EFFECTS OF LARGE TELESCOPE TEMPERATURE GRADIENTS ON VAS CALIBRATION ACCURACY

Ву

L. A. Sromovsky

CONTRACT NO. NAS5-21965

for

National Aeronautics and Space Administration Goddard Space Flight Center Glen Dale Road Greenbelt, Maryland 20771

4 September 1975

The University of Wisconsin
Space Science and Engineering Center
1225 West Dayton Street
Madison, Wisconsin 53706

TABLE OF CONTENTS

1.	Introduction	1
2.	Radiative Model of the VAS Telescope	1
3.	Parameterization of the Internal Radiance References	4
4.	A Lumped Description of the VAS Instrument	6
5.	Description of the Two Methods for In-Flight VAS Calibration	8
6.	Relative Sensitivities to Errors	10
7.	Combined Effects of Random Errors	13
8.	Effects of Systematic Errors	17
9.	Conclusions and Recommendations	19

Effects of Large Telescope Temperature Gradients on VAS Calibration Accuracy

1. Introduction

Since the launch of SMS-A it has been found from in-orbit measurements that VISSR telescope temperature gradients greatly exceed what was previously thought to be the worst case condition. It is apparent that the VAS in-orbit gradients will be, without any thermal redesign, similar to those measured for the VISSR.

We could thus expect temperature gradients two to three times larger than those of the day 172 case which has so far served as the extreme case in all the VAS calibration analyses. The impact of such large temperature gradients on the VAS calibration accuracy is sufficiently great to warrant a serious effort to achieve better thermal control of the VAS, probably through a redesign of the thermal interface between the VAS and the spacecraft.

In addition to presenting the results of the calibration study of large telescope gradients, this report also updates the UW calibration model to exclude many optical parameters which are no longer relevant. A specific example is the elimination of field lens parameters which are no longer significant because of the repositioning of the calibration target in front of the field lens.

2. Radiative Model of the VAS Telescope

Radiation entering the relay exit pupil during external target measurements is a weighted average of radiation from the target (attenuated by the telescope) and radiation from optical components. If N_{T} is the

radiance of the target and $B(T_i)$ is the blackbody radiance of the ith optical component within the relay exit pupil, then the average radiance within the relay exit pupil can be expressed as:

$$N_{E} = \tau N_{T} + \sum_{i=1}^{6} a_{i} B(T_{i})$$

$$(1)$$

where

$$\tau = 1 - \sum_{i=1}^{6} a_i \tag{2}$$

and $B(T_i)$ is the Planck radiance at temperature T_i . The six effective emissivity coefficients a_i and the corresponding component temperatures T_i are identified in Table 1. A schematic diagram of the VAS telescope is presented in Figure 1.

TABLE 1. Components of the VAS Telescope Radiative Model

<u>i</u>	COMPONENT NAME	TEMPERATURE	a _i _
1	scan mirror	T ₁	$(1-R_1)R_2R_3(1-K_4-K_6-K_7)$
2	primary mirror	^T 2	$(1-R_2)R_3(1-K_4-K_6-K_7)$
3	secondary mirror	^T 3	$(1-R_3)(1-K_7)$
4	central obscuration	^T 4	K4R3
5	primary mask	^T 5	^K 6 ^R 3
6	secondary mask	^T 6	K ₇

Parameters used to describe the a coefficients are defined below (see Figure 1).

 R_1 , R_2 , R_3 = reflectivities of the scan mirror, primary mirror, and secondary mirror respectively

