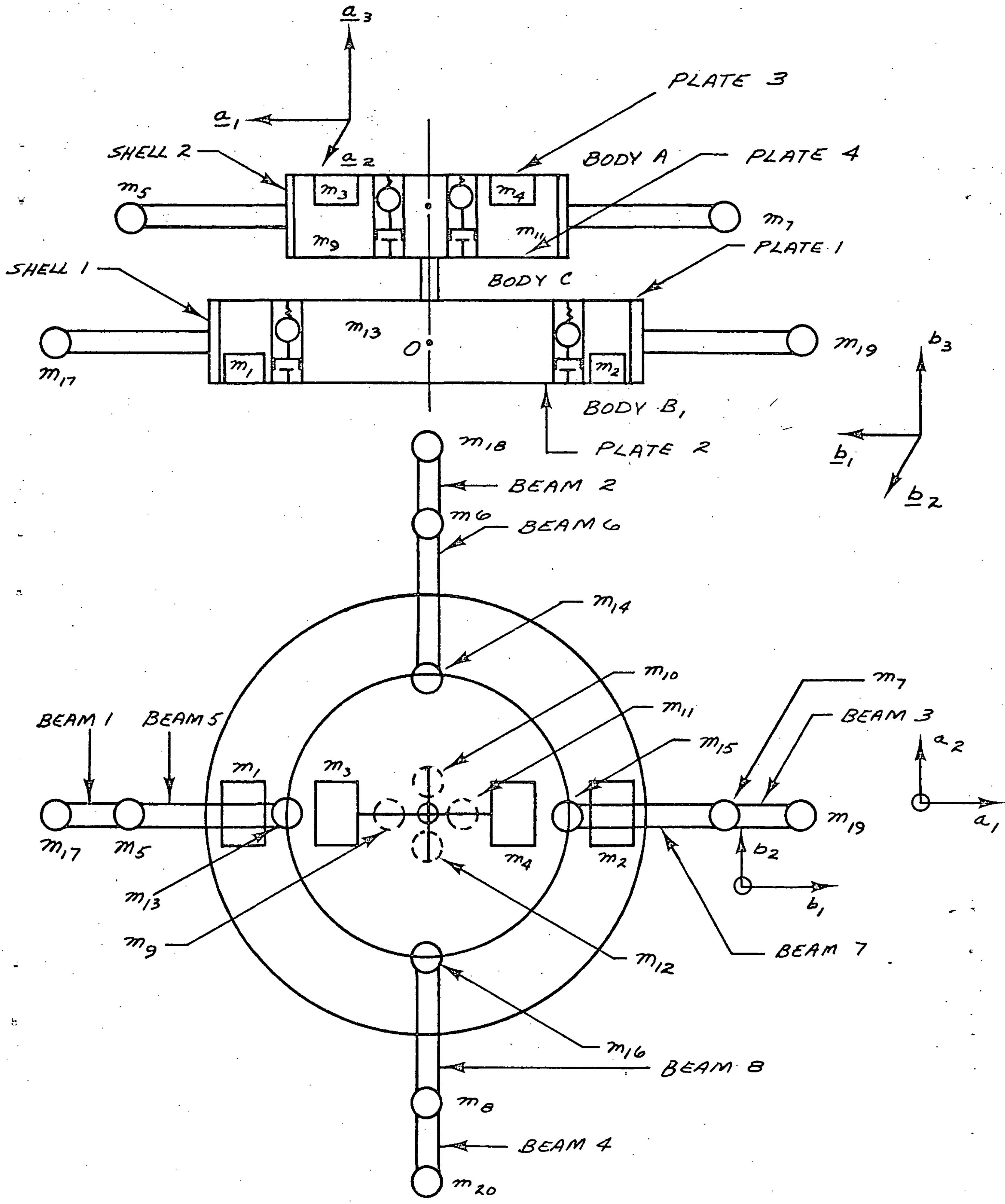

FIGURE 1

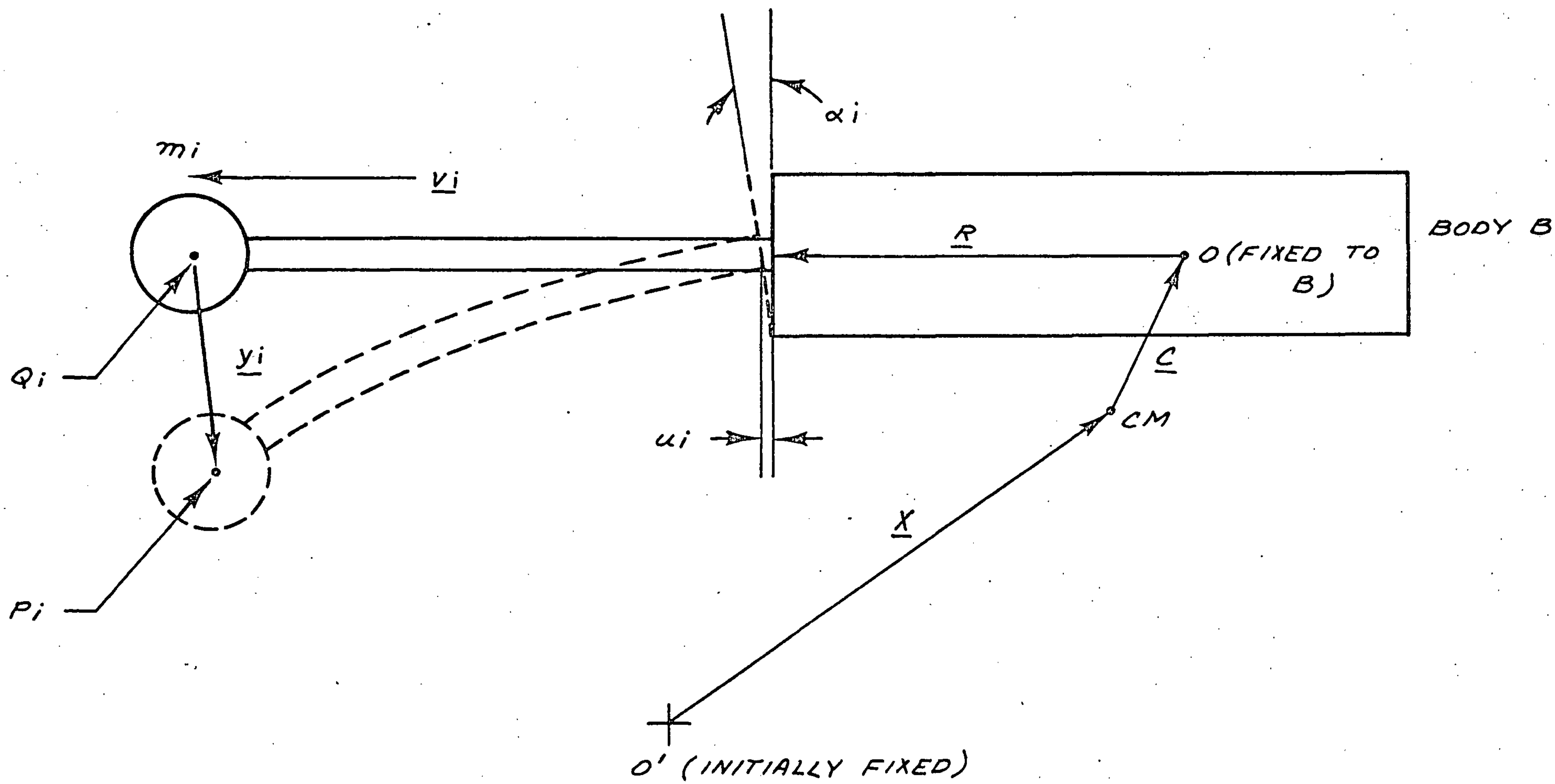

FIGURE 2

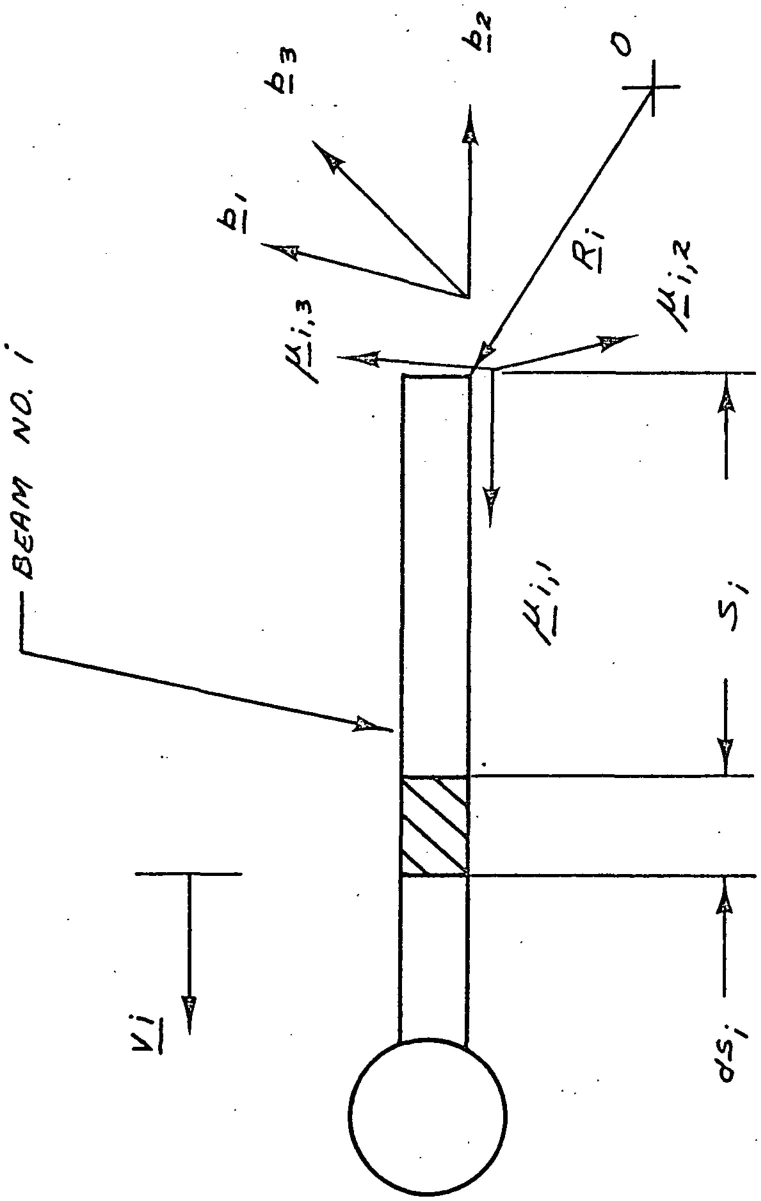

FIGURE 3

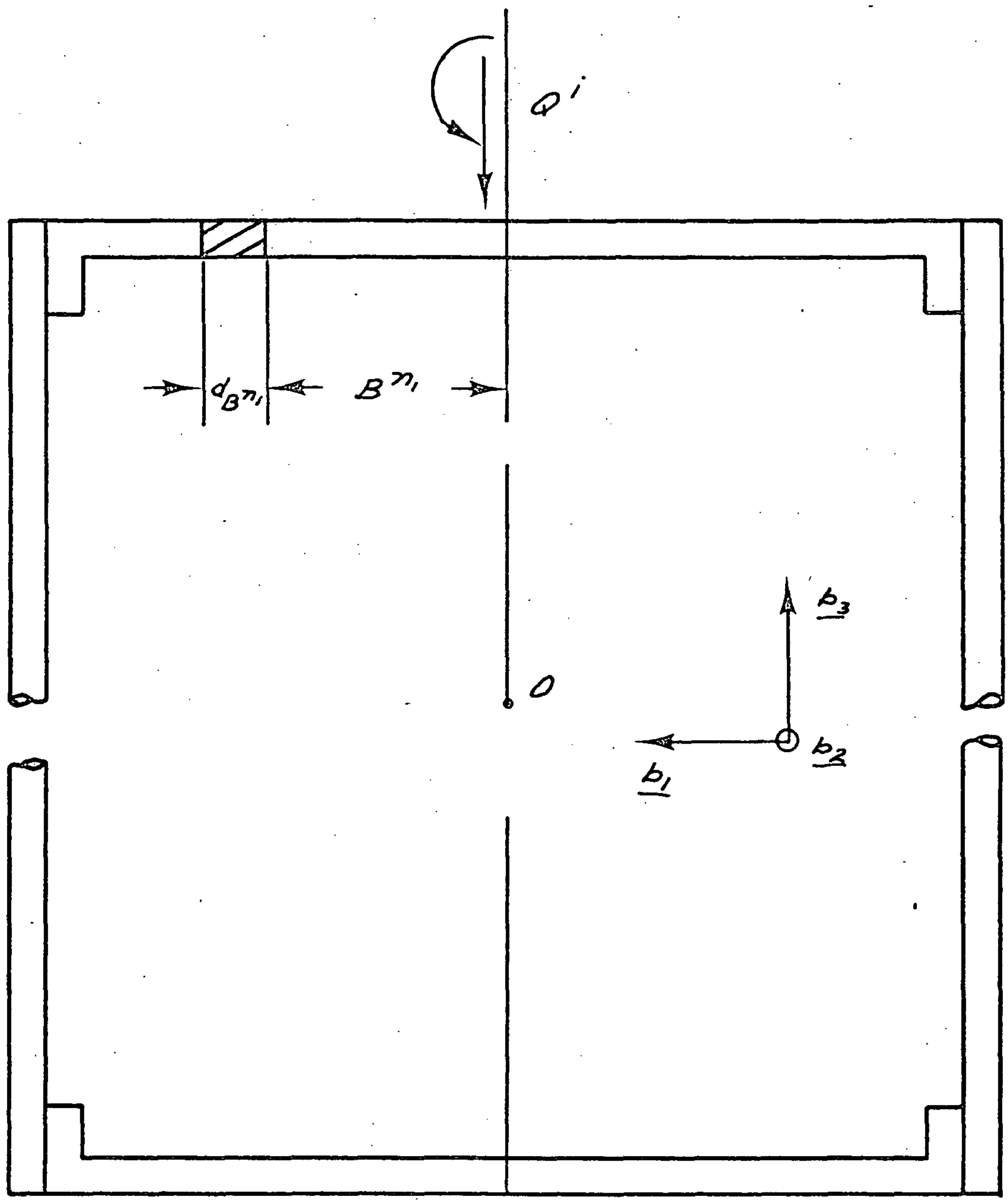

FIGURE 4.

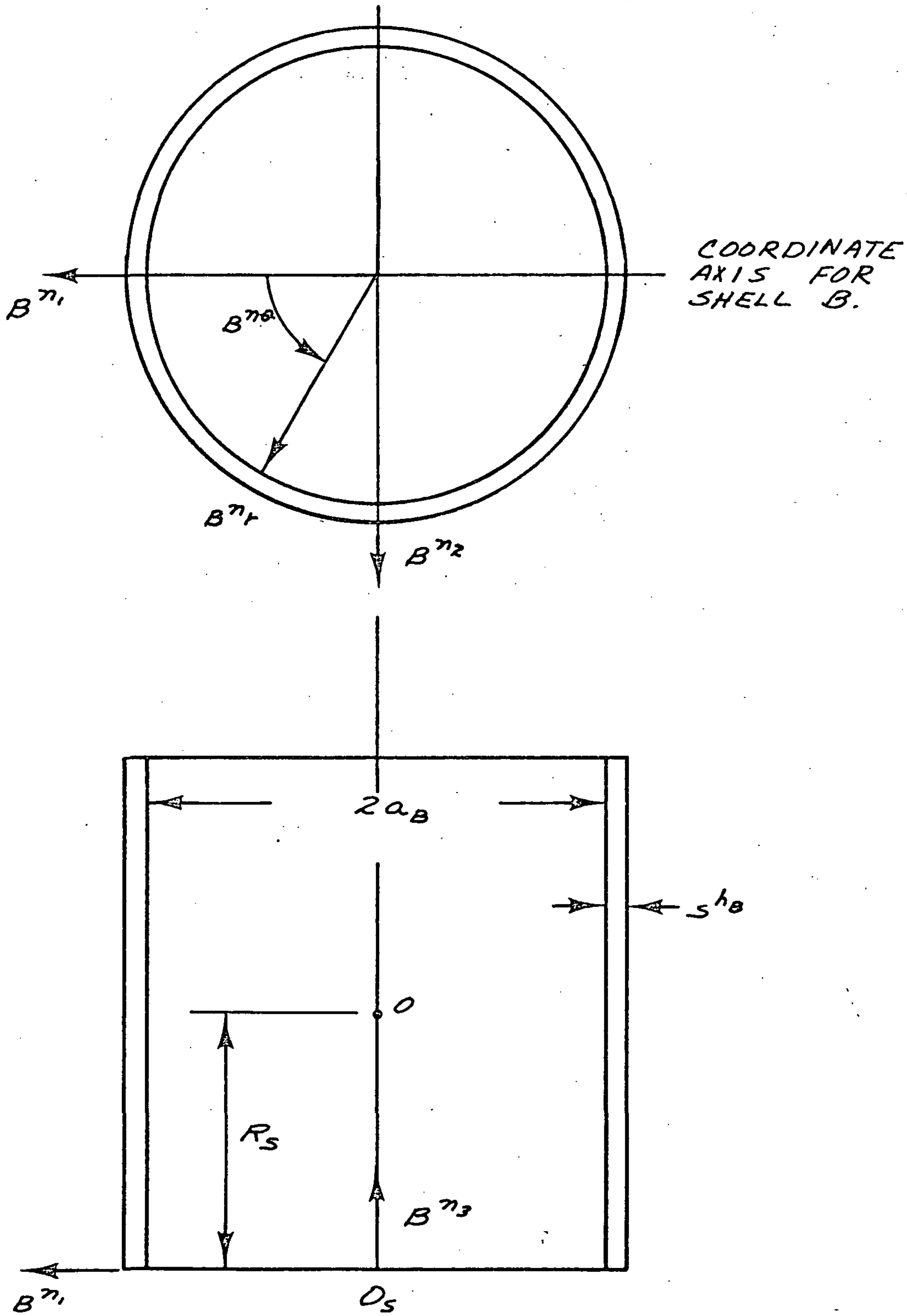
COORDINATE
AXIS FOR
SHELL B.
B^{n_1}
B^{n_θ}
B^{n_r}
B^{n_2}
$2a_B$
$_Sh_B$
O
R_S
B^{n_3}
B^{n_1}
O_S

FIGURE 5

Evaluation of Satellite Configurations for Optimum Pointing Accuracy

Introduction

The two principal requirements for scientific synchronous satellites are:

a) Constant attitude angles.

b) Precise determination and control of the attitude angles.

The possible satellite configurations are:

a) A spinning satellite.

b) A three axes stabilized satellite.

c) A dual-spin satellite, which contains a spinning part and a depsun part. The problems influencing the choice are as follows:

a) For a three-axis stabilized satellite, a very precise determination of instantaneous attitude angles is possible with interferometric methods. But its motion at a subsequent time and corresponding control is very uncertain.

b) A spinning satellite provides a very stable platform in space. But the attitude measurement creates a great deal of uncertainty because of the flexibility of the structure and the antennas attached to it.

Our present study is to determine the configuration of the satellite offering a stability and accuracy within certain limits.

The Simulation Problem

The development of an attitude control system necessarily involves a dynamic simulation of the vehicle being controlled, but the accuracy required of that simulation varies widely from system to system. When space vehicies missions do not impose stringent attitude control requirements, and when the vehicle is essentially rigid, the simulation of the vehicle as a fully rigid body is more than satisfactory. But modern space vehicles are very flexible with low natural frequencies. At the same time severe demands are being made on attitude control and attitude pointing error analysis. This is true, especially for remote sensing operations and optical observations. So a more improved dynamic simulation is required.

In the past Landon [16] and Iorillo [11] pioneered the analysis of the stability problem of nonrigid masses. They were closely followed by Karymov [13], Rossi [14] and Mingori [12], where the principal stress was given on the stability of motion. But the most important contributions are by Likins [1, 2, 4, 5, 6, 9]. He has shown a method in which the vibration frequencies, and modes of flexible satellites can be analyzed. Though he mostly uses the lumped mass discrete model, he has shown the usefulness of using synthetic modes where a structure can be treated as a combination of rigid and flexible masses [6].

