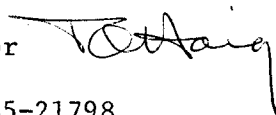


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THE UNIVERSITY OF WISCONSIN

13 March 1973

TO: Contracting Officer, Code 245, NASA/GSFC
Technical Officer, Code 651, NASA/GSFC

FROM: Thomas O. Haig
Executive Director 

REFERENCE: Contract NAS5-21798

SUBJECT: Monthly Progress Report for "Studies of Soundings and Imaging Measurements from Geostationary Satellites"

Task A Investigation of Meteorological Data Processing Techniques

Work on the correction of ATS line start errors using earth edge detection has resulted in considerable progress in the past month. Six digital image tapes from day 204, 1969, have been rewritten with line start corrections included. The displayed images show remarkable improvements in landmark stability and cloud motion continuity. Cloud tracers for these images have been selected and wind vectors will be available soon (a card punch failure has caused minor delays in processing). We believe that previous failures to obtain agreement between WINDCO lag residuals and timing errors was due to an error in navigation which has now been corrected.

We are also developing an algorithm to detect the position of the right-hand earth edge, a difficult task because of the large transmission noise in this portion of the images (for day 204, 1969). Initial tests indicate that an algorithm for the right edge correction will be at least partially successful. The ultimate accuracy possible for this kind of correction has not yet been determined.

Task B Sun Glitter

Sun glitter studies have followed two paths: (1) an investigation of the utility and technique of obtaining sun glitter measurements from a geostationary satellite images, and (2) a study of the problems of geometry (pointing angle, etc.) from a near earth orbiting satellite.

Kornfield has almost completed a Ph. D. thesis on the first subject. (He has one more chapter to go.) He shows that one can indeed obtain excellent estimates of surface wind velocity (we already knew we could get good estimates of wind speed)

Task B Sun Glitter (Continued)

from sun glitter measurements. There are two ways to get the wind direction information. One can use the mean square shape in the cross wind and the downwind directions separately. The sun glitter intensity is a measure of the slope probability--but it is also affected by the intervening atmosphere, scatter by the white caps, scatter by the ocean, and also glitter arising from the sky dome. He shows how to account for all of these and is able to get wind direction for typical conditions, but not the very light winds or very strong winds. He has also found--and this may be more important--that the displacement of the most probable slope; i.e., the brightest part of the sun glitter, from the position the sun's image would occupy if the sea were calm, is directly proportional to the square of surface wind. Indeed, it appears to be directly proportional to the wind stress on the sea surface. This kind of information is difficult to extract from geostationary satellite images because they cover such a wide area. This data would be easy to extract from near earth orbiting satellites. (Kornfield started this work with support from another sponsor, but his support for the last six months has been under this contract.)

There is no specific progress to report on item (2)--the geometry for the near earth satellite--except that the computer programming effort continues.

Task D Cloud Growth Rate

One of the most obvious uses for the geostationary meteorological satellite is surveillance of severe weather. Rapid growth of convective clouds can be very dramatic on time lapse movie loop displays. The real question however is, "How quantitative can we be?" The growth rate of convective clouds is an indicator of the intensity of the conversion. The growth rate can be attacked two ways: (1) from the change in brightness (cloud thickness) with time, and (2) from the expansion of the cirrus shield. The first technique requires a relationship between cloud thickness and brightness. Parks has completed a masters thesis where this approach is investigated. His results are surprisingly good. A simple, nearly linear, relationship exists between brightness and thickness. His results differ sharply from theory and agree with other observations (Kandratiev), although there is a fixed difference attributable to the lack of an absolute calibration of the ATS camera photo multiplier. Mosher is attempting to explain the difference between observation and theory by investigating the scattering characteristics of a mixed cloud containing both droplets and ice particles. Thus, we have a very useful tool--but we can't explain why it is so good.

Task D Cloud Growth Rate (Continued)

The second approach (cirrus shield expansion) is a very good measure of the mass flux through the cloud, but one must know the cross sectional area of the cloud column in order to calculate the vertical velocities. One way is to use the enhancement technique to identify the core area, but it is sensitive to the brightness criteria one chooses to identify the core. The brightness gradient should be better since this is less sensitive to camera gain settings, sun angle, etc. but so far we have not been successful using this approach (noise in the data causes great difficulty). Auvine and Sikdar are using a different approach. Other ground based studies relating mass flux to cloud size (Auer and Merwitz, JAM 7, 196-198; 1968) have been used as a means to calibrate the mass flux measurements describable from cirrus expansion measurements. These again show quite consistent results but differ by a fixed value from the ground based measurements. We are seeking to explain this difference because the ground based measurements tend to be made on small convective systems where the satellite observations have tended to concentrate on the large dramatic easily visible storms.

