VISSR ATMOSPHERIC SOUNDER

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I. Introduction

Work during the past month has been directed at reviving and improving the SMS-VAS simulator program and at improving spin budget calculations. In the first area revised instrument specs and improved methods for spatial weighting function calculations are being incorporated. Details of this effort will be reported on later. In the second area an effort has been made to resolve the differences between University of Wisconsin and Santa Barbara Research Center spin budget estimates and to update University of Wisconsin estimates with revised transmission estimates.

II. Comparison of UW and SBRC Calculations of Improvement Factors

1. UW Method

The error is the mean of a set of N samples of detector output is reduced below that of a single sample. However, since detector noise is not completely uncorrelated the reduction is less than $N^{-1/2}$. The exact value of the noise reduction is expressed as the ratio σ_M/σ where σ_M is the standard deviation of the means of N samples and σ is the standard deviation of single samples. It is straightforward to show that this ratio is related to the auto covariance function $C(\tau)$ through the relation

$$\frac{\sigma_M^2}{\sigma^2} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{i=j}^{N} \frac{1}{\sigma^2} C\,(t_i - t_j) \tag{1}$$

where

$$t_i = \text{the time of the } i^{th} \text{ sample,} \tag{2}$$

$$C(\tau) = \int_0^\infty P(f)\ \cos(2\pi f \tau)\ T(f)\ df, \tag{3}$$

$T(f)$ = power transfer function of the electronic filter, and (4)

$P(f)$ = detector noise power spectral density at frequency f. (5)

Calculation of $C(\tau)$ using equation (3) is greatly simplified by expanding the filter function as a series of bandpass filters, i.e.

$$T(f) \approx \sum_{k=1}^{n} W_k T_k(f), \text{ where} \tag{6}$$

$$T_k(f) = \begin{cases} o & f<f_{1,k} \text{ or } f>f_{2,k} \\ 1 & f_{1,k} \leq f \leq f_{2,k} \end{cases} \tag{7}$$

Specific values for W_k and $f_{1,k}$, $f_{2,k}$ can be found in previous reports. The detector noise power spectral density is assumed to have the form

$$P(f) = K\,(1 + \frac{f_c}{f}) \tag{8}$$

where f_c is the crossover frequency at which g-r and 1/f noise components are equal.

Inserting (8) and (6) into 3 yields

$$C(\tau) = K \sum_{k=1}^{n} W_k \int_{f_{1,k}}^{f_{2,k}} (1 + \frac{f_c}{f}) \cos(2\pi f\tau)\, df, \text{ and} \tag{9}$$

$$\sigma^2 = K\, \Delta f_N = K \sum_{k=1}^{n} W_k \,[f_{2,k} - f_{1,k} + f_c \ln \frac{f_{2,k}}{f_{1,k}}]. \tag{10}$$

Thus $C(\tau)/\sigma^2$ is independent of the constant K.

2. SBRC Method

The SBRC Method is an approximation which assumes that the effects of digitally averaging sequential samples can be simulated by an appropriate low pass filter. If α is the linear angular geometrical resolution of the detector, α_s the angular spacing between consecutive samples, and $\alpha_M = N\alpha_s$ the angular distance covered by N consecutive samples, the equivalent low pass filter cutoff is assumed to be

$$f_M = \frac{\alpha}{N\alpha_s} f_{3DB} = \frac{\alpha}{\alpha_M} f_{3DB}, \tag{11}$$

where f_{3DB} is the upper half power point of $T(f)$.

Given f_M the ratio of variances is calculated to be

$$\frac{\sigma_M^2}{\sigma^2} = \frac{(\Delta f_N)_M}{(\Delta f_N)_s}, \tag{12}$$

where M refers to the mean of successive samples,

$$(\Delta f_N)_M = f_M - f_L + f_c \ln \frac{f_M}{f_L}, \tag{13}$$

$$(\Delta f_N)_s = f_{3DB} - f_L + f_c \ln \frac{f_{3DB}}{f_L}, \tag{14}$$

and where f_L is the effective low frequency cutoff of the DC restore filter.

Two problems with this approach are: (1) a digital average is poorly represented by a low pass filter, and (2) equation (11) is not a good estimate of f_M.