As the present analysis is oriented towards pointing error studies, the principal stress is given on the mode shapes rather than on the stability of the motion. For an accurate modal analysis, discrete mass approximations are not satisfactory. Also the thermal stress effects, left out by earlier writers will be taken into consideration. The technique of coupling together the equations for the individual appendages through suitable continuity conditions follows that shown by Huang and others [17, 18, 19, 20]. The analysis will be made for force free motion as well as with self-disturbing and environmental torques [21, 22].

The principal problems that are faced in this analysis are: a) the accommodation of the nonvibratory motion of the flexible appendages, and b) the complete withdrawal of all restrictions on the angular velocities of bodies. The first condition introduces inertial coupling in the system, leading to time-varying inertia matrices. The second consideration brings in nonlinearities due to the centripetal and Coriolis accelerations. The Coriolis terms bring in a skew-symmetric coefficient matrix. And this is completely different from the classical "damping" matrix. So none of the advantages of the classical modal coordinate analysis are available because of the unrestricted rotation of the components.

Objectives:

The objective of this analysis is to estimate the pointing error. This is to be obtained in the following method. For a particular base line configuration, one or more of the rigid bodies m_i $(i = 1 - 4)$ will be the model of the attitude determination sensors. The rest will be modelled to be the imaging or sounding sensors. So this analysis will provide:

a) the extremes of the attitude error between the different sensors;

b) the extremes of the phase difference of the attitude error;

c) a probabilistic time history of the phase and magnitudes of the error for the transient zone after every control torque pulse;

d) a computer program to plot out the pitch, roll and yaw limit cycles for a 90 percent probability density band width;

e) an estimate of the stiffness requirement and the number and positions of the antennae and other flexible elements for a given maximum error limit;

f) comparison of the error limits for 3-axes stabilized, spinning and dual-spin configurations for the same base-line configurations.