Either of the indicators described above could be extremely valuable for mesoscale studies be they in GATE or severe storms of the midwest. Auvine, for example, is writing a Ph. D. thesis where he is testing hypothesis put forth by Johnson et. al that the severe weather one finds associated with the jet stream is dynamically related to the super gradient winds one finds down stream from the region of maximum winds. According to this view, one should find areas of enhanced and suppressed convection in areas on each side of the jet upstream and downstream from the jet maximum. Unfortunately, these effects cannot be studied adequately from the radiosonde network because the time interval between data sets is much too long (12 hours) and the geographical spacing is too large also. The geostationary satellite data could provide data on the instability as just mentioned, as well as winds from the drift of clouds.

To see if the scattering properties of the cloud particles differ significantly in the updraft region of a cloud system from its edges, a number of cloud brightness gradient fields have been analyzed in the time domain using McIDAS.

Analysis program designed for this investigation determines the angle between sun, cloud, and satellite, and computes the contrast ratio of the brightest part of the cloud to the rest of it. Analysis of three cloud clusters which remained active throughout the day, and of five clouds which were active only a part of the day in the Caribbean Sea area on 15 September 1969, show the contrast increasing at angle close to back scattering. More data are being processed for different times and locations to improve upon the statistics.

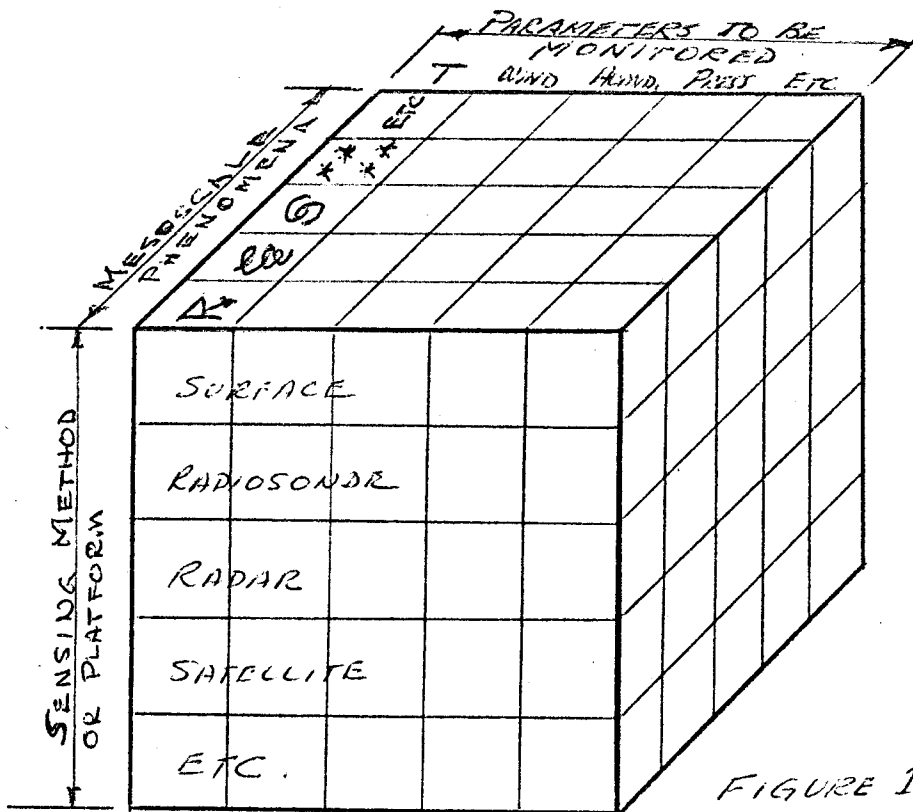
Task E Comparative Studies in Satellite Stability

Appendices 6 and 7 to Das' technical report are attached and can be collected with previously submitted parts of his paper. Coding of the model for the 1108 is progressing satisfactorily.

Task F High Resolution Optics Studies

We have been asked by Dr. William Nordberg to investigate the utility of a large operative telescope in geostationary orbit for surveillance of severe weather and other related meso scale phenomena. Work is progressing from both ends: (1) requirements studies and (2) hardware capabilities studies.

In the requirements study a matrix approach is being used as shown in Figure 1.