3. Digital Averaging as a Frequency Filter

Let the noise signal at frequency f and time t be represented by

$$e(f,t,\phi) = A(f) \cos (2\pi ft + \phi) \tag{15}$$

where the phase ϕ is a random number uniformly distributed over the interval $o \leq \phi < 2\pi$. The time average of this error signal over the interval τ is

$$\bar{e}(f,\phi) = \frac{1}{\tau} \int_o^\tau e(f,t,\phi)\, dt = \frac{A(f)}{2\pi f\tau} [\sin(2\pi f\tau + \phi) - \sin \phi] \tag{16}$$

for each phase ϕ. The average noise power is obtained by averaging $\bar{e}(f,\phi)^2$ over ϕ, i.e.

$$\bar{e}(f)^2 = \frac{1}{2\pi} \int_o^{2\pi} \bar{e}(f,\phi)^2 d\phi = \frac{A^2(f)}{(2\pi f\tau)^2}[1 - \cos 2\pi f\tau]. \tag{17}$$

Expanding $\cos 2\pi f\tau$ yields the expression $\cos 2x = 1 - 2\sin^2 x$

$$\bar{e}(f)^2 = \frac{1}{2}A^2(f) \left[\frac{\sin \pi f\tau}{\pi f\tau}\right]^2. \tag{18}$$

Since $P(f) = \frac{1}{2}A^2(f)$ we can rewrite (18) as

$$\overline{e}(f)^2 = P(f)\ [\frac{\sin\pi f\tau}{\pi f\tau}]^2, \quad (19)$$

where $\overline{e}(f)^2$ is the noise power spectral density of signals averaged over the time interval τ. For $\tau = o$, $\overline{e}(f)^2 = P(f)$. The total noise power in a mean over the time τ is given by

$$\sigma_M^2 = \int_o^\infty T(f)\overline{e}(f)^2\, df = \int_o^\infty T(f)P(f)\ [\frac{\sin\pi f\tau}{\pi f\tau}]^2 df \quad (20)$$

In order to reduce (20) to the form of equation (12) we must approximate to $\sin^2 x/x^2$ factor by an equivalent ideal low pass filter. If we choose the cutoff frequency of this filter f_M to yield the same information bandwidth as $\sin^2 x/x^2$, then f_M is given by

$$f_M = \int_o^\infty [\frac{\sin\pi f\tau}{\pi f\tau}]^2\, df = \frac{1}{2\tau}. \quad (21)$$

Figure 1 displays an example of $(\sin \pi f\tau/\pi f\tau)^2$ for $\tau = 4 \times 10^{-4}$ sec which corresponds to a scan length of 150 Km at the VAS subsatellite point. Also indicated are equivalent low pass filters calculated by equation (11) (SBRC) and equation (21) (UW). Note that SBRC and UW values for f_M differ substantially. For the given example we find

$$(f_M)_{SBRC} = \frac{.384\ mr}{4.190\ mr} \times f_{3DB} = 2.38\ KHz \quad (22)$$

$$(f_M)\ UW = \frac{1}{8 \times 10^{-4} sec} = 1.25\ KHz \quad (23)$$

where $f_{3DB} = 26\ KH_z$ is the upper 3DB frequency of the VAS electronics filter. Inserting these values of f_M into equation (13) yield the results