Nomenclature

CM	=	Vehicle center of mass
A, B, C	=	Satellite sub-assemblies
m_i, $i = 5\text{-}20$	=	Point masses (scalar)
m_i, $i = 1\text{-}4$	=	Rigid bodies having inertia tensors.
$\underline{a}_1, \underline{a}_2, \underline{a}_3$	=	Orthogonal unit vectors fixed in A.
$\underline{b}_1, \underline{b}_2, \underline{b}_3$	=	Orthogonal unit vectors fixed in B.
$\underline{m}_1, \underline{m}_2, \underline{m}_3$	=	Inertially fixed orthogonal unit vectors
$\lVert M \rVert$	=	Total vehicle mass (scalar)
O	=	Nominal location of center of mass in B
O'	=	Inertially fixed point
Q	=	Reference point, fixed in B

$\underset{\sim}{R}$, R = Position vector and B-basis maxtrix of Q relative to O

$\underset{\sim}{X}$, X = Inertial position of the center of mass and inertial-basis matrix

$\underset{\sim}{C}$, C = Vector and B-basis matrix for CM motion in B

$\underline{\omega}_B$, ω_B = Inertial angular velocity vector and B-basis matrix for B

$\underline{\omega}_A$, ω_A = Inertial angular velocity vector and A-basis matrix for A

A^{x_1}, A^{x_2}, A^{x_3} = Coordinate measure numbers in A-basis

B^{x_1}, B^{x_2}, B^{x_3} = Coordinate measure numbers in B-basis

$y_{i,1}$, $y_{i,2}$, $y_{i,3}$ (i = 1, -20) = Displacements of masses m_i in B-basis

$A^{y_{i,1}}$, $A^{y_{i,2}}$, $A^{y_{i,3}}$ (i = 1 to 20) = Displacements of masses m_i in A-basis

$\theta_{i,1}$; $\theta_{i,2}$; $\theta_{i,3}$ (i = 1-4) = Rotations of rigid bodies m_1, m_2, m_3 and m_4 in B-basis

$A^{\theta_{i,1}}$; $A^{\theta_{i,2}}$; $A^{\theta_{i,3}}$ (i = 1-4) = Rotations of rigid bodies m_1, m_2, m_3 and m_4 in A-basis

$\underset{\sim}{F}$, F = Vector and inertial-basis matrix of force on vehicle

$\underline{F}_i, F_i$ (i = 1-20)	=	Vector and B-basis matrix of force on m_i (i = 1-20)
$\underline{T}, T$	=	Vector and B-basis matrix of torque on vehicle
$\underline{T}_i, T_i$ (i = 1-4)	=	Vector and B-basis matrix of torque on mass m_i (i = 1-4)
$\underline{H}$	=	Vehicle angular momentum about CM
$\underline{H}_i$ (i = 1-4)	=	Angular momentum of masses m_i (i = 1-4) about its own c. m., P_i (i = 1-4)
Q_i	=	Position of m_i (nominal)
P_i	=	Mass center of m_i
$\underline{r}_i, r_i$	=	Position vector and B-basis matrix of Q_i relative to Q
$A^{\underline{r}_i}, A^{r_i}$	=	Position vector and A-basis matrix of Q_i relative to Q
$N^{\underline{a}_i}$	=	Inertial acceleration of masses m_i
Θ	=	Transformation matrix of direction cosines which transforms the inertial basis to B-basis
$N(\overset{\circ}{\ })$	=	Inertial time derivative of vector
$(\overset{\circ}{\ })$	=	Time derivative of vector in ref. frame B
$(\tilde{\ })$	=	Skew symmetric matrix operator defined by eq. (3).
η_i (i = 1-8)	=	Displacements of beams no. 1 - 8 in B-basis
A^{η_i} (i = 1-8)	=	Displacements of beams no. 1 - 8 in A-basis

ξ_B	=	Displacements of cylindrical shell B in B-basis
ξ_A	=	Displacements of cylindrical shell A in B-basis
${}_A\xi_B$	=	Displacements of shell B in A-basis
${}_A\xi_A$	=	Displacements of shell A in A-basis
χ_i $(i = 1-4)$	=	Displacements of plates no. 1 to 4 in B-basis
${}_A\chi_i$ $(i = 1-4)$	=	Displacements of plates no. 1 to 4 in A-basis
$\rho_B{}^i$ $(i = 1-4)$	=	Mass per unit length of beams, $i = 1-4$
ρ_{sA}, ρ_{sB}	=	Mass per unit area of shells A and B
ρ_{P_i} $(i = 1-4)$	=	Mass per unit area of plates no. 1 - 4
e_1	=	Mass centre shift for beam, plate and shell deformation in B-basis
e_2	=	Mass center shift for beam, plate and shell deformation in A-basis
$\mu_{i,1}$; $\mu_{i,2}$, $\mu_{i,3}$	=	Local orthogonal coordinates for axes for beams, and fixed to B
${}_A\mu_{i,1}$, ${}_A\mu_{i,2}$, ${}_A\mu_{i,3}$	=	Local orthogonal coordinate axes for beams and fixed to A
$q_{i,1}$, $q_{i,2}$, $q_{i,3}$	=	Beam elastic deformation in μ_i-axis in B
${}_Aq_{i,1}$, ${}_Aq_{i,2}$, ${}_Aq_{i,3}$	=	Beam elastic deformation in ${}_A\mu_i$-axis in A

μ_i^B = Transformation matrix for μ_i-axes to B-basis

${}_A\mu_i^B$ = Transformation for ${}_A\mu_i$-axes to A-basis

${}_b\vec{F}_{B_i}$ (i = 1-4) = Inertia force on the element of beam no. i (1-4) in B-basis

${}_b\vec{F}_{\mu_i}$ (i = 1-4) = Inertia force on the element of the i^{th}-beam in μ-basis, i.e. local coordinates

s_i, s_i^B, s_i^A = Position vectors of i^{th} beam element from the reference end in local, B-basis and A-basis coordinates respectively

R_i = B-basis position vector to the reference end of the i^{th} beam

AR_i = A-basis position vector to the reference end of the i^{th} beam

μR_i = μ-basis position vector, transformed from R_i

μAR_i = μ-basis position vector, transformed from AR_i

k_{B_i} = $-\dfrac{\rho_{B_i}}{\|M\|}$

$M_{T_{i,2 \text{ or } 3}}$ = Thermal bending moment in the i^{th} beam

E = Modulus of elasticity

$bI_{i,2}$, $bI_{i,3}$ = Moments of inertia of the i^{th} beam in the direction of $\vec{\mu}_{i,2}$ and $\vec{\mu}_{i,3}$ respectively

$\mathcal{K}_{T_{i,2}}$ and $\mathcal{K}_{T_{i,3}}$	=	Thermal curvature of the i^{th} beam in the direction of $\vec{\mu}_{i,2}$ and $\vec{\mu}_{i,3}$ respectively
K_i	=	Thermal bending constant for the i^{th} beam
τ_i	=	Characteristic time for heat transfer across the i^{th} beam
$\alpha_{i,2}$ and $\alpha_{i,3}$	=	Beam attitude angles w. r. to the sum
$\mathcal{K}^*_{T_{i,2 \text{ or } 3}}$	=	Thermal curvature maximum values for i^{th} beam
$rI^A_{i,jk}$	=	Moment of inertia sensor of the i^{th} rigid body in A-basis
$rI^B_{i,jk}$	=	Moment of inertia sensor of the i^{th} rigid body in B-basis
D_{p_i}	=	Stiffness of the i^{th} plate.
${}_pE_i$	=	Mod. of elasticity of the i^{th} plate.
${}_ph_i$	=	Thickness of the i^{th} plate.
${}_p\mu_i$	=	Poisson's ratio of the i^{th} plate.
${}_pT_i$	=	Differential temperature distribution of the i^{th} plate.
${}_p\alpha_i$	=	Thermal coeff. of expansion for the i^{th} plate.
k_{p_i}, τ_{p_i}, $pT_{i,0}$	=	Thermal constants for the i^{th} plate.
β_{p_i}	=	Attitude of the sun from the plate nominal normal vector.

$\beta^{*}_{p_i}$	=	Flexural attitude change of plate element from the nominal normal.
$\xi_{B,r}$, $\xi_{B,\theta}$, $\xi_{B,3}$	=	Radial, tengential and axial deformation of an element of shell B.
${}_sF_{B,1}$, ${}_sF_{B,2}$, ${}_sF_{B,3}$	=	Inertial force components per unit area of shell B in b-basis.
${}_sh_B$	=	Thickness of shell B.
${}_Bx_r$, ${}_Bx_\theta$, ${}_Bx_3$	=	Polar cylindrical coordinates for shell B.
a_B	=	Nominal radius of shell B.
${}_s\mu_B$	=	Poisson's ratios for shell B.
E_{sB}	=	Modulus of elasticity for shell B.
$\mathcal{I}$	=	Identity matrix.
${}_sT_B$	=	Differential temperature distribution of shell B.
β_{sB}	=	Attitude of the sun from the nominal normal of the shell element.
β^{*}_{sB}	=	Flexural change of attitude of shell B element from the nominal normal.
k_{sB}, τ_{sB}, ${}^{s}T_{B,0}$	=	Thermal constants for shell B.

Flexible Appendage Equations of Motion:

A) Equation of motion of particles m_i, i = 17, 18, 19, 20.

The equations of motion for a particle are

$$\underline{F}_i = m_i \cdot {}_N\underline{a}_i \qquad A(1)$$

Now ${}_N\underline{a}_i$ = the inertial second derivative of the sum of the displacement vectors $(\underline{X} + \underline{C} + \underline{R} + \underline{r}_i + \underline{y}_i)$. Vectors $\underline{C}$ and $\underline{y}_i$ are assumed to be continuous and small, such that the terms containing square and higher powers of these and their derivatives are neglected. The vector $\underline{X}$ establishes the trajectory of the vehicle mass center (See Figure 2).

Equation (1) in the B-basis is given by

$$\underline{F}_i' = m_i[{}^N\ddot{\underline{X}} + \ddot{C} + \ddot{y}_i + 2\underline{\omega}_B x(\dot{C} + \dot{y}_i) + {}^N\dot{\underline{\omega}}_B x(C + R + r_i + y_i)$$

$$+ \underline{\omega}_B x\{\underline{\omega}_B x(C + R + r_i + y_i)\}] \qquad A(2)$$

where ${}^N(\ddot{\ })$ = differentiation in the interial frame, with time.

$$\text{Now defining } \tilde{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \qquad A(3)$$

where $v \equiv [v_1, v_2, v_3]^T$, then $\underline{v} \times \underline{w} = \tilde{v}w$.

$\therefore$ Equation (2) becomes

$$F_i = m_i[\Theta\overset{\circ\circ}{X} + \overset{\circ\circ}{C} + \overset{\circ\circ}{y}_i + 2\tilde{\omega}_B(\overset{\circ}{C} + \overset{\circ}{y}_i) + \tilde{\overset{\circ}{\omega}}_B(C + R + r_i + y_i) + \tilde{\omega}\tilde{\omega}[C + R + r_i + y_i]]$$

A(4)

Aa) Expression for C (Shift of mass center)

$$C = -\frac{1}{\|M\|}\left[\sum_{i=1}^{20} m_i y_i + \sum_{j=1}^{4}\int_0^{\ell} \rho_{B_i}\eta_i ds + \iint \rho_{sA}\cdot \xi_A dA + \iint \rho_{sF}\xi_B dA\right.$$

$$\left. + \sum_{k=1}^{4}\iint \rho_{P_i} \chi_i dA\right] \qquad \text{A(5)}$$

$$= -\frac{1}{\|M\|}\sum_{i=1}^{20} m_i y_i + e_1 \quad \text{(say)} \qquad \text{(5a)}.$$

∴ Equation (2) becomes:

$$F_i = m_i[\Theta\overset{\circ\circ}{X} + \overset{\circ\circ}{e}_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j\cdot \overset{\circ\circ}{y}_j + 2\tilde{\omega}_B(e_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j\overset{\circ}{y}_j + \overset{\circ}{y}_i) + \ddot{y}_i$$

$$+ \tilde{\overset{\circ}{\omega}}_B(e_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j y_j + R + r_i + y_i) + \tilde{\omega}_B\tilde{\omega}_B(e_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j y_j + R + r_i + y_i)].$$

A(6)

This is the guiding equation in B-basis for masses

<u>m_{17}, m_{18}, m_{19}, m_{20}</u> and <u>m_{13}, m_{14}, m_{15}, m_{16}</u>.

B. Equation of motion for beams no. 1, 2, 3, 4

In the local coordinate frame μ_i, let $\mu_{i,1}$ be along the axis of the beam as shown. The transformation matrix to the b-frame is given by μ_i^B where

$$\mu_i^B = \begin{bmatrix} \mu_{i,1} \circ b_1 & \mu_{i,1} \circ b_2 & \mu_{i,1} \circ b_3 \\ \mu_{i,2} \circ b_1 & \mu_{i,2} \circ b_2 & \mu_{i,2} \circ b_3 \\ \mu_{i,3} \circ b_1 & \mu_{i,3} \circ b_2 & \mu_{i,3} \circ b_3 \end{bmatrix} \qquad B(1)$$

μ_i^B is a constant for the configuration.