 K_4 = ratio of the central obscuration solid angle to the exit pupil solid angle

 K_6 = fraction of the exit pupil solid angle obscured by the primary mirror mask

 K_7 = fractional solid angle obscured by the secondary mirror mask



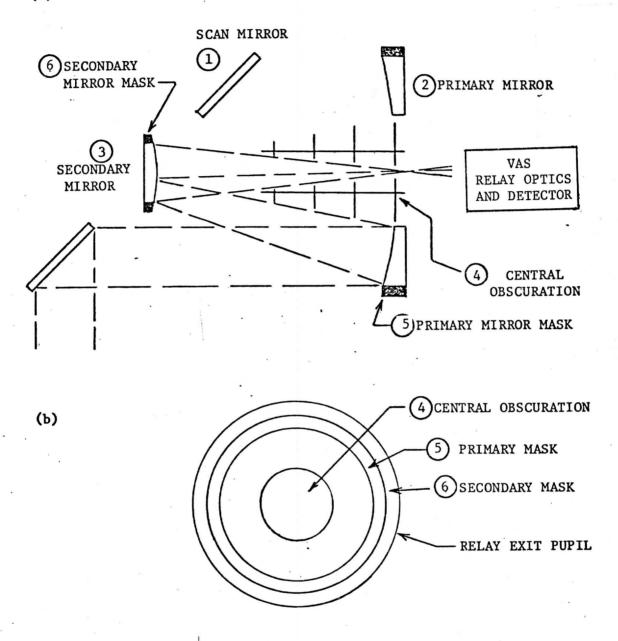


Figure 1. Schematic Diagram of VAS Telescope (a) and Obscurations within the Relay System Field of View (b)

Nominal values assumed for the fundamental optical constants are:

$$R_1$$
, R_2 , $R_3 = 0.96$,
 $K_4 = 0.131$,
 $K_6 = 0.060$, and
 $K_7 = 0.121$.

Given the specific values stated in (3) we find that the total fractional obscuration is:

$$K_4 + K_6 + K_7 = 0.312$$
 (4)

and the net telescope transmission is:

$$\tau = R_1 R_2 R_3 (1 - K_4 - K_6 - K_7) = 0.6087.$$
 (5)

3. Parameterization of the Internal Radiance References

The most general description of an internal radiation reference for the VAS is indicated in Figure 2. The basic components of the reference system are: (1) a blackbody cavity and (2) an oscillating reflective shutter. The parameters describing this system are:

 ϵ_c = emissivity of the blackbody cavity

 $T_c = temperature of the cavity.$

 R_{s} = reflectivity of the shutter

 $T_s = temperature of the shutter$

 T_{B} = mean effective radiating temperature corresponding to the reference radiance.

The general expression for the radiance input (N_c) to the VAS detector system when the shutter is in view is given by the following expression:

$$N_{c} = (1-R_{s})B(T_{s}) + R_{s}[\varepsilon_{c}B(T_{c}) + (1-\varepsilon_{c})B(T_{L})]$$
(6)

where ${\tt B}({\tt T}_{L})$ is the average ambient temperature of the VAS aft optics cavity.

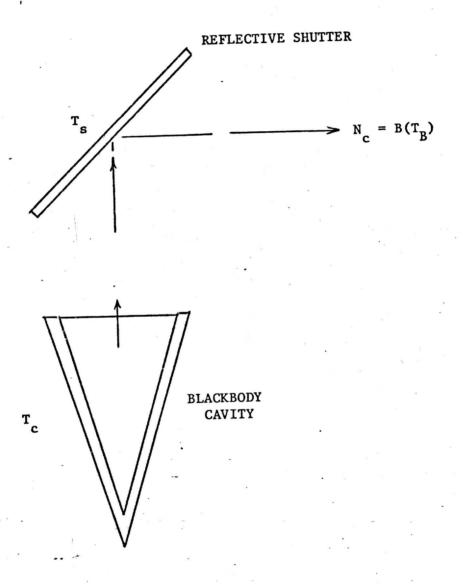


Figure 2. Components of a Generalized VAS Internal Radiance Reference. $T_{\rm B}$ is the Effective Blackbody Radiating Temperature of the Complete Reference System.

4. A Lumped Description of the VAS Instrument

A condensed description of the VAS and its radiation sources and targets is presented in Figure 3. The three qualitatively different measurements which must be considered in the VAS calibration are described in terms of lumped parameters in the following list.

VIEW SIGNAL OUTPUT

space through
the VAS telescope
$$V_1 = \alpha(1-\tau)B(T_A) + V_o$$
 (7)

internal
$$V_2 = \alpha B(T_B) + V_o$$
 (8)

external target
$$V_4 = \alpha[\tau N_T + (1-\tau)B(T_A)] + V_o$$
 (9)

The parameters used in expressions for the VAS signal output are defined as follows:

 α = responsivity of the VAS detector system (volts/unit) radiance.

 τ = transmission of the VAS telescope (equation 6).

 T_A = weighted average temperature of the VAS telescope.

T_B = weighted average radiating temperature of the VAS internal reference.

B(T) = Planck spectral radiance of a blackbody at temperature T (dependence on wavelength is implicit).