$$(\Delta f_N)_{M,SBRC} = 7.69\ KHz \quad (24)$$

$$(\Delta f_N)_{M,UW} = 6.08\ KHz \quad (25)$$

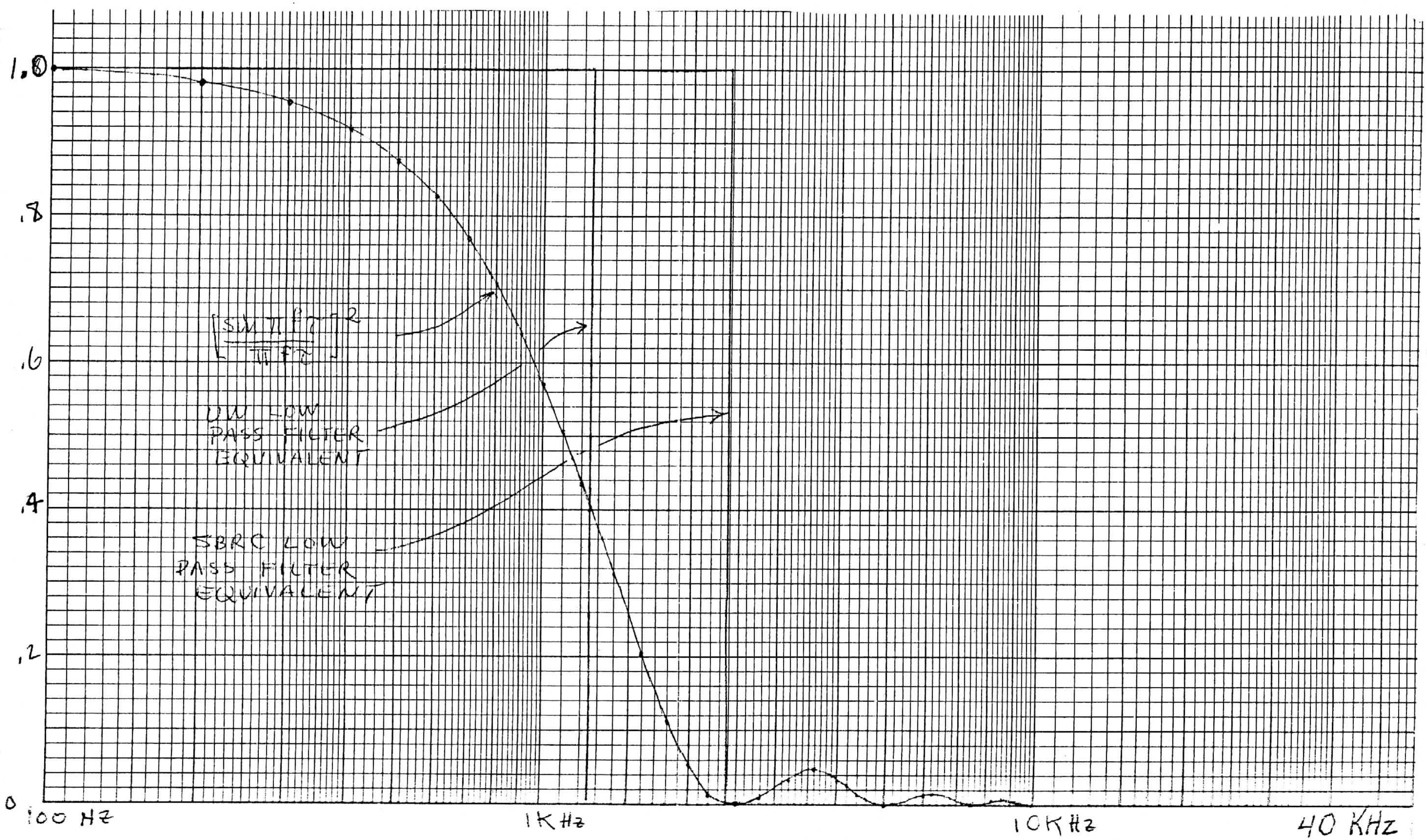

FIGURE 1. FREQUENCY TRANSFORM OF DIGITAL AVERAGING FOR $\tau = 4 \times 10^{-4}$ sec (150 Km) AND LOW PASS FILTER APPROXIMATIONS

where we have assumed the values

$$f_L = 2 \text{ Hz}, \; f_c = 750 \text{ Hz}. \tag{26}$$

Since equation (14) yields

$$(\Delta f_N)_s = 33.1 \text{ KHz}, \tag{27}$$

we find that, for a linear 150 Km average,

$$\left(\frac{\sigma_M^2}{\sigma^2}\right)_{SBRC} = 0.232 \tag{28}$$

$$\left(\frac{\sigma_M^2}{\sigma^2}\right)_{UW} = 0.184 \tag{29}$$

and for a 150 Km x 150 Km average both results are reduced by a factor of 11 (the number of scan lines which must be averaged). This results in improvement factors of

$$\left(\frac{\sigma}{\sigma_M}\right)_{150 \times 150,\ SBRC} = 6.89 \tag{30}$$

$$\left(\frac{\sigma}{\sigma_M}\right)_{150 \times 150,\ UW} = 7.73 \tag{31}$$

where both factors are calculated by means of a low pass filter approximation to digital averaging. The next section demonstrates that this approximation always underestimates the improvement factors.