$\therefore\ [\vec{q}_i] = [\mu_i^B][\vec{\eta}_i]$. B(2) $\longrightarrow$ $\vec{q}_i = [\mu_i^B][\vec{\eta}_i]$

and $s_i \mu_{i,1} = [\mu_i^B][s_i^B]$ B(3),

where s_i and s_i^B are the position vectors of the beam element from one end in local and B-basis coordinates respectively. Let $dm = \rho_{B_i} \cdot ds_i^B$ = the elemental mass $= \rho_{B_i} \cdot ds_i$.

The inertia force on the beam element, $\vec{F}_{B_i}$, in B-basis is given by

$$_b\vec{F}_{B_i} = \rho_{B_i} \cdot ds_i^B[\Theta \mathring{\mathring{X}} + \mathring{\mathring{C}} + \mathring{\mathring{\eta}}_i + 2\tilde{\omega}_B[\mathring{C} + \mathring{\eta}_i] + \mathring{\tilde{\omega}}_B[C + R + S_i^B + \eta_i] + \tilde{\omega}_B\tilde{\omega}_B[C + R + S_i^B + \eta_i]]$$

$\therefore$ In local coordinates, the force is given by

$$ {}_bF_{\mu_i} = [\mu_i^B]\cdot {}_bF_{B_i} = dm \cdot \mu_i^B[\Theta\ddot{X} + \ddot{C} + \ddot{\eta}_i + 2\tilde{\omega}_B[\dot{C} + \dot{\eta}_i] + \tilde{\dot{\omega}}_B[C + R_i + S_i^B + \eta_i] $$

$$ + \tilde{\omega}_B\tilde{\omega}_B[C + R_i + S_i^B + \eta_i]] $$

$$ = dm[\mu_i^B\Theta\ddot{X} + \mu_i^B\ddot{C} + \mu_i^B\ddot{\eta}_i + 2\mu_i^B\tilde{\omega}_B\dot{C} + 2\mu_i^B\tilde{\omega}_B\dot{\eta}_i $$

$$ + \mu_i^B\tilde{\dot{\omega}}_BC + \mu_i^B\tilde{\dot{\omega}}R_i + \mu_i^B\tilde{\dot{\omega}}_B S_i^B + \mu_i^B\tilde{\dot{\omega}}_B\eta_i + \mu_i^B\tilde{\omega}_B\tilde{\omega}_BC $$

$$ + \mu_i^B\tilde{\omega}\tilde{\omega}_BR_i + \mu_i^B\tilde{\omega}_B\tilde{\omega}_BS_i^B + \mu_i^B\tilde{\omega}_B\tilde{\omega}_B\eta_i], $$

where Ri = B-basis position vector to the reference end of the beam.

$$ \text{Now} \quad e = -\frac{1}{\|M\|}\left[\sum_{j=1}^{20} m_jy_j + \sum_{k=1}^{4}\int_0^{\ell}\rho_{B_k}\eta_k ds + \iint\rho_{SA}\xi_A dA \right. $$

$$ \left. + \iint\rho_{SB}\xi_B dA + \sum_{m=1}^{4}\iint\rho_{p_m}\chi_m dA\right] $$

$$ \text{and let } \vec{C} = -\frac{1}{\|M\|}\cdot\rho_{B_i}\int_0^{\ell}\vec{\eta}_i ds + \vec{e}_2 \quad \text{(say)} \qquad \text{B(2)} $$

$$ = -\frac{\rho_{B_i}}{\|M\|}[\mu_i^B]^T\int_0^{\ell}\vec{q}_i ds + \vec{e}_2. \qquad \text{(B2a)} $$

$$ \therefore \; {}_bF_{\mu_i} = dm[\mu_i^B\Theta\ddot{X} + \mu_i^B\ddot{e}_2 - \frac{\rho_{B_i}}{\|M\|}\mu_i^B\cdot(\mu_i^B)^T\cdot\int_0^{\ell}\ddot{\vec{q}}_i ds + \mu_i^B(\mu_i^B)^T\ddot{q}_i $$

$$ + 2\mu_i^B\tilde{\omega}_B\left\{-\frac{\rho_{B_i}}{\|M\|}(\mu_i^B)^T\int_0^{\ell}\dot{\vec{q}}_i ds + \dot{e}_2\right\} + 2\mu_i^B\tilde{\omega}_B(\mu_i^B)^T\dot{q}_i $$

$$ + \mu_i^B\tilde{\dot{\omega}}_B\left\{-\frac{\rho_{B_i}}{\|M\|}[\mu_i^B]^T\int_0^{\ell}\vec{q}_i ds + \vec{e}_2\right\} + \mu_i^B\tilde{\dot{\omega}}R_i + \mu_i^B\tilde{\dot{\omega}}_B(\mu_i^B)^T\cdot S_i\cdot\vec{\mu}_{i,}\, i\cdot $$

$$+ \mu_i^B \cdot \mathring{\tilde{\omega}}_B (\mu_i^B)^T q_i + \mu_i^B \tilde{\omega}\tilde{\omega}_B [- \frac{\rho_{B_i}}{\|M\|} (\mu_i^B)^T \int_0^\ell \vec{q}_i ds + \vec{e}_2] + \mu_i^B \tilde{\omega}_B \tilde{\omega}_B R_i$$

$$+ \mu_i^B \tilde{\omega}_B \tilde{\omega}_B (\mu_i^B)] S_i \cdot \vec{\mu}_{i,1} + \mu_i^B \tilde{\omega}_B \tilde{\omega}_B (\mu_i^B)^T q_i]. \qquad B(3)$$

$$\text{Let } \mu R_i = [\mu_i^B][\vec{R}_i], \qquad \therefore \vec{R}_i = (\mu_i^B)^T \cdot (\mu R_i). \qquad B(4)$$

$$\left.\begin{array}{l} \text{Let } \mu_i^B \tilde{w} (\mu_i^B)^T \text{ be denoted by } w^* \text{ and} \\ \mu_i^B \tilde{w} \tilde{w} (\mu_i^B)^T \text{ be denoted by } w^{**} \text{ where } w \text{ is any matrix.} \end{array}\right\} \qquad B(5)$$

$\therefore$ Equation (3) can be written as

$$\begin{aligned} {}_bF_{\mu_i} = dm \Big[& \mu_i^B \{ \Theta \ddot{X} + \ddot{e}_2 + 2\tilde{\omega}_B \dot{e}_2 + \mathring{\tilde{\omega}}_B (e_2 + R_i) + \tilde{\omega}_B \tilde{\omega}_B (e_2 + R_i) \} \\ & + \ddot{q}_i - \frac{\rho_{B_i}}{\|M\|} \int_0^\ell \ddot{q}_i ds + 2\omega_B^* \mathring{q}_i - 2 \frac{\rho_{B_i}}{\|M\|} \omega_B^* \int_0^\ell \mathring{q}_i ds + \mathring{\omega}_B^* q_i \\ & - \mathring{\omega}_B^* \cdot \frac{\rho_{B_i}}{\|M\|} \int q_i ds + \mathring{\omega}_B^* S_i \cdot \mu_{i,1} + \mathring{\omega}_B^{**} q_i - \frac{\rho_{B_i}}{\|M\|} \omega_B^{**} \int_0^\ell q_i ds \\ & + \omega_B^{**} S_i \cdot \mu_{i,1} \Big], \end{aligned}$$

$$\begin{aligned} \text{or } {}_bF_{\mu_i} = dm \Bigg[& \mu_i^B \{ \Theta \ddot{X} + \ddot{e}_2 + 2\tilde{\omega}_B \dot{e}_2 + \mathring{\tilde{\omega}}_B (e_2 + R_i) + \tilde{\omega}_B \tilde{\omega}_B (e_2 + R_i) \} \\ & + \left\{ \frac{d^2}{dt^2} + 2\omega_B^* \frac{d}{dt} + (\mathring{\omega}_B^* + \omega_B^{**}) \right\} \left\{ q_i + k_{B_i} \int_0^\ell q_i ds \right\} \\ & + (\mathring{\omega}_B^* + \omega_B^{**}) \begin{Bmatrix} S_i \\ 0 \\ 0 \end{Bmatrix} \Bigg] \end{aligned} \qquad B(6)$$

where $k_{B_i} = -\frac{\rho_{B_i}}{\|M\|}$.

B(a). Thermo-elastic considerations on the beams

The thermal oscillations of the outstretched booms cause considerable changes in the attitudes of the spacecraft. The thermal curvature of the boom will be assumed to bear a linear relation to the local heat input. This assumption is made with success by Y. Y. Yu [23] and Etkin [24].

The effect of heat transfer across the beam is given by the following equation. The inertia force on the beam

$$\equiv {}_bF_{\mu_{i,2}} = [E(bI_{i,3}]\cdot q_{i,2}^{(iv)} + M''_{T_{i,2}}]dm = \left[E(bI_{i,3})\frac{\partial^4 q_{i,2}}{\partial S_i^4} + \frac{\partial^2 M_{T_{i,2}}}{\partial S_i^2}\right]dm$$

and $${}_bF_{\mu_{i,3}} = \left[E(bI_{i,2})\cdot\frac{\partial^4 q_{i,3}}{\partial S_i^4} + \frac{\partial^2 M_{T_{i,2}}}{\partial S_i^2}\right]dm \qquad \text{B(7) and (8)}$$

where $$M_{T_{i,2}} = -E(bI_{i,3})\cdot \varkappa_{T_{i,2}} \qquad \text{B(9)}$$

and $$M_{T_{i,3}} = -E(bI_{i,2})\cdot \varkappa_{T_{i,3}} . \qquad \text{B(10)}$$

Also $$\frac{\partial \varkappa_{T_{i,2}}}{\partial t} = -\frac{\varkappa_{T_{i,2}}}{\tau} + K_i \cos(\alpha_{i,2} + \theta_{i,2})$$

where $$\theta_{i,2} = \frac{\partial q_{i,2}}{\partial S_i} \qquad \text{B(11)}$$

and $$\frac{\partial \varkappa_{T_{i,3}}}{\partial t} = -\frac{\varkappa_{T_{i,3}}}{\tau} + K_i \cos(\alpha_{i,3} + \theta_{i,3}),$$

where $\theta_{i,3} = \frac{\partial q_{i,3}}{\partial S_i}$ B(12)

Solutions of (B 11 and 12) are taken from Yu [23] and, for small $\theta_{i,2}$ and $\theta_{i,3}$, are given by

$$\mathcal{X}_{T_{i,2}} = \mathcal{X}^*_{T_{i,2}} \cdot \cos(\alpha_{i,2} + \frac{\partial q_{i,2}}{\partial S_i} - \tau \frac{\partial^2 q_{i,2}}{\partial S_i, \partial t}) \qquad B(13)$$

and

$$\mathcal{X}_{T_{i,3}} = \mathcal{X}^*_{T_{i,3}} \cos(\alpha_{i,3} + \frac{\partial q_{i,2}}{\partial S_i} - \tau \frac{\partial^2 q_{i,2}}{\partial S_i \partial t}) \qquad B(14)$$

Equations (13) and (14) are approximated by

$$\mathcal{X}_{T_{i,2}} = \mathcal{X}^*_{T_{i,2}} \left[\cos \alpha_{i,2} - (\frac{\partial q_{i,2}}{\partial S_i} - \tau \frac{\partial^2 q_{i,2}}{\partial S_i \partial t}) \cdot \sin \alpha_{i,2}\right] \qquad B(15)$$

$$\mathcal{X}_{T_{i,3}} = \mathcal{X}^*_{T_{i,3}} \left[\cos \alpha_{i,3} - (\frac{\partial q_{i,3}}{\partial S_i} - \tau \frac{\partial^2 q_{i,3}}{\partial S_i}) \cdot \sin \alpha_{i,3}\right] \qquad B(16)$$

