VAS Detector Noise Reduction Analysis

UW/SSEC

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Temperature soundings of the atmosphere can be made with VAS only if clear column spectral radiances over the initial regions of interest are determined within a standard error of .25 erg/(sec-ster-cm^2-cm^{-1}). Instrumentation specified for sounding with VAS can achieve this only by averaging many individual measurements, each with a standard error larger than that required for successful sounding. Therefore it is necessary to understand precisely the effectiveness of such averaging procedures in reducing detector noise.

If all the radiance measurements contained only random independent noise, then the standard error of the mean of N measurements, each with standard error σ, would just be the familiar $\sigma/\sqrt{N}$. However, the infrared detectors on VAS have substantial amounts of low frequency noise so that samples taken at higher frequency will experience noise contributions with significant correlation. This implies that the standard error of the average of N VAS IR measurements will be larger than $\sigma/\sqrt{N}$. To determine how much larger, consider the expected variance of the means of samples of size N

$$\sigma_M^2 = E[(M-\mu)^2] = E(M^2) - \mu^2 \tag{1}$$

where μ is the population mean (assumed to be zero for simplicity), and M is the mean of N detector noise voltage samples, $e(t_i)$, which we write as

$$M = \frac{1}{N} \sum_{i=1}^{N} e(t_i), \tag{2}$$

and E denotes the expectation value

$$E[e(t)e(t+\tau)] = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} e(t)e(t+\tau)dt \tag{3}$$

which is just the autocovariance function $C(\tau)$.

Then, we can write

$$\sigma_M^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} E[e(t_i)e(t_j)]$$

$$= \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} C(t_j-t_i). \tag{4}$$

But the signal variance is defined as

$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} [e(t)]^2\, dt$$

$$= C(0), \tag{5}$$

so we have

$$\sigma_M^2 = \frac{\sigma^2}{N} + \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j\neq i}^{N} C(t_j-t_i). \tag{6}$$

This shows explicitly the effect of noise correlation. If $e(t_i)$ and $e(t_j)$ were random independent variables then we would have $C(t_j-t_i) = 0$ for $i\neq j$ and $\sigma_M^2 = \sigma^2/N$. Since $e(t_i)$ and $e(t_j)$ are correlated a considerably different result is obtained.

To evaluate explicitly the variance of the means of many detector samples, one must take into account the spectral distribution of the detector noise power and the spatial distribution of the samples to be averaged.

For the purposes of this discussion, the detector noise can be completely described by the noise power spectral density

$$P(f) = \lim_{T\to\infty} \frac{2}{T} \left| \int_{-\frac{T}{2}}^{+\frac{T}{2}} e(t)\exp(-2\pi ft)dt \right|, \tag{7}$$

where f is frequency, and the autocovariance function

$$C(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} e(t)e(t+\tau)dt. \tag{8}$$

The two functions can be related through a cosine transform

$$C(\tau) = \int_0^\infty P(f)\cos(2\pi f\tau)df, \tag{9}$$

implying that the signal variance can be written

$$\sigma^2 = C(0) = \int_0^\infty P(f)df. \tag{10}$$

Clearly $P(f)df$ represents the contribution to the signal variance from frequencies between f and f+df.

P(f) appropriate to the VAS detector comes from two independent processes. The generation-recombination noise which is independent of frequency and the 1/f noise. The frequency at which these two are equal, f_c, is called the crossover frequency. Filtering the detector signal to eliminate all frequencies below f_{min} and above f_{max} yields an effective noise power spectrum

$$P(f) = \sigma^2(1 + f_c/f)/\Delta f_n \qquad f_{min} \le f \le f_{max}$$
$$= 0 \quad \text{otherwise,}$$

where the noise equivalent bandwidth

$$\Delta f_n = f_{max} - f_{min} + f_c \ln(f_{max}/f_{min}). \tag{11}$$

We are now almost equipped to evaluate the autocovariance function and σ_M^2. First, however, the autocovariance should be rewritten to include a power transfer function of the electronic filter, T(f), so equation (9) now reads

$$C(\tau) = \int_0^\infty T(f)\; P(f)\; \cos(2\pi f\tau)df. \tag{9'}$$

T(f) is evaluated as the product of the power transfer function of the presampling filter (modelled to be a 5 pole Bessel filter) and the power transfer function of the DC restore circuit (which behaves like an RC circuit).

The transfer function for a 5 pole low pass (LP) Bessel filter can be written

$$T_{LP}(f_{LP}) = 945/[(945-420f_{LP}^2+15f_{LP}^4)^2 + (945f_{LP}-105f_{LP}^3+f_{LP}^5)^2]^{1/2},$$

where f_{LP} is the low pass frequency. To translate this into a band pass filter transfer function, scale to band pass frequencies as follows

$$\frac{f_{LP}}{f_{max,LP}} = \frac{f_0}{f_{max} - f_{min}} \left[\frac{f}{f_0} - \frac{f_0}{f}\right]$$

where $f_0 = \sqrt{f_{max} f_{min}}$, and write

$$T_{BP}(f) = T_{LP}(f_{LP}). \tag{12}$$

The VAS DC restore noise transfer function can be written

$$T_{DCR}(f) = 1 + \frac{1}{1+(2\pi fT_2)^2} - \frac{2}{(2\pi fT_1)\sqrt{1+(2\pi fT_2)^2}} \times [\sin(2\pi fT_1-\theta) + \sin\theta] \tag{13}$$

where T_1 is the data gathering time, T_2 is the time constant of the RC circuit, and $\theta = -\arctan(2\pi fT_2)$.