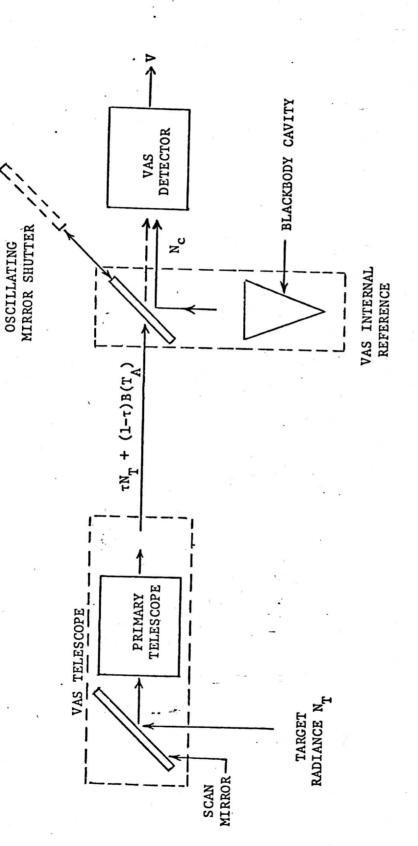
The weighting used to obtain $\boldsymbol{T}_{\boldsymbol{\Delta}}$ is defined by the condition

$$(1-\tau)B(T_{A}) = \sum_{i=1}^{6} a_{i}B(T_{i}).$$
 (10)

The basic calibration equation relating target radiance to line-byline voltage measurements (voltages measured for every scan line) is just

$$N_{T} = B(T^{*}) (V_{4} - V_{1}) / (V_{2} - V_{1}), \tag{11}$$

where, in this case, V_2 is measured for T_c at ambient. The formal expression for $B(T^*)$, the radiance of an external blackbody at temperature T^* which would



= Space, earth, or equivalent external blackbody.

= Scan mirror and primary telescope.

VAS TELESCOPE

TARGET

VAS DETECTOR

= Relay optics, detectors, and electronics. Final output is a voltage signal V. VAS INTERNAL REFERENCE = High emissivity cone and oscillating shutter used to insert a known radiance level into the VAS detectors.

Figure 3. Lumped Description of the VAS Radiometer

produce the same response as the internal blackbody at temperature T_c , is obtained from equations (7) through (9), i.e.

$$B(T^*) = \frac{1}{\tau} [B(T_L) - (1-\tau)B(T_A)]. \tag{12}$$

Note that since this quantity is only used while the cavity is at ambient temperature \mathbf{T}_L , the parameter $\mathbf{B}(\mathbf{T}_B)$ has been replaced by the simple form

$$B(T_B) = B(T_L) \tag{13}$$

where we have justifiably assumed that

$$T_s = (T_c)_{ambient} = T_L.$$
 (14)

In the subsequent analysis only the B(T*) factor of equation (11) will be considered in estimating calibration errors (the voltage factors have errors which are more properly described as measurement errors). Furthermore, equation (12) will be used exclusively to define B(T*).

5. Description of the Two Methods for In-Flight VAS Calibration

In general we must consider two different sets of calibration measurements taken at different times and at different internal reference radiances. In both approaches there are calibration measurements made during every scan line while the cavity is at ambient temperature. In the case of Method 1 (the original VAS calibration approach) these are the only in-flight calibration measurements made. In the case of Method 2 (using an internal high temperature reference) there is another set of measurements made at a different time using a non-ambient reference in addition to the ambient reference. The first set of measurements is obviously made very frequently, while the second set is made only occasionally. Thus, the second set can in general be affected by different parameter values than the first. Both sets are defined below.

TIME STATUS	VIEW	SIGNAL OUTPUT	
CURRENT (LINE-BY-LINE)	Space (through telescope)	$V_1 = \alpha(1-\tau)B(T_A) + V_0$	(15)
	Ambient Reference	$V_2 = \alpha B(T_L) + V_o$	(16)
	Space (through telescope)	$V_1' = \alpha'(1-\tau')B(T_A') + V_0$	(17)
OCCASIONAL	Ambient Reference	$V_2' = \alpha' B(T_L') + V_0'$	(18)
	Non-Ambient Reference	$V_3' = \alpha' B(T_N') + V_0'$	(19)

For Method 1 (ambient blackbody) only current measurements are available, i.e. V_1 , V_2 and component temperatures. Estimated quantities are a_i , i=1, 6 and $\tau=1-\frac{6}{i=1}a_i$. The solution for B(T*) is found using equation (12) and the estimated quantities, i.e.