Task F High Resolution Optics Studies (Continued)

Extensive literature on mesoscale phenomena, such as the thunderstorm, the tornado, hurricane and so on, have been assembled and summarized. From this the key observables which describe the motion field, thermal field, moisture field, by hydrometeors or other factors will be identified in detail. Since the most powerful observing system to monitor these phenomena will probably be a composite system based on both conventional and space based technology, the observables best handled by each system--and their limitations will be identified. From this extensive series of tables, the key requirements can be drawn up.

During the next period, the main effort will be to carefully identify the key parameters that need to be monitored--irrespective of how one might do it.

New sensing systems, such as the charge coupled line scanner or the "bucket brigade" system which make use of a line of silicon photo diodes or a target plane of them could be ideally suited to high resolution imaging systems from geostationary altitudes. A study showing approach concepts on how these might be used is now underway. Numerous trade-off on size resolution, raster format, and communication bandwidth are possible and are being studied.

TOH/jz

APPENDIX 6

Reduced Equations for the Shell

Equation (1.59) is now substituted into Eq. (1.55), and the terms independent of θ are set equal to zero. This produces the following equation:

$$\begin{aligned}
& \frac{s_B^2 h_B^2}{12a_B^2} \{ [(z_1 - \rho'') (z_1 + \rho')] (p_2^8 + 2p_2^6 + p_2^4) + (2z_1 + \rho' - \rho'') (8p_2^7 + 12p_2^5 + 4p_2^3) \\
& + (72p_2^6 + 60p_2^4 + 12p_2^2)] e^{P_2 z_1} b_2 + [(z_1 - \rho'') (z_1 + \rho')] (p_3^8 + 2p_3^6 + p_3^4) \\
& + (2z_1 + \rho' - \rho'') (8p_3^7 + 12p_3^5 + 4p_3^3) + (72p_3^6 + 60p_3^4 + 12p_3^2)] e^{P_3 z_1} b_3 \} \\
& - \frac{s_B^2 h_B^2}{6a_B^2} (1 - s_B^{\mu_B}) \{ (z_1 - \rho'') (z_1 + \rho') (b_2 p_2^6 e^{P_2 z_1} + b_3 p_3^6 e^{P_3 z_1}) \\
& + 6(2z_1 + \rho' - \rho'') (b_2 p_2^5 e^{P_2 z_1} + b_3 p_3^5 e^{P_3 z_1}) + 30(b_2 p_2^4 e^{P_2 z_1} + b_3 p_3^4 e^{P_3 z_1}) \} \\
& + (1 - s_B^{\mu_B^2}) \{ (z_1 - \rho'') (z_1 + \rho') (b_2 p_2^4 e^{P_2 z_1} + b_3 p_3^4 e^{P_3 z_1}) + 4(2z_1 + \rho' - \rho'') \\
& \cdot (b_2 p_2^3 e^{P_2 z_1} + b_3 p_3^3 e^{P_3 z_1}) + 12(b_2 p_2^2 e^{P_2 z_1} + b_3 p_3^2 e^{P_3 z_1}) \} \\
& + \frac{a_B^2 (1 - s_B^{\mu_B^2}) (s_B^{\rho_B})}{s_B^E \cdot s_B h_B} \left[\frac{d^2}{dt^2} - \omega_{B,3}^2 - \frac{1}{2} (\omega_{B,1}^2 + \omega_{B,2}^2) \right] \{ (z_1 - \rho'') (z_1 + \rho') \\
& \cdot (b_2 p_2^4 e^{P_2 z_1} + b_3 p_3^4 e^{P_3 z_1}) + 4(2z_1 + \rho' - \rho'') (b_2 p_2^3 e^{P_2 z_1} + b_3 p_3^3 e^{P_3 z_1}) \\
& + 12(b_2 p_2^2 e^{P_2 z_1} + b_3 p_3^2 e^{P_3 z_1}) \} + \frac{a_B^2 (1 - s_B^{\mu_B^2}) (s_B^{\rho_B})}{s_B^E \cdot s_B h_B} \left\{ \frac{1}{4} (\omega_{B,1}^2 - \omega_{B,2}^2) \left[\left(\frac{\pi^4}{4} - 32 \right) \frac{1}{\ell_1} \right. \right. \\
& \cdot b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left. \left. (16 \frac{\pi^4}{4} - 32) b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \right. \\
& \left. + \frac{1}{2} \omega_{B,1} \omega_{B,2} \left[\left(\frac{\pi^4}{4} - 82 \right) b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(16 \frac{\pi^4}{4} - 32 \right) b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{48a_B} (1 + s^{\mu_B}) (s^h_B)^2 (\sin \beta) \{ (\sin \omega_{B,3} t) (1 - s^{\tau_B} \frac{d}{dt}) \\
& \cdot \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + 4 \frac{\pi^2}{\ell_1^2}\right)^3 b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right. \\
& + \frac{\pi}{\ell_1} \left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_4 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + \frac{2\pi}{\ell_1} \left(1 + 4 \frac{\pi^2}{\ell_1^2}\right)^3 b_5 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} \left. \right] \\
& + (\cos \omega_{B,3} t) (1 - s^{\tau_B} \frac{d}{dt}) \left[\frac{\pi}{\ell_1} \left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_6 \cos \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& + \frac{2\pi}{\ell_1} \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^3 b_7 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} - \left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \\
& \left. - \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^3 b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \} = 0 \tag{A6.1}
\end{aligned}$$