4. Direct Derivation of the Low Pass Filter Approximation

Recalling the exact expressions

$$\sigma_M^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} C\,(t_i - t_j), \text{ and} \tag{32}$$

$$C(\tau) = \int_0^\infty T(f)P(f) \cos 2\pi f\tau \, df, \tag{33}$$

we shall approximate the double sum in (32) by the double integral

$$\sigma_M^2 \approx \frac{1}{\tau^2} \int_0^\tau dt \int_0^\tau dt' \; C(t-t') \tag{34}$$

where τ is the time corresponding to N sequential samples. Since the sampling rate is significantly higher than f_{3DB} of T(f), this is a good approximation.

Since C(t-t') depends only (t-t') the double integral can be converted to the single integral

$$\sigma_M^2 = \frac{2}{\tau} \int_0^\tau C(\eta) \; (1 - \frac{\eta}{\tau}) \; d\eta. \tag{35}$$

Inserting (33) into (35) and interchanging the order of integrations yields

$$\sigma_M^2 = \frac{2}{\tau} \int_0^\infty T(f)P(f) \; [\int_0^\tau \cos 2\pi f\eta \; (1 - \frac{\eta}{\tau}) d\eta] df. \tag{36}$$

Since the η integration yields

$$\int_0^\tau \cos 2\pi f\eta \; (1 - \frac{\eta}{\tau}) d\eta = \frac{1}{\tau} \frac{[1 - \cos 2\pi f\tau]}{(2\pi f)^2}, \tag{37}$$

equation (36) reduces to

$$\sigma_M^2 = \int_0^\infty T(f)P(f) \; [\frac{\sin\pi f\tau}{\pi f\tau}]^2 df, \tag{38}$$

which duplicates the result given by equation (20). If we replace T(f) by its equivalent ideal bandpass filter with cutoff frequencies $f_1 = f_c$, $f_2 = f_{3DB}$ (a reasonably accurate approximation), we can write (38) in the form

$$\sigma_M^2 = \int_{f_1}^{f_2} P(f) \; [\frac{\sin\pi f\tau}{\pi f\tau}]^2 df. \tag{39}$$

Since P(f) is a monotonically decreasing function of f and $(\sin x/x)^2$ is always positive we conclude that

$$\int_{f_1}^{\frac{1}{2\tau}} P(f) df > \int_{f_1}^{f_2} P(f) \; [\frac{\sin\pi f\tau}{\pi f\tau}]^2 df. \tag{40}$$

Thus the low pass filter approximation will always yield more variance for a given average than the exact method, i.e. the exact method always yields larger improvement factors than the low pass filter approximation.

III. Revised Spin Budget Calculations

Relative standard deviations calculated by the exact method are presented in Table 1 for the averaging areas of interest. (The improvement factor is the reciprocal of the relative standard deviation σ_M/σ).

Table 1: Standard Deviation for Integrating Squares Relative to Standard Error of a Single Sample

SIZE OF INTEGRATING SQUARE (Km x Km)	LINES x SAMPLES AVERAGED	RELATIVE STANDARD DEVIATIONS f_c= 750Hz	f_c=10Hz
27.5 x 27.0	2 x 9	.706	.639
151.2 x 150.0	11 x 50	.119	.066

New transmission values obtained from SBRC, new single sample NEN values and corresponding values of required spins are presented in Table 2. The spin requirement is obtained from single sample NEN values, the results of Table 1, and the required NEN values at stated resolutions.

Table 2. Revised Spin Budget Estimates

BAND (cm^{-1})	τ_ν	NEN FOR LARGE DETECTOR (Ergs/etc.)	AVERAGING RESOLUTION	NEN @ RES.	REQUIRED NEN	SPINS REQUIRED
680	.22	4.7	150 Km	.56	.25	5
692	.27	2.4	30 Km	1.0	.25	16
703	.31	2.0	30 Km	.84	.25	11
715	.35	1.4	30 Km	.59	.25	6
745	.35	1.5	30 Km	.63	.25	6
760	.35	1.5	30 Km	.63	.25	6
790	.35	1.6	30 Km	.67	.25	7
895	.39	.20	30 Km	.08	.25	1
1380	.35	1.8	30 Km	.76	.15	26
1490	.34	.53	30 Km	.22	.10	5
2335	.24	.05	150 Km	.003	.002	2
2680	.26	.006	30 Km	.002	.002	1
					TOTAL SPINS	92