Method 1:
$$B(T_1^*) = \frac{1}{\tau} [B(T_L) - (1-\tau)B(T_A)]$$
 (20)

where $B(T_A)$ is given by equation (10) in terms of the a_i and T_i . the subscript on T_1^* is used to denote the T^* value derived by Method 1. $B(T_1^*)$, V_1 and V_2 are then inserted in equation (11).

For Method 2 both current and occasional measurements are used. The occasional measurements are used to derive an improved estimate of τ^{\prime} which has the formal solution

$$\tau' = 1 - \frac{B(T_L')}{B(T_A')} + \frac{V_2' - V_1'}{V_2' - V_3'} \cdot \frac{B(T_L') - B(T_N')}{V(T_A')}. \tag{21}$$

Since in practice all terms in equation (21) will have either measurement or estimation errors (or both) it is useful to define a new parameter γ :

 $\gamma \equiv (\tau')_{estimated with equation (21).}$

The corresponding value of T^* is determined by inserting the γ value in equation (12) in place of τ , i.e.

Method 2:
$$B(T_2^*) = \frac{1}{\gamma_2} [B(T_L) - (1-\gamma_2)B(T_A)].$$
 (23)

It should be noted that although we have allowed for $\alpha' \neq \alpha$, it is assumed that the optical constants are unchanged. Cases for which $a_i' \neq a_i$ and thus $\tau^{\, {}^{}} \neq \tau$ are not dealt with in this analysis. The average temperatures are obtained using the same optical parameters in all cases, i.e.

$$B(T_{A}) = \begin{bmatrix} c & c & c \\ c & 1 & c \end{bmatrix} = \begin{bmatrix} c & c & c \\ c & 1 & c \end{bmatrix} = \begin{bmatrix} c & c & c \\ c & 1 & c \end{bmatrix} = \begin{bmatrix} c & c & c \\ c & 1 & c \end{bmatrix} = \begin{bmatrix} c & c & c \\ c & 1 & c \end{bmatrix} = \begin{bmatrix} c & c & c \\ c & 1 & c \\ c & 1 & c \end{bmatrix} = \begin{bmatrix} c & c & c \\ c & 1 & c \\ c &$$

$$B(T'_{A}) = \begin{bmatrix} 6 \\ 1 = 1 \end{bmatrix} a_{i} B(T'_{i}) / \begin{bmatrix} 6 \\ 1 = 1 \end{bmatrix} a_{i}.$$
 (25)

Relative Sensitivities to Errors

In determining T_{1}^{\star} and T_{2}^{\star} a large number of parameters must be either estimated or measured. A complete list is provided below: ESTIMATED OPTICAL CONSTANTS $R_{1}, R_{2}, R_{3}, K_{4}, K_{6}, K_{7}, R_{5}, \varepsilon_{c}$

OCCASIONAL TEMPERATURE

$$T_{i}'$$
 (i=1,6), T_{c}', T_{s}', T_{L}'

CURRENT TEMPERATURE

MEASUREMENTS

MEASUREMENTS

$$T_i$$
 (i=1,6), T_L

OCCASIONAL VOLTAGE

$$v_1', v_2', v_3'$$

 v_1', v_2', v_3' MEASUREMENTS

Since all of these parameters will contain errors there will also be errors in T_1^* and T_2^* . If x_k denotes the k^{th} parameter of the 27 just tabulated then the error in T_1^* , T_2^* , or T_3^* can be obtained from the following expressions

SYSTEMATIC ERROR
$$\delta T_{j}^{*} = \sum_{k=1}^{27} \frac{\partial T_{j}^{*}}{\partial x_{k}} \delta x_{k}$$
 (26)

RANDOM ERROR
$$\sigma T_{\mathbf{j}}^* = \begin{bmatrix} 27 & \frac{\partial T_{\mathbf{j}}^*}{\partial \mathbf{x}_{\mathbf{k}}} & 2 & 2 & 1/2 \\ \frac{\Sigma}{\partial \mathbf{x}_{\mathbf{k}}} & \sigma \mathbf{x}_{\mathbf{k}} & 1 & 1/2 \end{bmatrix}$$
(27)

where $\delta x_k^{}$ and $\sigma x_k^{}$ are systematic and random errors respectively in the parameter $x_k^{}$ and j takes on the values 1 and 2.