From Eqs. (1.55) and (1.59), setting the coefficient of $\cos \theta$ equal to zero, we obtain the following equation:

$$\begin{aligned}
& \frac{s^h_B \cdot \pi^4}{12a_B^2 \ell_1^4} \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^2 b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 16 \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^2 b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& - \frac{s^h_B (1 - s^{\mu_B}) \pi^2}{6a_B^2 \ell_1^2} \left[\left(1 - \frac{\pi^4}{\ell_1^4}\right) b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 4 \left(1 - \frac{16\pi^4}{\ell_1^4}\right) b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& + (1 - s^{\mu_B}) \frac{\pi^4}{\ell_1^4} \left[b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 16 b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& + \frac{a_B^2 (1 - s^{\mu_B}) (s^{\rho_B})}{s_B \cdot s_B} \left\{ \left[\frac{d^2}{dt^2} - \omega_{B,3}^2 - \frac{1}{2} (\omega_{B,1}^2 + \omega_{B,2}^2) \right] \right. \\
& \cdot \left. \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^2 b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^2 b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4}(\omega_{B,1}^2 - \omega_{B,2}^2) \left[\left(\frac{\pi^4}{\ell_1^4} + \frac{2\pi^2}{\ell_1^2} - 31 \right) b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& + \left. \left(16 \frac{\pi^4}{\ell_1^4} + 8 \frac{\pi^2}{\ell_1^2} - 31 \right) b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& + \frac{1}{2} \omega_{B,1} \omega_{B,2} \left[\left(\frac{\pi^4}{\ell_1^4} + \frac{2\pi^2}{\ell_1^2} - 31 \right) b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& + \left. \left(\frac{16\pi^4}{\ell_1^4} + 8 \frac{\pi^2}{\ell_1^2} - 31 \right) b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] + f_1 \Big\} \\
& + \frac{1}{12} (1 + s^{\mu_B}) (s^h_B)^2 \sin \beta \left\{ \cos \omega_{B,3} t + \frac{1}{4a_B} (\sin \omega_{B,3} t) (1 - s^{\tau_B} \frac{d}{dt}) \right. \\
& \cdot [2(z_1 - \rho') (z_1 + \rho') (b_2 p_2^7 e^{p_2 z_1} + b_3 p_3^7 e^{p_3 z_1}) + 14(2z_1 + \rho' - \rho'') \\
& \cdot (b_2 p_2^6 e^{p_2 z_1} + b_3 p_3^6 e^{p_3 z_1}) + 84(b_2 p_2^5 e^{p_2 z_1} + b_3 p_3^5 e^{p_3 z_1}) \\
& - 2(4 + \frac{\pi^2}{\ell_1^2})^3 b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} - 128(1 + \frac{\pi^2}{\ell_1^2})^3 b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \\
& - \frac{\pi}{\ell_1} (4 + \frac{\pi^2}{\ell_1^2})^3 b_8 \cos \frac{\pi(z_1 + \rho')}{\ell_1} - 128 \frac{\pi}{\ell_1} (1 + \frac{\pi^2}{\ell_1^2})^3 b_9 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} \Big] \\
& + \frac{1}{4a_B} (\cos \omega_{B,3} t) (1 - s^{\tau_B} \frac{d}{dt}) \left[2(4 + \frac{\pi^2}{\ell_1^2})^3 b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& + 128(1 + \frac{\pi^2}{\ell_1^2})^3 b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} - \frac{\pi}{\ell_1} (4 + \frac{\pi^2}{\ell_1^2})^3 b_{10} \cos \frac{\pi(z_1 + \rho')}{\ell_1} \\
& \left. - 128 \frac{\pi}{\ell_1} (1 + \frac{\pi^2}{\ell_1^2})^3 b_{11} \cos \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \Big\} = 0
\end{aligned} \tag{A6.2}$$