B(b). Final guiding equations for the beams

$$\begin{bmatrix} A_i E \dfrac{\partial^2 q_{i,1}}{\partial S_i^2} \\ E(bI_{i,3})\left[\dfrac{\partial^4 q_{i,2}}{\partial S_i^4} - \mathcal{X}^*_{T_{i,2}}\left\{\cos \alpha_{i,2} - (\dfrac{\partial^3 q_{i,2}}{\partial S_i^3} - \tau \dfrac{\partial^4 q_{i,2}}{\partial S_i^3 \partial t})\right\} \sin\alpha_{i,2}\right] \\ E(bI_{i,2})\left[\dfrac{\partial^4 q_{i,3}}{\partial S_i^4} - \mathcal{X}^*_{T_{i,3}}\left\{\cos \alpha_{i,3} - (\dfrac{\partial^3 q_{i,2}}{\partial S_i^3} - \tau \dfrac{\partial^4 q_{i,2}}{\partial S_i^3 \partial t})\right\} \sin \alpha_{i,3}\right] \end{bmatrix} =$$

$$= \left[\frac{d^2}{dt^2} + 2\omega_B^* \frac{d}{dt} + (\mathring{\omega}_B^* + \omega_B^{**}) \right] \cdot \left[q_i + k_{B_i} \int_0^{\ell} q_i ds \right]$$

$$+ (\mathring{\omega}_B^* + \omega_B^{**}) \begin{Bmatrix} S_i \\ 0 \\ 0 \end{Bmatrix} + \mu_i^B [\Theta \overset{\circ\circ}{X} + \overset{\circ\circ}{e}_2 + 2\tilde{\omega}_B \mathring{e}_2$$

$$+ \tilde{\mathring{\omega}}_B (e_2 + R_i) + \tilde{\omega}_B \tilde{\omega}_B (e_2 + R_i)] \,. \qquad \text{B(17)}$$

In this equation the gravity gradient torque on the beam is not considered, as that would make this equation nonlinear. The gravity gradient torque is assumed constant for small deflections of the beam, and is so will be considered directly in the vehicle equation of motion. These equations are simultaneous linear fourth order integral equations of the Fredholm type. The kernel for a physical object can be separated into the spatial and time dependent functions. The solution technique will be shown in a later chapter.

C. <u>Equations of Motion for Rigid Bodies</u> m_1 <u>and</u> m_2

A rigid body of finite dimensions has six degrees of freedom. So to describe the motions of the bodies m_i $(i = 1, 2)$, both the force and the torque equations are to be considered.

The force equation is formally the same as the equation (A6) derived for point masses.

$$F_i = m_i[\Theta\overset{\circ\circ}{X} + \overset{\circ\circ}{e}_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j\overset{\circ\circ}{y}_j + \overset{\circ\circ}{y}_i + 2\tilde{\omega}_B(\mathring{e}_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j\mathring{y}_j + \mathring{y}_i)$$

$$+ \tilde{\mathring{\omega}}_B(e_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j y_j + R + r_i + y_i) + \tilde{\omega}_B\tilde{\omega}_B(e_1 - \frac{1}{\|M\|}\sum_{j=1}^{20} m_j y_j + R + r_i + y_i)]$$

$$(i = 1, 2). \qquad \text{(C1)}$$

The absolute angular velocity $r\omega_i$ of the rigid bodies are given by

$$r\omega_i = \omega_B + \mathring{\theta}_i \, . \qquad \text{(C2)}$$

$\therefore$ As derived by Likins and Gale (1),

$$T_i = rI^B_{i,\,jk}(\mathring{\omega}_B + \overset{\circ\circ}{\theta}_i + \tilde{\omega}_B\mathring{\theta}_i) + \tilde{\omega}_B \cdot rI^B_{i,\,jk} \cdot \omega_B - (\widetilde{rI^B_{i,\,jk} \cdot \omega_B})\mathring{\theta}_i$$

$$+ \tilde{\omega}_B \cdot rI^B_{i,\,jk} \cdot \mathring{\theta}_i + [rI^B_{i,\,jk} \cdot \tilde{\mathring{\omega}}_B - (\widetilde{rI^B_{i,\,jk} \cdot \omega_B}) - \tilde{\omega}_B(\widetilde{rI^B_{i,\,jk} \cdot \omega_B})$$

$$+ \tilde{\omega}_B \cdot rI^B_{i,\,jk}\,\tilde{\omega}\,]\theta_i \; ; \qquad (i = 1, 2) . \qquad \text{(C3)}$$

Equations (C1) and (C2) are the guiding equations for the rigid bodies m_1 and m_2 .

D. Equation for Thermal and Flexural Motion for Plates no. 1 and 2

The load-system on the plates is shown in Fig. 4.

As in the equations for the beams, the general solution for the plates under inertial and thermal loads will be found first. These solutions will then be coupled to the solutions for the attached rigid bodies and shells through suitable continuity conditions.

To keep the governing equations in deflections linear, the extensions of the plate will be considered to be decoupled from the flexural motion. The coordinate system for each plate is stationary and parallel with respect to the b-basis and have the origins located at the nominal center of the plates. The axis ${}_B x_1$ passes through the mass center of the attached rigid body m_1.

The elastic forces acting perpendicular to the nominal plate surface on the elemental mass of sides $d\,{}_B x_1$ and $d\,{}_B x_2$ =

$$= D_{p_i}\left[\frac{\partial^4 x_{i,3}}{\partial\,{}_B x_1^4} + 2\frac{\partial^4 x_{i,2}}{\partial\,{}_B x_1^2\,\partial\,{}_B x_2^2} + \frac{\partial^4 x_{i,3}}{\partial\,{}_B x_2^2}\right] d\,{}_B x_1 \cdot d\,{}_B x_2$$

where D_{p_i} = the stiffness of the plate $= \dfrac{({}_pE_i)\cdot({}_ph_i)^3}{12(1 - {}_p\mu_i^2)}$;

${}_pE_i$ = Mod. of elasticity of the i^{th} plate

${}_ph_i$ = thickness of the i^{th} plate

${}_p\mu_i$ = Poisson's ratio of the i^{th} plate.

Let ${}_pT_i$ be the difference of temperature between the two faces of the i^{th} plate at any point. For a thin plate, a linear temperature distribution across the thickness of the plate can be assumed. Also, the plates are assumed homogeneous, so that the thermal bending moments at a point are equal in two orthogonal directions.

So the thermoelastic forces perpendicular to the plate

$$= \frac{{}_p\alpha_i}{{}_ph_i} D_{p_i} (1 + {}_p\mu_i) \cdot \nabla^2 \, {}_pT_i \cdot d\,{}_Bx_1 \cdot d\,{}_Bx_2$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial\, {}_Bx_1^2} + \frac{\partial^2}{\partial\, {}_Bx_2^2} .$$

Then the guiding equation for the plate becomes, considering only $\chi_{i,3} = \chi_i$

$$D_{p_i} \cdot \nabla^4 \chi_i + \frac{{}_p\alpha_i (1 + {}_p\mu_i)}{h_i} D_{p_i} \nabla^2 \, {}_pT_i =$$

$$= \rho_{p_i} [\Theta \ddot{X} + \ddot{e}_3 - \sum_{j=1}^{4} \frac{\rho_{p_j}}{\|M\|} \iint \ddot{\chi}_j \, dA + \ddot{\chi}_i$$

$$+ 2\tilde{\omega}_B (\mathring{e}_3 - \sum_{j=1}^{4} \frac{\rho_{p_j}}{\|M\|} \iint \mathring{\chi}_j \, dA + \mathring{\chi}_i) + (\mathring{\tilde{\omega}}_B + \tilde{\omega}_B \tilde{\omega}_B)(e_3 - \sum_{j=1}^{4} \frac{\rho_{p_j}}{\|M\|} \iint \chi_j dA +$$

$$+ R + r_i + y_i)] \circ \underline{b_3} \qquad \text{(D 1)}$$

where

$$c = e_3 - \sum_{j=1}^{4} \left[\frac{\rho_{p_i}}{\|M\|} \iint \chi_i \, dA \right] \qquad \text{(D2)}$$

A model of the variation of ${}_pT_i$ along the mid-plane of the plate, i.e. in the ${}_Bx_1$, ${}_Bx_2$ plane is obtained as follows. It is a first order time dependent model made in a way similar to that used for the beam.

Let β_{p_i} be the attitude of the sun from the nominal normal of the plate. Then for small deflections, let $\beta^*_{p_i}$ be the rotation of the plate surface normal due to flexure of the i^{th} plate.

$$\therefore \quad \frac{\partial\, {}_pT_i}{\partial t} + \frac{{}_pT_i}{\tau_{p_i}} = K_{p_i} \cos(\beta_{p_i} + \beta^*_{p_i}) \,. \tag{D3}$$

Then the solution of (D3) is given by

$${}_pT_i = k_{p_i}\tau_{p_i} \cos \beta_{p_i} - k_{p_i}\tau_{p_i} \sin[\beta_{p_i}(\beta^*_{p_i} - \tau_{p_i}\mathring{\beta}^*_{p_i} + \tau^2_{p_i}\overset{\circ\circ}{\beta}{}^*_{p_i} - \tau^3_{p_i}\overset{\circ\circ\circ}{\beta}{}^*_{p_i} + \ldots)].$$

Keeping only the first power of τ_{p_i}, the solution becomes

$${}_pT_i = {}_pT_{i,0} \cos[\beta_{p_i} + \beta^*_{p_i} - \tau_{p_i}\mathring{\beta}^*_{p_i}] \tag{D4}$$

where ${}_pT_{i,0} = k_{p_i} \cdot \tau_{p_i}$ = the maximum value of ${}_pT_i$.

In this analysis, k_{p_i} and τ_{p_i} are thermodynamic constants for the i^{th} plate. For small values of

$$\frac{\partial \chi_{i,3}}{\partial\, {}_Bx_1} \text{ and } \frac{\partial \chi_{i,3}}{\partial\, {}_Bx_2}, \quad \mathring{\beta}^*_{p_i} = \left(\frac{\partial^2 \chi_{i,3}}{\partial\, {}_Bx_1 \partial t} + \frac{\partial^2 \chi_{i,3}}{\partial\, {}_Bx_2, \partial t}\right) \text{ and}$$

$$\beta_{p_i} = \left[\left(\frac{\partial \chi_{i,3}}{\partial\, {}_Bx_1}\right)^2 + \left(\frac{\partial \chi_{i,3}}{\partial\, {}_Bx_2}\right)^2\right]^{1/2}.$$ Hence, on further linearization,

$$ {}_pT_i = {}_pT_{i,0}\cos\beta_{p_i} + {}_pT_{i,0}\cdot\tau_{p_i}\left(\frac{\partial^2\chi_{i,3}}{\partial\,{}_Bx_1\,\partial t} + \frac{\partial^2\chi_{i,3}}{\partial\,{}_Bx_2\,\partial t}\right)\cdot\sin\beta_{p_i} \quad \text{(D5)}. $$

So equations (Dl) and (D5) together govern the thermoelastic flexure of the plates.

E. Equation of Motion for Shell B

The shell B is assumed to be a uniform, thin, isotropic, circular cylindrical shell. For the elastic analysis, the linear equations of Vlasov (25) will be used. The analysis of thermal effects follows that made by Kraus (26).

The orientation of the cylindrical polar coordinates ${}_Bx_r$, ${}_Bx_\theta$ and ${}_Bx_3$ is shown in Fig. 5. Let $\xi_{B,r}$ be the radial displacement of the shell. $\xi_{B,\theta}$ and $\xi_{B,3}$ are the displacements in the tangential and axial directions.