The partial derivatives have been numerically evaluated for Method 1 using equation (20) and for Method 2 using equation (23). Results for both methods each for three different temperature gradient conditions are presented in Table 3. These results are valid only for the specific temperature and parameter values identified. The temperature conditions studied are listed in Table 3 (the primed values are only used in Method 2).

The day 172 temperature gradients are a worst case condition based on VISSR thermal model calculations. The other two cases, which have two and three times the day 172 gradients, are more nearly representative of the in-orbit measurements for SMS-A.

An additional implicit parameter for these calculations is the wave number ν . Although results in Table 3 apply for ν = 680 cm⁻¹, the temperature equivalent errors will not change significantly for other VAS spectral intervals. Assumed responsivity values are as follows

 α = 0.024 V/(erg/etc.) α' = 0.012 V/(erg/etc.) (28) Several general characteristics of the derivative values listed in

Table 4 should be noted: (1) for both methods the significance of the telescope optical parameter (k=1,6) increases almost linearly with the size of the temperature gradients; (2) for both methods the temperatures used on a line by line basis (k=9,15) do not significantly change their influence on T* for any of the gradients for either method; and (3) all the parameters peculiar to method 2 (k=7,8; k=16,27) do show a roughly linear increase in influence with increasing gradients, all resulting from the increasing sensitivity of T* to errors in γ.

Table 3. Temperature Values Assumed for the Three Gradient Conditions Studied.

TEMPERATURE PARAMETER		DAY 172 GRADIENTS	2X DAY 172 GRADIENTS	3X DAY 172 GRADIENTS	
$^{\mathrm{T}}{_{\mathrm{L}}}$		300°K	300°K	300°K	
T _L - T ₁	6 M .	3.34°K	6.68°K	10.02°K	
$T_L - T_2$	PM .	2.16°K	4.32°K	6.48°K	
$T_L - T_3$	SM.	8.43°K	16.86°K	25.29°K	
T _L - T ₄	3	6.47°K	12.94°K	19.41°K	
T _L - T ₅	MM	2.16°K	4.32°K	6.48°K	
T _L - T ₆	SMS	8.54°K	17.08°K	25.62°K	
T'c		340°K	340°K	340°K	
T's		300°K	300°K	300°K	
s T'L		300°K	300°K	300°K	
T' _L - T' ₁		3.34°K	6.68°K	10.02°K	
T' _L - T' ₂		-2.16°K	4.32°K	6.48°K	
$T_L' - T_3'$		8.43°K	16.86°K	25.29°K	
$T_L' - T_4'$		6.47°K	12.94°K	19.41°K	
T' _L - T' ₅		2.16°K	4.32°K	6.48°K	
T' _L - T' ₆		8.54°K	17.08°K	25.62°K	

7. Combined Effects of Random Errors

The effect of randomly distributed parameter uncertainties on T* errors can be calculated using equation (27) and the derivatives listed in Table 4. Although there are 27 parameters to be accounted for, for the moment we will consider only three different standard deviations associated with three different parameter groups: optical constants, temperature measurements, and voltage measurements. These are defined as follows:

$$\sigma_{R} = \sigma_{R_{1}} = \sigma_{R_{2}} = \sigma_{R_{3}} = \sigma_{K_{4}} = \sigma_{K_{6}} = \sigma_{K_{7}} = \sigma_{R_{5}} = \sigma_{\varepsilon_{c}}$$
(29)

$$\sigma_{T} = \sigma_{T_{L}} = \sigma_{T_{L}'} = \sigma_{T_{S}'} = \sigma_{T_{I}'} = \sigma_{T_{I}'$$

$$\sigma_{\mathbf{V}} = \sigma_{\mathbf{V}_{1}^{\prime}} = \sigma_{\mathbf{V}_{2}^{\prime}} = \sigma_{\mathbf{V}_{3}^{\prime}} \tag{31}$$

Equation (16) can then be written in the two forms

$$\sigma_{\mathbf{T_1^*}} = \left[\sum_{k=1}^{6} \left(\frac{\partial \mathbf{T_1^*}}{\partial \mathbf{x_k}} \right)^2 \sigma_{\mathbf{R}}^2 + \sum_{k=9}^{15} \left(\frac{\partial \mathbf{T_1^*}}{\partial \mathbf{x_k}} \right)^2 \sigma_{\mathbf{T}}^2 \right]$$
(32)

$$\sigma_{\mathbf{T}_{2}^{*}} = \begin{bmatrix} \sum_{k=1}^{18} \left(\frac{\partial \mathbf{T}_{2}^{*}}{\partial \mathbf{x}_{k}} \right) & \sigma_{\mathbf{R}^{2}} + \sum_{k=9}^{24} \left(\frac{\partial \mathbf{T}_{2}^{*}}{\partial \mathbf{x}_{k}} \right)^{2} & \sigma_{\mathbf{T}^{2}} + \sum_{k=25}^{27} \left(\frac{\partial \mathbf{T}_{2}^{*}}{\partial \mathbf{x}_{k}} \right)^{2} & \sigma_{\mathbf{V}^{2}} \end{bmatrix}^{1/2}.$$
(33)

Inserting derivative values from Table 4 yields the following results.