Proceeding similarly, the coefficients of $\sin\theta$ in Eq. (1.55) generate the following equation:

$$\begin{aligned}
& \frac{s_B^2}{12a_B^2} \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^2 \frac{\pi^4}{\ell_1^4} b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + \frac{4\pi^2}{\ell_1^2}\right) \frac{16\pi^4}{\ell_1^4} b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& - \frac{s_B^2(1 - s_B^{\mu_B})}{6a_B^2} \left[\frac{\pi^2}{\ell_1^2} \left(1 - \frac{\pi^4}{\ell_1^4}\right) b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \frac{4\pi^2}{\ell_1^2} \left(1 - \frac{16\pi^4}{\ell_1^4}\right) b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& + (1 - s_B^{\mu_B}) \left[\frac{\pi^4}{\ell_1^4} b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \frac{16\pi^4}{\ell_1^4} b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& + \frac{a_B^2(1 - s_B^{\mu_B})(s_B^{\rho_B})}{s_B^E \cdot s_B^h} \left\{ \left[\frac{d^2}{dt^2} - \omega_{B,3}^2 - \frac{1}{2}(\omega_{B,1}^2 + \omega_{B,2}^2) \right] \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^2 b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \right. \\
& + \left. \left. \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^2 b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] - \frac{1}{4}(\omega_{B,1}^2 - \omega_{B,2}^2) \left[\left(\frac{\pi^4}{\ell_1^4} + 2\frac{\pi^2}{\ell_1^2} - 31\right) b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \right. \\
& + \left. \left. \left(16\frac{\pi^4}{\ell_1^4} + 8\frac{\pi^2}{\ell_1^2} - 31\right) b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] + \frac{\omega_{B,1}\omega_{B,2}}{2} \left[\left(\frac{\pi^4}{\ell_1^4} + 2\frac{\pi^2}{\ell_1^2} - 31\right) \right. \right. \\
& \cdot \left. \left. b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(16\frac{\pi^4}{\ell_1^4} + 8\frac{\pi^2}{\ell_1^2} - 31\right) b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] + f_2 \right\} \\
& - \frac{1}{12} (1 + s_B^{\mu_B}) \left(\frac{h_B^2}{s_B}\right) \sin 8 \left\{ \sin \omega_{B,3} t + \frac{1}{4a_B} (\cos \omega_{B,3} t) (1 - s_B^{\tau_B} \frac{d}{dt}) \right. \\
& \cdot \left. [2(z_1 - \rho'')(z_1 + \rho')] (b_2 p_2^7 e^{P_2 z_1} + b_3 p_3^7 e^{P_3 z_1}) \right. \\
& + \left. 14(2z_1 + \rho' - \rho'') (b_2 p_2^6 e^{P_2 z_1} + b_3 p_3^6 e^{P_3 z_1}) + 84(b_2 p_2^5 e^{P_2 z_1} + b_3 p_3^5 e^{P_3 z_1}) \right.
\end{aligned}$$

$$\begin{aligned}
& + 2(4 + \frac{\pi^2}{\ell_1^2})^3 b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 128(1 + \frac{\pi^2}{\ell_1^2})^3 b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \\
& + \frac{\pi}{\ell_1} (4 + \frac{\pi^2}{\ell_1^2})^3 b_8 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + 128 \frac{\pi}{\ell_1} (1 + \frac{\pi^2}{\ell_1^2})^3 b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1}] \\
& + \frac{1}{4a_B} (\sin \omega_{B,3} t) (1 - s^{\tau_B} \frac{d}{dt}) [2(4 + \frac{\pi^2}{\ell_1^2})^3 b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \\
& + 128(1 + \frac{\pi^2}{\ell_1^2})^3 b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} - \frac{\pi}{\ell_1} (4 + \frac{\pi^2}{\ell_1^2})^3 b_{10} \cos \frac{\pi(z_1 + \rho')}{\ell_1} \\
& - \frac{128\pi}{\ell_1} (1 + \frac{\pi^2}{\ell_1^2})^3 b_{11} \cos \frac{2\pi(z_1 + \rho')}{\ell_1}] \} = 0 . \tag{A6.3}
\end{aligned}$$