${}_sT_B$ is the temperature distribution on the mid-plane of the shell. The distribution of temperature across the thickness of the shell is assumed to be linear, with a constant gradient over the mid surface.

Let ${}_sF_{B,1}$, ${}_sF_{B,2}$ and ${}_sF_{B,3}$ be the inertia forces per unit area on a shell element along ${}_Bx_1$, ${}_Bx_2$ and ${}_Bx_3$ respectively.

$$ \therefore \begin{Bmatrix} {}_sF_{B,1} \\ {}_sF_{B,s} \\ {}_sF_{B,3} \end{Bmatrix} = [\Theta\overset{\circ\circ}{X} + \ddot{e} + \overset{\circ\circ}{\xi}_B + 2\tilde{\omega}_B(\mathring{c} + \mathring{\xi}_B) + \tilde{\mathring{\omega}}_B(c + R + \xi_B) + \tilde{\omega}_B\tilde{\omega}_B(c + R + \xi_B)]\,{}_sh_B\cdot\rho_{sB} \quad \text{(E 1)} $$

In equation (E 1), $\vec{R}$ is the position vector of the plate element from the mass center 0 of the spacecraft in the b-basis,

Let $\vec{c} = \vec{e}_4 - \frac{\rho_{sB}}{\|M\|} \iint \vec{\xi}_B dA$. (E2)

Let μ^{Br} be the transformation matrix for changing the b-basis vectors to the normal, tangential and axial components.

$$\therefore \quad \mu^{Br} = \begin{bmatrix} \cos {}_B x_\theta & \sin {}_B x_\theta & 0 \\ -\sin {}_B x_\theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{E3}$$

$$\therefore \vec{\xi}_B^{\,r} = \begin{Bmatrix} \xi_{B,r} \\ \xi_{B,\theta} \\ \xi_{B,3} \end{Bmatrix} = \mu^{Br} \begin{Bmatrix} \xi_{B,1} \\ \xi_{B,2} \\ \xi_{B,3} \end{Bmatrix} \quad \text{and} \quad {}_s\vec{F}_B^{\,r} = \begin{Bmatrix} {}_sF_{B,r} \\ {}_sF_{B,\theta} \\ {}_sF_{B,3} \end{Bmatrix} = \mu^{Br} \begin{Bmatrix} {}_sF_{B,1} \\ {}_sF_{B,2} \\ {}_sF_{B,3} \end{Bmatrix}$$

where ${}_sF_{B,r}$ and ${}_sF_{B,\theta}$ and ${}_sF_{B,3}$ are the radial, tangential and axial components of the force vector ${}_sF_B$.

$$\therefore \quad {}_s\vec{F}_B^{\,r} = [\,\mu^{Br} \cdot \Theta\ddot{X} + \mu^{Br}\ddot{c} + \ddot{\xi}_B^{\,r} + 2\mu_B^r\, \omega_B(\mathring{c} + \mathring{\xi}_B) + \mu_B^r\, \tilde{\mathring{\omega}}_B(c + R + \xi_B)$$

$$+ \mu_B^r\, \tilde{\omega}_B\, \tilde{\omega}_B(c + R + \xi_B)]\, {}_sh_B \cdot \rho_{sB}$$

or

$$ {}_s\vec{F}_B^{\,r} = [\mu^{Br}\{\Theta\ddot{X} + \ddot{e}_4 + 2\tilde{\omega}_B\mathring{e}_4 + \tilde{\mathring{\omega}}_B(e_4 + R) + \tilde{\omega}_B\tilde{\omega}_B(e_4 + R)\}$$

$$+ (\ddot{\xi}_B^{\,r} - \frac{\rho_{sB}}{\|M\|} \iint \ddot{\xi}_B^{\,r} dA) + 2\{\mu_B^r\, \tilde{\omega}_B(\,{}_B^r)^{-1}\}\, (\mathring{\xi}_B^{\,r} - \frac{\rho_{sB}}{\|M\|} \iint \mathring{\xi}_B^{\,r}\, dA)$$

$$+ \mu_B^r \tilde{\mathring{\omega}}_B (\mu_B^r)^{-1} \left(\xi_B^r - \frac{\rho_{sB}}{\|M\|} \iint \xi_B^r dA\right) + \left\{\mu_B^r \tilde{\omega}_B \tilde{\omega}_B (\mu_B^r)^{-1}\right\} \left(\xi_B^r - \frac{\rho_{sB}}{\|M\|} \iint \xi_B^r dA\right)\Big]\, {}_s h_B \cdot \rho_{sB}$$

Settling $\mu_B^r \tilde{\omega} (\mu_B^r)^{-1} = \omega_{Br}^*$; $\mu_B^r \tilde{\mathring{\omega}} (\mu_B^r)^{-1} = \mathring{\omega}_{Br}^*$ and $\mu_B^r \tilde{\omega}\, \tilde{\omega} (\mu_B^r)^{-1} = \omega_{Br}^{**}$,

$$
\begin{aligned}
{}_s\vec{F}_B^r = {}_s h_B \cdot \rho_{sB} \Big[& \mu_B^r \left\{\Theta \ddot{X} + \ddot{e}_4 + 2\tilde{\omega}_B \dot{e}_4 + (\tilde{\mathring{\omega}}B + \tilde{\omega}_B \tilde{\omega}_B)(e_4 + R)\right\} \\
& + \left(\ddot{\xi}_B^r - \frac{\rho_{sB}}{\|M\|} \iint \ddot{\xi}_B^r dA\right) + 2\omega_{Br}^* \left(\dot{\xi}_B^r - \frac{\rho_{sB}}{\|M\|} \iint \dot{\xi}_B^r dA\right) \\
& + (\mathring{\omega}_{Br}^* + \omega_{Br}^{**}) \left(\xi_B^r - \frac{\rho_{sB}}{\|M\|} \iint \xi_B^r \, dA\right) \Big]
\end{aligned}
\tag{E4}
$$

As in the case for the plates, to reduce the high-frequency response, the contributions of $\xi_{B,\theta}$ and $\xi_{B,3}$ in the inertia force are neglected.

So the equationsof motion are given by

$$
\begin{aligned}
\xi_{B,3} = & \frac{{}_s h_B^2}{12 a_B^2} \left[\frac{\partial^5 \phi_B}{\partial\, {}_B x_3^5} - \frac{\partial^5 \phi_B}{\partial\, {}_B x_3 \cdot \partial\, {}_B x_\theta^4} \right] + \frac{1}{a_B^2} \cdot \frac{\partial^3 \phi_B}{\partial\, {}_B x_3 \cdot \partial\, {}_B x_\theta^2} \\
& - \frac{{}_s\mu_B}{a_B^2} \cdot \frac{\partial^3 \phi_B}{\partial\, {}_B x_3^3}
\end{aligned}
\tag{E5}
$$

$$\xi_{B,\theta} = \frac{{}_s h_B^2}{6a_B^2}\left[\frac{\partial^5 \phi_B}{\partial\, {}_B x_3^4 \cdot \partial\, {}_B x_\theta} + \frac{\partial^5 \phi_B}{\partial\, {}_B x_3^2 \cdot \partial\, {}_B x_\theta}\right]$$

$$- \frac{(2 + {}_s\mu_B)}{a_B^2}\,\frac{\partial^3 \phi_B}{\partial\, {}_B x_3^2\, \partial\, {}_B x_\theta} - \frac{1}{a_B^2}\,\frac{\partial^3 \phi_B}{\partial\, {}_B x_\theta} \qquad \text{(E6)}$$

$$\xi_{B,r} = \frac{1}{a_B^2}\nabla^4 \phi_B = \frac{1}{a_B^4}\left[\frac{\partial^4 \phi_B}{\partial\, {}_B x_3^4} + 2\,\frac{\partial^4 \phi_B}{\partial\, {}_B x_3^2\, \partial\, {}_B x_\theta^2} + \frac{\partial^4 \phi_B}{\partial\, {}_B x_\theta^4}\right] \qquad \text{(E7)}$$

and

$$\frac{{}_s h_B^2}{12\, a_B^2}(\nabla^4 + 2\nabla^2 + 1)\nabla^4 \xi_{B,r} - \frac{2\, {}_s h_B^2}{12\, a_B^4}(1 - {}_s\mu_B)\left(\frac{\partial^4}{\partial\, {}_B x_3^4} - \frac{\partial^4}{\partial\, {}_B x_3^2\, \partial\, {}_B x_\theta^2}\right)\nabla^2 \xi_{B,r}$$

$$+ \frac{(1 - {}_s\mu_B^2)}{a_B^4}\,\frac{\partial^4 \xi_{B,r}}{\partial\, {}_B x_3^4} = \frac{(1 - {}_s\mu_B^2)}{{}_s h_B \cdot E_{sB}\, a_B^2}\nabla^4\, {}_s F_{B,r} + \frac{(1 - {}_s\mu_B)}{a_B^2}\, {}_s\alpha_B \cdot \nabla^4\left(\frac{{}_s T_B}{a_B}\right)$$

Setting $\frac{{}_s h_B^2}{12\, a_B^2} = k_B^2$, $\underline{\nabla}^2 = a_B^2\, \nabla^2$, ${}_B x_3 = a_B \cdot {}_B \underline{x}_3$; such that

$$\underline{\nabla}^2 = \frac{\partial^2}{\partial\, {}_B x_3^2} + \frac{\partial^2}{\partial\, {}_B x_\theta^2}\,, \qquad \nabla^4 = \nabla^2\nabla^2 \text{ and } \nabla^8 = \nabla^4\nabla^4\,,$$

then the last equation of motion becomes

$$k_B^2(\underline{\nabla}^4 + 2\underline{\nabla}^2 + 1)\underline{\nabla}^4\, \xi_{B,r} - 2k_B^2(1 - {}_s\mu_B)\left[\frac{\partial^4}{\partial\, {}_B x_3^4} - \frac{\partial^4}{\partial\, {}_B \underline{x}_3^4\, \partial\, {}_B x_\theta^2}\right]\underline{\nabla}^2\, \xi_{B,r}$$

$$+ (1 - {}_s\mu_B^2) \frac{\partial^4 \xi_{B,r}}{\partial\, {}_B x_3^4} = a_B \nabla^4 [(1 + {}_s\mu_B) {}_s\alpha_B \; {}_sT_B] + \frac{(1 - {}_s\mu_B^2)\, {}_s h_B}{12\, E_{sB} \cdot k_B^2} \nabla^4 \, {}_sF_{B,r} \, . \qquad \text{(E8).}$$

E(A). Equation for ${}_sT_B$

In this case also, the method of modelling ${}_sT_B$ is analogous to that used for the beam and plate.

Let β_{sB} be the attitude of the sun from the nominal normal at an arbitrary point of the shell. Also, let β^*_{sB} be the rotation of the shell element from the normal.

$$\therefore \quad \frac{\partial\, {}_sT_B}{\partial t} + \frac{{}_sT_B}{\tau_{sB}} = k_{sB} \cos(\beta_{sB} + \beta^*_{sB}) \, . \qquad \text{(E9)}$$

Neglecting all terms containing τ^2_{sB}, τ_{sB}, and other higher powers of τ_{sB}, the solution to (E9) is

$$ {}_sT_B = {}_sT_{B,0} \cos[\beta_{sB} + \beta^*_{sB} - \tau_{sB} \cdot \mathring{\beta}^*_{sB}] \qquad \text{(E10)}$$

where ${}_sT_{B,0} = k_{sB} \cdot \tau_{sB}$ = the maximum value of ${}_sT_B$.

k_{sB} and τ_{sB} are thermodynamic constants for the shell B.