DAY 172:

$$\sigma_{\mathbf{T}_{1}^{*}} = [(30.30)^{2} \sigma_{\mathbf{R}}^{2} + (1.64)^{2} \sigma_{\mathbf{T}}^{2}]^{1/2}$$
(34)

$$\sigma_{\text{T}_{2}^{*}} = [(18.96)^{2} \sigma_{\text{R}}^{2} + (1.73)^{2} \sigma_{\text{T}}^{2} + (27.5^{\circ} \text{K/V})^{2} \sigma_{\text{V}}^{2}$$
(35)

2X DAY 172:

$$\sigma_{\mathbf{T}_{1}^{*}} = [(58.37)^{2} \sigma_{\mathbf{R}}^{2} + (1.60)^{2} \sigma_{\mathbf{T}}^{2}]^{1/2}$$
(36)

$$\sigma_{\mathbf{T}_{2}^{*}} = [(40.75)^{2} \sigma_{\mathbf{R}}^{2} + (1.98)^{2} \sigma_{\mathbf{T}}^{2} + (58.5^{\circ} \text{K/V})^{2} \sigma_{\mathbf{V}}^{2}$$
(37)

Table 4. T* Derivatives for Methods 1 and 2 for Three Gradient Conditions. 2X DAY 172 3X DAY 172 **DAY 172** GRADIENTS GRADIENTS GRADIENTS 3T* 9T% ∂T5 ∂T* ∂T* ∂Τ* - $\overline{\partial x}_{\underline{k}}$ 9x1/5 $\frac{\partial \mathbf{x}}{\mathbf{k}}$ $\frac{\partial x}{\partial k}$ $\frac{\partial \mathbf{x}}{\mathbf{k}}$ $\frac{\partial \mathbf{x}}{\mathbf{k}}$ ^{x}k k 1 R_1 -7.35+3.14 -14.27+6.68 -20.72+10.70 R₂. +9.73 -17.232 -6.12+4.54 -11.86+15.61 3 R3 -14.04-4.35-26.93-9.19-38.76-14.694 +0.99 +29.11 +2.17 +42.11 +3.58 K, +15.07 5 +16.56 +24.07 -21.74K₆ -6.29-13.54+8.54 6 K₇ +18.16 +4.43 +34.87 +9.46 +50.36 +15.05 7 Rs -11.41 -24.57-39.79ε c 8 -10.95-23.57-38.19+1.58 +1.58 +1.56 +1.56 9 T_L +1.61 +1.61 -.04 -.03 10 T₁ -.04 -.04 -.04 -.04 T₂ -.04 -.04 -.04 -.04 11 -.04 -.05 12 T3 -.05 -.05 -.05 -.05 -.05 -.05 T₄ -.18 -.20-.19 -.19-.18 13 -.20 14 T₅ -.09 -.09 -.09 -.09 -.08 -.08 -.16 T₆ -.19 -.17 -.17 -.1615 -.19 T'c -.29 -.64 -1.0316 -.02 -.04 17 -.01 T'L +.97 +1.54 18 +.45 -.015 19 T'1 -.004 -.008 T'2 -:008 -.008 -.015 20 T'₃ 21 -.008 -.012-.019 T'4 -.046 -.073 22 -.023 T5 23 -.011 -.019 -.034 T'6 . 24 -.023 -.042 -.069 ٧'1 25 +9.54°K/V +19.94°K/V +31.13°K/V 26 V'2 -22.43°K/V -47.55°K/V -75.81°K/V 27 V' +12.83°K/V +27.70°K/V +44.85°K/V