The equation obtained from the coefficients of $\cos 2\theta$ is

$$\begin{aligned}
& \frac{s^h_b}{12a_B} [(4 + \frac{\pi^2}{\ell_1^2})^2 (3 + \frac{\pi^2}{\ell_1^2})^2 b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 16(1 + \frac{\pi^2}{\ell_1^2})^2 (3 + \frac{4\pi^2}{\ell_1^2})^2 \\
& \cdot b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1}] - \frac{s^h_B (1 - s^{\mu_B})}{6a_B^2} [\frac{\pi^2}{\ell_1^2} (16 - \frac{4}{\ell_1^4}) b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \\
& + 64 \frac{\pi^2}{\ell_1^2} (1 - \frac{4}{\ell_1^4}) b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1}] + (1 - s^{\mu_B}) [\frac{4}{\ell_1^4} b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \\
& + 16 \frac{\pi^4}{\ell_1^4} b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1}] + \frac{a_B^2 (1 - s^{\mu_B}) (s^{\rho_B})}{s^E_B \cdot s^h_B} \\
& \cdot \left\{ [\frac{d^2}{dt^2} - \omega_{B,3}^2 - \frac{1}{2}(\omega_{B,1}^2 + \omega_{B,2}^2)] [(4 + \frac{\pi^2}{\ell_1^2})^2 b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& \left. + 16(1 + \frac{\pi^2}{\ell_1^2})^2 b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1}] + \frac{1}{4}(\omega_{B,1}^2 - \omega_{B,2}^2) [-32b_1 \right.
\end{aligned}$$

$$\begin{aligned}
& + 2(z_1 - \rho'')(z_1 + \rho')(p_2^4 - 8p_2^2 - 16)b_2 e^{p_2 z_1} + 2(2z_1 + \rho' - \rho'')(4p_2^3 - 16p_2)b_2 e^{p_2 z_1} \\
& + 8(3p_2^2 - 4)b_2 e^{p_2 z_1} + 2(z_1 - \rho'')(z_1 + \rho')(p_3^4 - 8p_3^2 - 16)b_3 e^{p_3 z_1} \\
& + 8(2z_1 + \rho' - \rho'')(p_2^3 - 4p_2)b_3 e^{p_3 z_1} + 8(3p_2^2 - 4)b_3 e^{p_3 z_1} \Big\} \\
& - \frac{1}{48a_B} (1 + s \mu_B) \left(\frac{h_B^2}{s} \right) \sin \beta \left\{ (\sin \omega_{B,3} t) \left(1 - s \tau_B \frac{d}{dt} \right) \right. \\
& \cdot \left[\left(1 + \frac{\pi^2}{\ell_1^2} \right)^3 b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + \frac{4\pi^2}{\ell_1^2} \right)^3 b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right. \\
& + \left. \frac{\pi}{\ell_1} \left(1 + \frac{\pi^2}{\ell_1^2} \right)^3 b_4 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + \frac{2\pi}{\ell_1} \left(1 + \frac{4\pi^2}{\ell_1^2} \right)^3 b_5 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \\
& - (\cos \omega_{B,3} t) \left(1 - s \tau_B \frac{d}{dt} \right) \left[\frac{\pi}{\ell_1} \left(1 + \frac{\pi^2}{\ell_1^2} \right)^3 b_6 \cos \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& + \frac{2\pi}{\ell_1} \left(1 + \frac{4\pi^2}{\ell_1^2} \right)^3 b_7 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} - \left. \left(1 + \frac{\pi^2}{\ell_1^2} \right)^3 b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \\
& \left. \left. - \left(1 + \frac{4\pi^2}{\ell_1^2} \right)^3 b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \right\} = 0 \tag{A6.4}
\end{aligned}$$