Now, if β^*_{sB} is small, then

$$\beta^*_{sB} = \left[\left(\frac{\xi_{B,\theta}}{a_B} - \frac{\partial \xi_{B,r}}{a_B\, \partial\, {}_B x_\theta} \right)^2 + \frac{\partial \xi_{B,r}}{\partial\, {}_B x_s}^{2} \right]^{1/2}$$

and

$$\mathring{\beta}^*_{sB} = \frac{1}{a_B}\left(\mathring{\xi}_{B,\theta} - \frac{\partial^2 \xi_{B,r}}{\partial\, {}_B x_\theta\, \partial t}\right) - \frac{\partial^2 \xi_{B,r}}{\partial\, {}_B x_3 \partial t} \triangleq -\frac{1}{a_B}\frac{\partial}{\partial t}\left[\frac{\partial \xi_{B,r}}{\partial\, {}_B x_\theta} + a_B \cdot \frac{\partial \xi_{B,r}}{\partial\, {}_B x_3}\right]$$

Neglecting β^*_{sB} in (η10), but keeping $\mathring{\beta}^*_{sB}$, we finally have

$$ {}_sT_B = {}_sT_{B,0} \cos\beta_{sB} - {}_sT_{B,0} \cdot \tau_{sB}\left(\frac{1}{a_B}\frac{\partial^2 \xi_{B,r}}{\partial\, {}_B x_\theta \partial t} + \frac{\partial^2 \xi_{B,r}}{\partial\, {}_B x\, \partial t}\right) \sin\beta_{sB} \,. \tag{B11}$$

Equations (E5), (E6), (E7), (E8) and (E11) are the required guiding equations for shell B.

F. Equations of Motion of the Elements of the Body A

The equations of motion of the elements of the body A are obtained in the a-basis in exactly the same way as that used to describe the motion of the elements in body B in the b-basis. In this case, all the angular velocities, transformation matrices etc. will relate to the a-basis. The final dynamic coupling of the bodies A, B and C is obtained by transforming the reaction forces and couples between the bodies A and C to the b-basis, and solving the resulting equations for the complete vehicle in the b-basis.

Let Θ_A be the transformation matrix for changing the inertial basis to the a-basis.

So if ${}_N\vec{F}$ be the force vector in n-basis corresponding to ${}_A\vec{F}$ in a-basis, then

$${}_A\vec{F} = \Theta_A \cdot {}_N\vec{F} \qquad \text{or} \quad {}_N\vec{F} = \Theta_A^{-1} \cdot {}_A\vec{F}\,, \quad \text{or}$$

$$_{B}\vec{F} = \Theta_{N}\vec{F} = \Theta\Theta_{A}^{-1} \cdot {}_{A}\vec{F} . \qquad \text{(F1)}$$

where $_{B}\vec{F}$ is the vector in b-basis corresponding to that in a-basis.

Let $\underline{\omega}_A = \mathring{\underline{\psi}}_A$ and $\underline{\omega}_B = \mathring{\underline{\psi}}_B$, where $\underline{\psi}_A$ and $\underline{\psi}_B$ are angular rotation vectors of bases A and B, respectively.

In the application of equation (F1) and the preceding equations of motion, two important cases can occur, and these are treated as follows:

Case I. The body is nominally inertially nonrotating.

Let the body A be nominally fixed in the n-basis. Then $\underline{\psi}_A$ are small, and the following approximation can be made:

$$\Theta_A \triangleq [\mathcal{J} - \tilde{\psi}_A] \qquad \text{(F2)}$$

where "$\mathcal{J}$" is the identity matrix. A similar argument holds if $\underline{\psi}_B$ are small.

Case II. The body is rotating.

Let the body B rotate with respect to the inertial frame having instantaneous Euler angles denoted by $\psi_{B,1}$, $\psi_{B,2}$, $\psi_{B,3}$.

$\psi_{B,3}$ is the spin angle. $\psi_{B,2}$ and $\psi_{B,1}$ are the precession and nutation respectively. Then the transformation equation is given as follows:

$$\begin{bmatrix} {}_{B}F_1 \\ {}_{B}F_2 \\ {}_{B}F_3 \end{bmatrix} = \begin{bmatrix} (\cos\psi_{B,1}\cdot\cos\psi_{B,2}\cdot\cos\psi_{B,3} & (-\cos\psi_{B,1}\cdot\sin\psi_{B,2}\cdot\cos\psi_{B,3} & (\sin\psi_{B,1}\cdot\cos\psi_{B,3}) \\ -\sin\psi_{B,2}\cdot\sin\psi_{B,3}) & -\cos\psi_{B,2}\cdot\sin\psi_{B,3}) & \\ (\cos\psi_{B,1}\cdot\cos\psi_{B,2}\cdot\sin\psi_{B,3} & (-\cos\psi_{B,1}\cdot\sin\psi_{B,2}\cdot\sin\psi_{B,3} & (\sin\psi_{B,1}\cdot\sin\psi_{B,3}) \\ \sin\psi_{B,2}\ \cos\psi_{B,3}) & \cos\psi_{B,2}\cdot\cos\psi_{B,3}) & \\ (-\sin\psi_{B,1}\cdot\cos\psi_{B,2}) & (\sin\psi_{B,1}\cdot\sin\psi_{B,2}) & (\cos\psi_{B,1}) \end{bmatrix} \begin{bmatrix} {}_{N}F_1 \\ {}_{N}F_2 \\ {}_{N}F_3 \end{bmatrix} \tag{F3}$$

A similar equation holds if the body A rotates.

Eqns. of Motion for the Composite Bodies

To describe the motion of the space vehicle, there are two quite different approaches. The first method is to write the equations involving the motion of every movable mass into one complex equation. The second method is to solve the equations of motion of the sub-bodies separately and then accommodate the interactions between the bodies as external forces and torques. For simpler configurations and lumped mass approaches, the first method is advantageous. But when the bodies perform large relative rotations, and when continuous mass distribution is assumed, computational efficiency increases greatly with the second method. In this analysis, it is the second method that will be used.

G. Eqn. of Motion for the Body B

Let $^N\underline{F}_{EB}$ be the external force in the inertial basis on body B. Also let $^N\underline{F}_{BC}$ be the reaction force of body C on body B in the inertial basis.

$$({}^N\underline{F}_{EB} + {}^N\underline{F}_{BC}) = ||M||_B \, {}^N\ddot{\underline{X}}_B \tag{G1}$$

where $||M||_B$ is the mass of the body B alone. $\ddot{\underline{X}}_B$ is the inertial acceleration of the mass centre CM_B of the body B.

$$\therefore \; {}^N\ddot{\underline{X}}_B = \frac{1}{||M||_B} [{}^N\underline{F}_{EB} + {}^N\underline{F}_{BC}] \; . \tag{G2}$$

Eqn. (G2) is the translational eqn. for body B.

The total torque on the body B = $^N\underline{T}_{EB} + {}^N\underline{T}_{BC} + ({}^N\underline{R}_{BC} \times {}^N\underline{F}_{BC})$. $^N\underline{T}_{EB}$ is the external torque applied on the body B, in the inertial basis. $^N\underline{T}_{BC}$ is the reaction torque applied on the body B by the body C, in the inertial basis. $^N\underline{R}_{BC}$ is the inertial position vector of the point of contact of the bodies B and C from the mass-centre of the body B.

Let the inertial angular momentum of the body B be $^N\underline{H}_B$.

The rotational eqn. of motion of the body B is

$$[{}^N\underline{T}_{EB} + {}^N\underline{T}_{BC} + ({}^N\underline{R}_{BC} \times {}^N\underline{F}_{BC})] = {}^N\dot{\underline{H}}_B = \text{the time rate of change of } {}^N\underline{H}_B \text{ in the inertial basis.} \tag{G3}$$

Let $-\underline{C}_B$ be the shift of the mass-centre CM_B from the nominal mass centre of the body B at 0_B. Let $\underline{p}_B$ be the position vector of a mass-element in body B .

$$^N\underline{H}_B = \int (p_B - C_B) \times ({}^N\dot{\underline{p}}_B - {}^N\dot{\underline{C}}_B) dm = \underline{I}_B \cdot \underline{\omega}_B + ||M||_B \cdot \dot{\underline{C}}_B \times \underline{C}_B$$

$$+ \int \underline{p}_B \times \dot{\underline{p}}_B \, dm.$$

As $\underline{C}_B$ is small, so $H_B \doteq \underline{I}_B \cdot \underline{\omega}_B + \int \underline{p}_B \times \dot{\underline{p}}_B \, dm.$

where I_B = the inertia dyadic of the body B with respect to 0_B.

If there are rigid rotating bodies inside the body B, like reaction wheels and motors, then

$$H_B = \underline{I}_B \cdot \underline{\omega}_B + \underline{h}_B + \int p_{\underline{B}} \times \dot{p}_{\underline{B}}\, dm. \tag{G4}$$

where $\underline{h}_B$ = the relative angular momentum ω.r. to B of the rotating wheels etc.

$$
\begin{aligned}
{}^N\dot{H}_B &= \dot{H}_B + \omega_B \times H_B \\
&= [\dot{I}_B \cdot \omega_B + I_B \cdot \dot{\omega}_B + \dot{h}_B + \int p_B \times \ddot{p}_B\, dm] \\
&\quad + \omega_B \times [I_B \cdot \omega_B + h_B + \int p_B \times \dot{p}_B\, dm].
\end{aligned}
$$

Neglecting $\dot{I}_B$, we get,

$$
\begin{aligned}
[N^T EB + N^T BC + (N^R BC \times N^F BC)] = [&I_B \cdot \dot{\omega}_B + \dot{h}_B + \int p_B \times \ddot{p}_B\, dm \\
&+ \omega_B \times h_B + \omega_B \times \int p_B \times \dot{p}_B\, dm.]
\end{aligned} \tag{G5}
$$

Neglecting $(y_i \times \ddot{y}_i)$, and the product of other flexible appendage displacements, eqn. (G5) becomes,

$$
\begin{aligned}
N^T EB + N^T BC + (N^R BC \times N^F BC) &= [I_B \cdot \dot{\omega}_B + \dot{h}_B + \omega_B \times h_B] \\
&+ \sum_{i=1}^{20} [m_i (R_i + r_i) \times \ddot{y}_i] + \sum_{i=1}^{20} [\tilde{\omega}_B \{m_i (R_i + r_i) \times \dot{y}_i\}] \\
&+ \sum_{i=1}^{4} \{\rho_{Bi} \int_0^{\ell} (R_i + S_i^B) \times \ddot{\eta}_i\, dS_i\} + \sum_{i=1}^{4} \tilde{\omega}_B [\rho_{Bi} \int_0^{\ell} (R_i + S_i^B) \times \dot{\eta}_i\, dS_i] \\
&+ \rho_{SB} \iint (R_S + \mu^{\eta} B) \times \ddot{\xi}_B dA + \rho_{SB} . \tilde{\omega}_B . \int (R_S + \mu^{\eta} B) \times \dot{\xi}_B dA\, . \\
&+ \sum_{i=1}^{2} \rho_{\phi i} (\iint (R_i + X_B) \times \ddot{X}_{i,B} dA) + \Sigma\, \rho_{pi} . \tilde{\omega}_B (\iint (R_i + \eta_B) \times \dot{X}_{i,B} dA)
\end{aligned} \tag{G6}
$$

Eqn. (G6) is the rotational equation of motion for body B. Two equations similar to (G2) and (G6) are also developed for body A.

H. Coupling equations for the Bodies A, B and C.

The body C is considered to be a mass-less, extensionally and torsionally rigid body. But C behaves as a combination of a linear spring and viscous damper against transverse linear and angular displacements of one side with respect to the other.