3X DAY 172:

$$\sigma_{\mathbf{T}_{1}^{*}} = [(84.36)^{2} \sigma_{\mathbf{R}}^{2} + (1.58)^{2} \sigma_{\mathbf{T}}^{2}]^{1/2}$$
(38)

$$\sigma_{\text{T}_{2}^{*}} = [(65.78)^{2} \sigma_{\text{R}}^{2} + (2.44)^{2} \sigma_{\text{T}}^{2} + (93.4 \, \text{°K/V})^{2} \sigma_{\text{V}}^{2}. \tag{39}$$

If we assume an RMS voltage error equal to the RMS 8-bit quantizing error of 5 x 10^{-3}V and the SBRC estimates of

$$\sigma_{R} = .01$$

$$\sigma_{T} = 0.13^{\circ} K,$$
(40)

then the T* standard deviations expressed by equations (34) through (39) take on the specific values

DAY 172:

$$\sigma_{\text{T}_{1}^{*}} = [(.303^{\circ}\text{K})^{2} + (.213^{\circ}\text{K})^{2}]^{1/2} = 0.37^{\circ}\text{K}$$
 (41)

$$\sigma_{\text{T}_{2}^{*}} = [(.190\,^{\circ}\text{K})^{2} + (.225\,^{\circ}\text{K})^{2} + (.138\,^{\circ}\text{K})^{2}]^{1/2} = 0.33\,^{\circ}\text{K}$$
 (42)

2X DAY 172:

$$\sigma_{T_1^*} = [(.584^{\circ}K)^2 + (.208^{\circ}K)^2]^{1/2} = 0.62^{\circ}K$$
 (43)

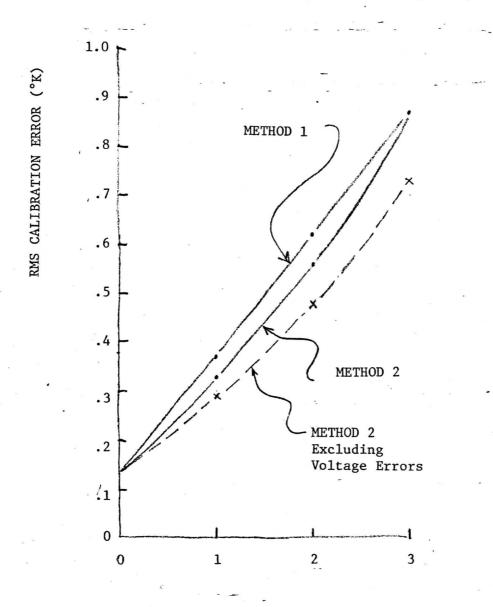
$$\sigma_{\text{T}_{2}^{*}}^{1} = \left[(.408^{\circ}\text{K})^{2} + (.257^{\circ}\text{K})^{2} + (.293^{\circ}\text{K})^{2} \right]^{1/2} = 0.56^{\circ}\text{K}$$
 (44)

3X DAY 172:

$$\sigma_{\text{T}_{1}^{*}} = [(.844^{\circ}\text{K})^{2} + (205^{\circ}\text{K})^{2}]^{1/2} = 0.87^{\circ}\text{K}$$
(45)

$$\sigma_{\text{T}_{2}^{*}} = [(.658^{\circ}\text{K})^{2} + (.317^{\circ}\text{K})^{2} + (.467^{\circ}\text{K})^{2}]^{1/2} = 0.87^{\circ}\text{K}.$$
 (46)

These results we also plotted against gradient size in Figure 4. Included in this plot is an additional curve for Method 2 which excludes voltage measurement errors. Since there will be an opportunity for considerable averaging of measurements it is possible that the voltage errors could be reduced well below the quantization noise. The case of zero voltage error (plotted as a dashed curve) thus represents a lower bound to the Method 2 result.



SIZE OF TEMPERATURE GRADIENTS (MULTIPLE OF DAY 172 CASE)

Figure 4. Dependence of VAS Calibration RMS Error on Size of Telescope Temperature Gradients.

8. Effects of Systematic Errors

The systematic T* errors δT_1^* and δT_2^* can be computed using equation (26), the derivatives in Table 4, and estimated systematic parameter errors δx_k . First let us consider the case of uniform systematic error, i.e.

$$\delta R_{1} = \delta R_{2} = \delta R_{3} = \delta R_{s} \equiv \delta$$

$$\delta K_{4} = \delta K_{6} = \delta K_{7} = \delta \varepsilon_{c} \equiv 0$$

$$\delta T = 0.$$
(47)

In this case the summations of equation (26) for each gradient and method yield the following results.