From the coefficients of $\sin 2\theta$, we obtain,

$$\begin{aligned}
& \frac{s h_B^2}{12a_B^2} \left[\left(4 + \frac{\pi^2}{\ell_1^2} \right)^2 \left(3 + \frac{\pi^2}{\ell_1^2} \right)^2 b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 16 \left(1 + \frac{\pi^2}{\ell_1^2} \right)^2 \left(3 + \frac{4\pi^2}{\ell_1^2} \right)^2 \right. \\
& \cdot \left. b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] - \frac{s h_B^2 (1 - s \mu_B)}{6a_B^2} \left[\frac{\pi^2}{\ell_1^2} \left(16 - \frac{\pi^4}{4} \right) b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{64\pi^2}{\ell_1^2} \left(1 - \frac{\pi^4}{\ell_1^4}\right) b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} + \frac{a_B^2 (1 - s_B^2) (s_B^{\rho_B})}{s_B^E \cdot s_B^h} \\
& \cdot \left\{ \left[\frac{d^2}{dt^2} - \omega_{B,3}^2 - \frac{1}{2}(\omega_{B,1}^2 + \omega_{B,2}^2) \right] \left[\left(4 + \frac{\pi^2}{\ell_1^2}\right)^2 b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \right. \\
& + 16 \left(1 + \frac{\pi^2}{\ell_1^2}\right)^2 b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} + 2\omega_{B,1}\omega_{B,2} [-16b_1 + (z_1 - \rho'')(z_1 + \rho')] \\
& \cdot (p_2^4 - 8p_2^2 - 16)b_2 e^{P_2 z_1} + 4(2z_1 + \rho' - \rho'')(p_2^3 - 4p_2)b_2 e^{P_2 z_1} \\
& + 4(3p_2^2 - 4)b_2 e^{P_2 z_1} + (z_1 - \rho'')(z_1 + \rho')(p_3^4 - 8p_3^2 - 16)b_3 e^{P_3 z_1} \\
& \left. + 4(2z_1 + \rho' - \rho''')(p_3^3 - 4p_3)b_3 e^{P_3 z_1} + 4(3p_3^2 - 4)b_3 e^{P_3 z_1} \right\} \\
& + \frac{1}{48a_B} (1 + s_B^{\mu_B}) (s_B^h)^2 \sin \beta \left\{ (\sin \omega_{B,3} t) (1 - s_B^{\tau_B} \frac{d}{dt}) \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \right. \right. \\
& + \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^3 b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} - \frac{\pi}{\ell_1} \left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_6 \cos \frac{\pi(z_1 + \rho')}{\ell_1} \\
& - \frac{2\pi}{\ell_1} \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^3 b_7 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} \left. \right] - (\cos \omega_{B,3} t) (1 - s_B^{\tau_B} \frac{d}{dt}) \\
& \cdot \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^3 b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right. \\
& \left. + \frac{\pi}{\ell_1} \left(1 + \frac{\pi^2}{\ell_1^2}\right)^3 b_4 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + \frac{2\pi}{\ell_1} \left(1 + \frac{4\pi^2}{\ell_1^2}\right)^3 b_5 \cos \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \left. \right\} = 0
\end{aligned}$$

(A6.5)

Equation (A6.1) is successively multiplied by 1, $(z_1 - \rho'')(z_1 + \rho')e^{p_2 z_1}$ and $(z_1 - \rho'')(z_1 + \rho')e^{p_3 z_1}$, and integrated between $(-\rho', \rho')$. This gives three equations. The other eight equations are obtained from Eqs. (A6.2) through (A6.5) by taking similar inner products with $\sin \frac{\pi(z_1 + \rho')}{\ell_1}$ and $\sin \frac{2\pi(z_1 + \rho')}{\ell_1}$.

The constants p_2 and p_3 are as given by Kraus [37], in his Eq. (5.17b). Thus

$$p_2 = -p_3 = \left[\frac{3(1 - s_B^2)}{a_B^2 \cdot h_B^2} \right]^{1/4} \quad (\text{A6.6})$$

APPENDIX 7

Shell Loading at the Beam-Shell Junctions

Using Eqs. (1.49), (1.45) and (1.46), Eq. (1.63) is rewritten as

$$q_i = - \frac{s_{E_B} \cdot h_B^3}{12(1-s_{\mu_B}^2)a_B^4} \left\{ \nabla^4 \xi_{B,r} + \frac{a_B^2}{s_{h_B}^2} \xi_{B,r} - (1 + s_{\mu_B}) a_B^2 (s_{\kappa_{B,0}})^2 \cos s_{\beta_B} \right. \\ \left. - \frac{(1+s_{\mu_B})a_B^2}{2} \nabla^2 [s_{\kappa_{B,0}} (\sin s_{\beta_B}) (1 - s_{\tau_B} \frac{d}{dt}) (\frac{\partial}{\partial \theta} + \frac{\partial}{\partial z}) \xi_{B,r}] \right\}_{\substack{z_1 = \rho_i \\ \theta = \theta_i}}$$

In the above equation, the form of $\xi_{B,r}$ given by Eq. (1.59) is substituted. Hence

$$q_i = - \frac{s_{E_B} \cdot h_B^3}{12(1-s_{\mu_B}^2)a_B^4} \left\{ (z_1 - \rho'') (z_1 + \rho') (b_2 p_2^4 e^{P_2 z_1} + b_3 p_3^4 e^{P_3 z_1}) \right. \\ + 4(2z_1 + \rho' - \rho'') (b_2 p_2^3 e^{P_2 z_1} + b_3 p_3^3 e^{P_3 z_1}) + 12(b_2 p_2^2 e^{P_2 z_1} + b_3 p_3^2 e^{P_3 z_1}) \\ + [(1 + \frac{\pi^2}{2})^2 b_4 \sin \frac{\pi(z_1 + \rho')}{l_1} + (1 + \frac{4\pi^2}{2})^2 b_5 \sin \frac{2\pi(z_1 + \rho')}{l_1}] \cos \theta \\ + [(1 + \frac{\pi^2}{2})^2 b_6 \sin \frac{\pi(z_1 + \rho')}{l_1} + (1 + \frac{4\pi^2}{2})^2 b_7 \sin \frac{2\pi(z_1 + \rho')}{l_1}] \sin \theta \\ + [(4 + \frac{\pi^2}{2})^2 b_8 \sin \frac{\pi(z_1 + \rho')}{l_1} + 16(1 + \frac{\pi^2}{2})^2 b_9 \sin \frac{2\pi(z_1 + \rho')}{l_1}] \cos 2\theta \\ + [(4 + \frac{\pi^2}{2})^2 b_{10} \sin \frac{\pi(z_1 + \rho')}{l_1} + 16(1 + \frac{\pi^2}{2})^2 b_{11} \sin \frac{2\pi(z_1 + \rho')}{l_1}] \sin 2\theta$$