Let ν_1, ν_2, ν_3, and ν_4 be the stiffness and damping constants for translation and rotation of the ends of the body C. So the coupling equations are developed as follows:

$$||M||X = ||M||_B X_B + ||M||_A X_A \tag{H1}$$

$$N^F BC = -N^F AC = \nu_1(X_B - X_A) + \nu_2(\dot{X}_B - \dot{X}_A) \tag{H2}$$

$$N^T BC = -N^T AC = \nu_3(\psi_B - \psi_A) + \nu_4(\omega_B - \omega_A) \tag{H3}$$

$$N^F EB + N^F EA = (||M||_B + ||M||_A)\ddot{X} \tag{H4}$$

$$N^T EB + N^T EA = {}^N\dot{H}_B + {}^N\dot{H}_A \tag{H5}$$

From the equilibrium condition in force-free motion, $N^F EB = N^F EA = 0$ and $X=0$.

$$\left.\begin{array}{l} \therefore\ N^F BC = ||M||_B\, {}^N\ddot{X}_B \quad \text{and} \quad N^F AC = ||M||_A\, {}^N\ddot{X}_A \\ \quad \text{and} \quad X_A = -\dfrac{||M||_B}{||M||_A} X_B \end{array}\right) \tag{H6}$$

Also if $\quad N^T EB = N^T EB^* + N^T EB^{**}$ (H7)

and $\quad N^T EA = N^T EA^* + N^T EA^{**}$ (H8)

where $(\ )^*$ is the environmental torque and $(\ )^{**}$ is the control torque, then

$$N^T EB^{**} = L_B(\underline{\psi}_B) \quad \text{and} \quad N^T EA^{**} = L_A(\underline{\psi}_A) \tag{H9}$$

L_B and L_A are the specific attitude control system operators.

$N^T EB^*$ and $N^T EA^*$ can be determined explicitly in terms of the vehicle geometry and angular rotations.

Eqns. (H1) through (H9) are sufficient to describe the dynamic system completely.

I: Models of the Environmental Torques.

1. Residual Magnetic Torque:

The residual magnetic torque exists in a spacecraft because of the interaction of the flow of current in the spacecraft electrical circuits, and the Earth's magnetic field vector B.

For a geosynchronous satellite, the torque will be time periodic. The time period will be equal to the spin-rate of the satellite. The magnitude of the torque has to be determined from experimental values. The torque model is

$$\vec{T}_{EBM} = \vec{T}_{EBM}^{*} \sin(\psi B_B t) \quad \text{and} \quad \vec{T}_{EAM} = \vec{T}_{EAM}^{*} \sin(\psi A_B t) \qquad (I1)$$

Approximate values will be of the order of 5×10^{-6} ft lbs.

2. Eddy Current Torque:

The eddy current torque on a body is given by

$$\vec{T}\ \text{e.c.} = \frac{1}{c^*} \int \vec{r} \times (\vec{J} \times \vec{H}) dv$$

where $\vec{H} = \frac{\vec{B}}{\mu o}$ = the Earth's magnetic field and μo = permeability of aluminium.

$\vec{J}$ = volume eddy current density.

$\vec{r}$ = position vector from the centre of mass.

c^* = speed of light in vacuum.

Also, $\vec{J} = \frac{1}{2\sigma} (\vec{\omega} \times \vec{H}) \times \vec{r} + \nabla d$

where σ = static electrical conductivity

and $\nabla^2 \phi = 0$ for the body under consideration with the condition that $\frac{\partial \phi}{\partial n} = 0$ on the boundary.

In this case, the field of ϕ will be taken as the thin shells and the plates. This assumption makes the Laplacian a two dimensional operator. Then the boundary condition that the slope is zero makes ϕ = a constant, so that $\nabla \phi = 0$.

$$\therefore \quad \vec{J} = \frac{1}{2\sigma} (\vec{w} \times \vec{H}) \times \vec{r}$$

$$\vec{T}e.c. = \frac{1}{2\sigma c^*} \cdot \int \vec{r} \times [\{(\vec{\omega} \times \vec{H}) \times \vec{r}\} \times \vec{H}] dv$$

$$= \frac{1}{2\sigma c^*} \int \vec{r} \times [\{(\vec{\omega} \cdot \vec{r})\vec{H} - (\vec{H} \cdot \vec{r})\vec{\omega}\} \times \vec{H}] dv$$

$$= \frac{1}{2\sigma c^*} \int [(\vec{\omega} \cdot \vec{r})[\vec{r} \times (\vec{H} \times \vec{H})] - (\vec{H} \cdot \vec{r})[\vec{r} \times (\vec{\omega} \times \vec{H})]] dv$$

$$= - \frac{1}{2\sigma c^*} \int (\vec{H} \cdot \vec{r})[(\vec{r} \cdot \vec{H})\vec{\omega} - (\vec{r} \cdot \vec{\omega})\vec{H}] dv$$

$$= - \frac{1}{2\sigma c^*} \int [(\vec{H} \cdot \vec{r})^2 \vec{\omega} + (\vec{r} \cdot \vec{H})(\vec{r} \cdot \vec{\omega})\vec{H}] dv$$

Now for the spacecraft, $\vec{H}, \vec{\omega}$ are constants. Also $\int \vec{r} \cdot \vec{k}\, dv = 0$ where $\vec{k}$ is constant, because the equations are ω.r. to the mass centre.

$$\therefore \quad Te.c. = - \frac{1}{2\sigma c^*} \int (\vec{H} \cdot \vec{r})^2 \vec{\omega}\, dv \qquad (I2)$$

$$= - \frac{\vec{\omega}}{2\sigma c^*} \int [H_x^2 r_x^2 + H_y^2 r_y^2 + H_z^2 r_z^2 + 2H_x H_y r_x r_y + \ldots] dv$$

$$= - \frac{\vec{\omega}}{2\sigma c^*} [H_x^2 I_{xx} + H_y^2 I_{yy} + H_z^2 I_{zz} + 2H_x H_y I_{xy} + 2H_y H_z I_{yz} + 2H_z H_x I_{zx}]$$

Assuming the spacecraft to be nominally symmetric,

$$Te.c. = - \frac{\vec{\omega}}{2\sigma c^*} [H_x^2 I_{xx} + H_y^2 I_{yy} + H_z^2 I_{zz}] \qquad (I3)$$

3. Torque due to Electromagnetic Radiation:

The equations for the solar torque are obtained from Baletskii (27). These are

$$\vec{T}_{sp} = Pe[(1-\varepsilon_o)\{\vec{\tau} \times \int_{S_i} \vec{r}(\vec{n}\cdot\vec{\tau})ds\} + \varepsilon_o\{2\int_{S_i} \vec{n} \times \vec{r}\ (\vec{n}\cdot\vec{\tau})^2 ds\}] \qquad (I4)$$

In these equations

Pe = constant solar pressure = 1×10^{-7} lbs/ft^2, for a surface normal to the sun.

ε_o = reflection coefficient

$\vec{n}$ = unit outward normal to the surface S_1 exposed to the sun.

$\vec{\tau}$ = unit vector directed from the sun.

$\vec{r}$ = position vector from the centre of mass

4. Gravity Gradient Torque:

The torque on a rigid body caused by the gravity gradient is given by $\vec{T}_G = \frac{(3\mu \vec{r})}{R^3} \times (\vec{\vec{I}} \cdot \vec{r})$, when $\vec{T}_G$ is expressed in the body fixed basis.

In this equation it is assumed that the Earth is spherical.

Also, μ = The Earth's gravitational constant = 1.4082×10^6 ft^3/sec.^{2}

$\vec{r}$ = unit vector in the direction of the Earth's radius vector

R = The distance from the CM to the centre of the Earth

$\vec{\vec{I}}$ = The inertia dyadic of the body.

For the body B, the expression for the gravient gradient torques becomes

$$T_{GB\ 1} = -\frac{3\mu}{R^3}\left[(I_{B22}-I_{B33})d_2d_3 + I_{B12}d_1d_3 - I_{B13}d_1d_2 + I_{B23}(d_3^2-d_2^2)\right]$$

$$T_{GB\ 2} = -\frac{3\mu}{R^3}\left[(I_{B33}-I_{B11})d_1d_3 + I_{B23}d_1d_2 - I_{B12}d_2d_3 + I_{B13}(d_1^2-d_3^2)\right] \quad \text{(I5)}$$

and

$$T_{GB\ 3} = -\frac{3\mu}{R^3}\left[(I_{B11}-I_{B22})d_1d_2 + I_{B13}d_2d_3 - I_{B23}d_1d_3 + I_{B12}(d_2^2-d_1^2)\right]$$

where d_1, d_2, d_3 are the dir cosines of $\vec{r}$.

For the nominally symmetric body, the torque expressions become

$$T_{GB\ 1} = -\frac{3\mu}{R^3}\left[(I_{B22}-I_{B33})d_2d_3\right]$$

$$T_{GB\ 2} = -\frac{3\mu}{R^3}\left[(I_{B33}-I_{B11})d_1d_3\right] \quad \text{(I6)}$$

and

$$T_{GB\ 3} = -\frac{3\mu}{R^3}\left[(I_{B11}-I_{B22})d_1d_2\right]$$

In terms of the Euler angles, d_1, d_2 and d_3 are given by the following equations:

$$d_1 = \cos\psi_{B,1}\cos\psi_{B,2}\cos\psi_{B,3} - \sin\psi_{B,2}\sin\psi_{B,3}$$

$$d_2 = \cos\psi_{B,1}\cos\psi_{B,2}\sin\psi_{B,3} + \sin\psi_{B,2}\cos\psi_{B,3} \tag{I7}$$

$$d_3 = -\sin\psi_{B,1}\cos\psi_{B,2}$$

where it is assumed that the axis $\vec{n}_1$ of the inertial basis is parallel to the radius vector of the Earth.

5. Control Torque Pulses:

The method of modelling the torque pulses depends on the frequency of free rigid body vibration of the spacecraft, as compared to the angular velocity of the rotor if there is any. If the two frequencies are close, an error based sampled data control system will be assumed. The jet torque pulses will then be considered to be series of gate-functions having a frequency, which is a multiple of the rotor spin rate. For the three axes actively controlled spacecraft, the pulsing frequency will be a parameter of the equations. It will be assumed that linear superposition of the solutions for individual gate-function-torques will hold.

But if the error sampling frequency is large compared to the spacecraft natural frequency, the sampling will be considered to be continuous and a Fourier's series will be assumed for modelling the train of torque pulses.

Conclusions:

The basic features of the present analysis in which it claims to be a more accurate model of any particular satellite is the following:

1) The hybrid formulation involving modal coordinates, as well as position and attitude coordinates of rigid elements.

2) The complete spacecraft structural flexibilities are considered. Structural damping can easily be taken into consideration by modeling the materials as linear viscoelastic and changing the elastic modulii into the corresponding complex modulii. The only limiting problem is the computer memory. For introduction of the complex modulii will double the number of coordinates.

3) The model is already large and flexible enough to accommodate a large class of satellites, which are structurally similar.

4) If stiffened plates and shells are used, which most probably is the case, then those stiffened elements will first have to be converted into regular elements by methods already well known.

5) The model can most easily be extended to nonsynchronous satellites.

6) The most important mathematical feature is that the solution bound is much less restricted than that shown by Likins, Kane and others. The existing models are almost wholly restricted to a rigid rotor with a constant angular velocity, together with the flexible elements having very low angular velocities. So their equations are all linear. In this solution, the angular velocities will be assumed partially unrestricted so that asymptotic

expansion methods will be used for each structural element. The first attempt at the asymptotic solution will be made by assuming the angular velocities to be of the form $\omega_i = \lambda_i + \varepsilon_i \sin p_i t$ where λ_i will be a completely unrestricted quantity. But ε_i will be considered small. This will require us to generate a new series of functions comparable to Matheu functions. After completion of the present work we hope to prepare a comprehensive table for such functions so that all future work in dynamics will be considerably simplified.

Finally, I thank you all to offer me this project which has given me quite a few new insights into the problem of modeling flexible bodies in motion.

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