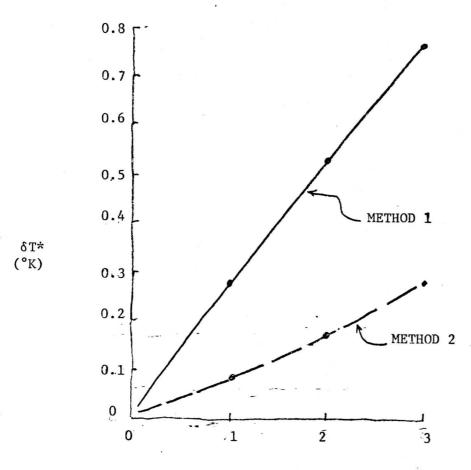
DAY 172:
$$\delta T_1^* = (-27.51^{\circ} \text{K}) \delta R$$
, $\delta T_2^* = (-8.08^{\circ} \text{K}) \delta R$ (48)

2X DAY 172:
$$\delta T_1^* = (-53.06^{\circ} \text{K}) \delta R$$
, $\delta T_2^* = (-17.35^{\circ} \text{K}) \delta R$ (49)

3X DAY 172:
$$\delta T_1^* = (-76.71^{\circ} K) \delta R$$
, $\delta T_2^* = (-28.17^{\circ} K) \delta R$. (50)

For the uniform systematic errors described by (47) Method 2 (using the auxiliary hot reference) appears to offer a factor of three reduction in resulting calibration errors over Method 1 (which uses only the ambient reference). For both methods the T* error increases approximately linearly with the size of the telescope temperature gradients. Specific values of T* bias errors (δ T*) for δ R = -.01 are presented in Figure 5. This case represents the result of a possible in-orbit degradation of optical components of .01 per element. The T* errors resulting from systematic errors of \pm .005 in laboratory measurements are just half those indicated in Figure 5 with the addition of a possible change in sign.

For the case of nonuniform systematic errors results vary considerably with the distribution of errors among optical components. In Table 5 results are presented for a wide variety of nonuniform degradations. In all cases those elements which degrade are assumed to change optical parameters by 0.01.



MAGNITUDE OF VAS TELESCOPE TEMPERATURE GRADIENTS (IN MULTIPLES OF DAY 172 GRADIENTS)

gure 5. Dependence of VAS Calibration Bias Error ($\delta T*$) on VAS Telescope Temperature Gradient Magnitude for a Uniform Systematic Optical Constant Error of $\delta R = -.01$.

Table 5. T* Bias Errors Resulting from 0.02 Degradation per Element as a Function of the Combination of Elements which Degrade. All Temperature Errors are in °K.

Parameters	DAY 172		2X	2X DAY 172		3X DAY 172	
which degrade by .01 each	$\frac{\delta T_1^*}{2}$	δ T *	δT*	δT*2	$\frac{\delta T_1^*}{1}$	δ Τ *	
R ₁	+.07	03	+.14	07	+.21	11	
R ₁ ,R ₃	+.21	+.01	+.41	02	+.59	04	
R ₁ ,R ₂ ,R ₃	+.28	03	+.53	08	+.76	12	
R ₁ ,R ₂ ,R ₃ ,R _s	+.28	+.08	+.53	+.17	+.76	+.28	
R _s	.00	+.11	.00	+.25	.00	+.40	
ε _c	.00	+.11	.00	+.24	.00	+.38	
R_s , ϵ_c	.00	+.22	.00	+.49	.00	+.78	
$R_1, R_2, R_3, R_s, \varepsilon_c$	+.28	+.19	+.53	+.41	+.76	+.66	

9. Conclusions and Recommendations

According to the calibration error estimates presented in the previous sections, temperature gradients between two and three times those of the DAY 172 case still do not produce absolute calibration errors in excess of the 1.5°K specification on absolute accuracy. However, two qualifications must be made to this statement: (1) absolute errors in T* can produce various kinds of relative errors in scene measurements both from band to band and as a function of time, and (2) since under some conditions almost all of the absolute T* error can manefest itself as some form of relative error, it is desirable at this point to make every easonable effort to keep absolute T* errors below the relative error specification of 0.5°K. Since, according to Figure 4, gradients in

excess of twice the DAY 172 values can produce absolute T* errors greater than 0.5°K (even using Method 2), it is recommended that VAS thermal control be improved to the point at which telescope temperature gradients do not exceed 1.5 to 2 times the day 172 values.