$$\begin{aligned}
& + \frac{a_B^2}{s_B^2} [b_1 + (z_1 + \rho')(z_1 - \rho'') (b_2 e^{P_2 z_1} + b_3 e^{P_3 z_1}) \\
& + b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \cos \theta + b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \cos \theta \\
& + b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \sin \theta + b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \sin \theta \\
& + b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} \cos 2\theta + b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \cos 2\theta \\
& + b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} \sin 2\theta + b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \sin 2\theta] \\
& - \frac{1}{2} (1 + s_B \mu_B) a_B^2 (s_B \kappa_{B,0}) \sin \beta [2 \cos(\omega_{B,3} t + \theta) \\
& + \sin(\omega_{B,3} t + \theta) (1 - s_B \tau_B \frac{d}{dt}) \{ (z_1 - \rho'')(z_1 + \rho') (b_2 P_2^3 e^{P_2 z_1} + b_3 P_3^3 e^{P_3 z_1}) \\
& + 3(2z_1 + \rho' - \rho'') (b_2 P_2^2 e^{P_2 z_1} + b_3 P_3^2 e^{P_3 z_1}) + 6(b_2 P_2 e^{P_2 z_1} + b_3 P_3 e^{P_3 z_1}) \\
& - \frac{\pi}{\ell_1} [(1 + \frac{\pi^2}{\ell_1^2}) b_4 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + 2(1 + \frac{4\pi^2}{\ell_1^2}) b_5 \cos \frac{2\pi(z_1 + \rho')}{\ell_1}] \cos \theta \\
& - \frac{\pi}{\ell_1} [(1 + \frac{\pi^2}{\ell_1^2}) b_6 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + 2(1 + \frac{4\pi^2}{\ell_1^2}) b_7 \cos \frac{2\pi(z_1 + \rho')}{\ell_1}] \sin \theta \\
& - \frac{\pi}{\ell_1} [(4 + \frac{\pi^2}{\ell_1^2}) b_8 \cos \frac{\pi(z_1 + \rho')}{\ell_1} + 8(1 + \frac{\pi^2}{\ell_1^2}) b_9 \cos \frac{2\pi(z_1 + \rho')}{\ell_1}] \cos 2\theta \\
& - \frac{\pi}{\ell_1} [(4 + \frac{\pi^2}{\ell_1^2}) b_{10} \cos \frac{\pi(z_1 + \rho')}{\ell_1} + 8(1 + \frac{\pi^2}{\ell_1^2}) b_{11} \cos \frac{2\pi(z_1 + \rho')}{\ell_1}] \sin 2\theta
\end{aligned}$$

$$\begin{aligned}
& + \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right) b_4 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + \frac{4\pi^2}{\ell_1^2}\right) b_5 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \sin \theta \\
& - \left[\left(1 + \frac{\pi^2}{\ell_1^2}\right) b_6 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + \left(1 + \frac{4\pi^2}{\ell_1^2}\right) b_7 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \cos \theta \\
& + 2 \left[\left(4 + \frac{\pi^2}{\ell_1^2}\right) b_8 \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 4 \left(1 + \frac{\pi^2}{\ell_1^2}\right) b_9 \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \sin 2\theta \\
& - 2 \left[\left(4 + \frac{\pi^2}{\ell_1^2}\right) b_{10} \sin \frac{\pi(z_1 + \rho')}{\ell_1} + 4 \left(1 + \frac{\pi^2}{\ell_1^2}\right) b_{11} \sin \frac{2\pi(z_1 + \rho')}{\ell_1} \right] \cos 2\theta \Bigg\} \\
& \hspace{20em} \text{(A7.1)}.
\end{aligned}$$

The above equation is to be evaluated at $z_1 = \rho_i$ and $\theta = \theta_i$.