Together then we have the effective power transfer function

$$T(f) = T_{BP}(f)\ T_{DCR}(f).$$

Figure 1 shows a plot of T(f) evaluated for $f_{min} = .026$ Hz, $f_{max} = 26$ KHz, $T_1 = 30$ msec, and $T_2 = 10$ msec. The oscillatory behavior of the transfer function near the low pass edge is due to the DC restore circuitry. Figure 2 shows a plot of the autocovariance $C(\tau)$ evaluated for the HgCdTe detector ($f_c = 750$ Hz) and the InSb detector ($f_c = 10$ Hz) with a sampling interval of 8 μsec. Figure 3 shows plots of σ_M^2/σ^2 as a function of N, the number of samples, for both detectors.

To see how multiple scanning reduces the VAS detector noise equivalent radiance (NER), consider the following example. Each sample of the radiation produced by one instantaneous geometric field of view (IGFOV) for a given detector has noise added onto the signal amounting to σ/IGFOV = 1.5 erg/etc. How many spins are required to view a 30x30 Km^2 area so that σ/Area $\leq$ .25 erg/etc? A scan of 30 Km lasts roughly 48 μsec so 6 samples are taken with correlated noise. Evaluating we find $\sigma_M^2/\sigma^2 = .448$, so σ/30 Km swath equals $1.5\sqrt{.448}$ or 1.00 erg/etc. Two 30 Km swaths displaced north-south from one another by an IGFOV cover an area of roughly 30x30 Km^2 (since IGFOV is 13.8x13.8 Km^2). The noise is uncorrelated from one spin to the next so σ/30x30 Km^2 = $1.00/\sqrt{2}$ = .707 erg/etc. This must be reduced by a factor of 2.83 implying 8 spins per swath. Assuming uncorrelated noise, we have 12 samples in the 30x30 Km^2 area hence σ/30x30 Km^2 = $1.5/\sqrt{12}$ = .433 erg/etc implying 3 spins per swath are needed. This is a much different result.

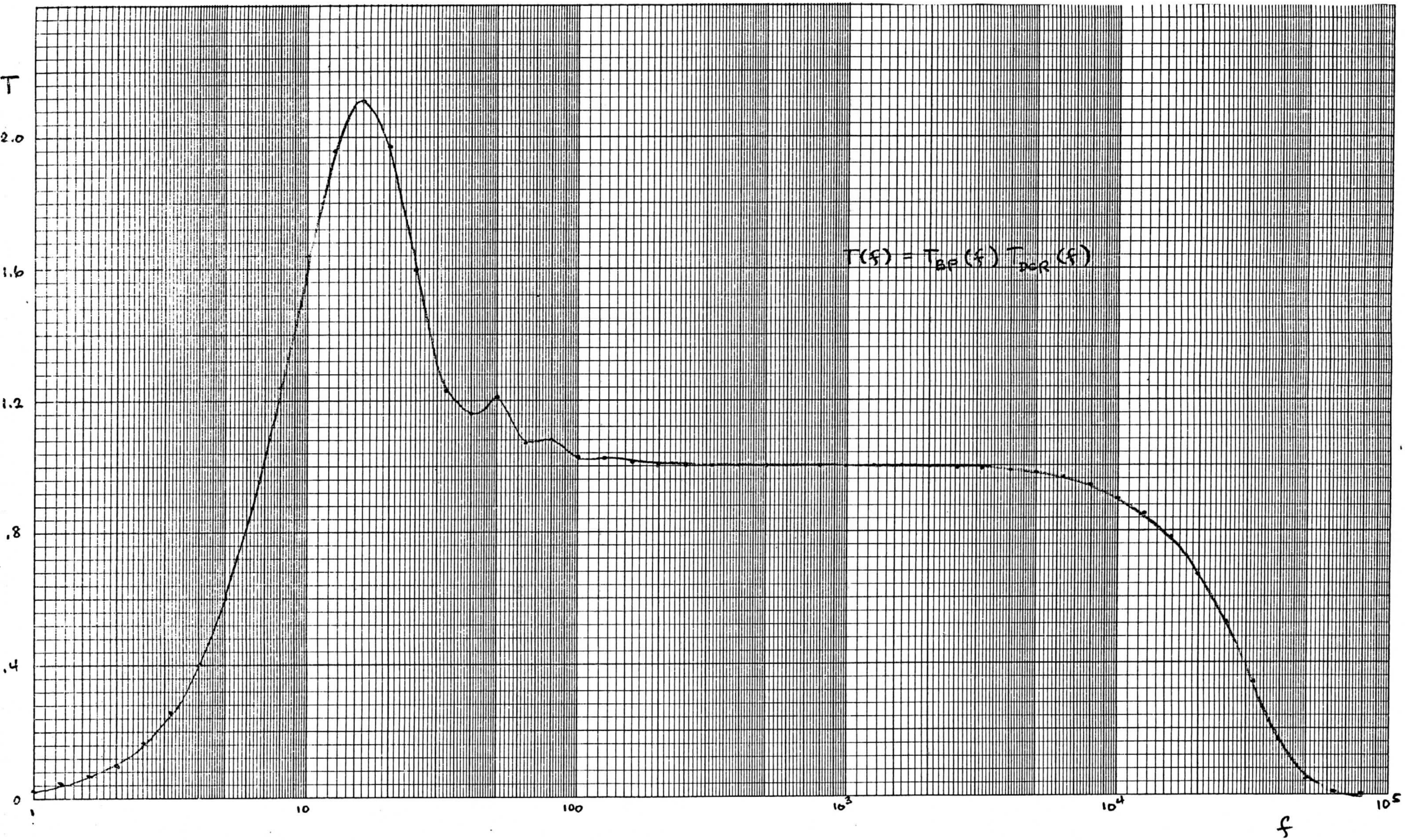

Figure 1. VAS Power Transfer Function

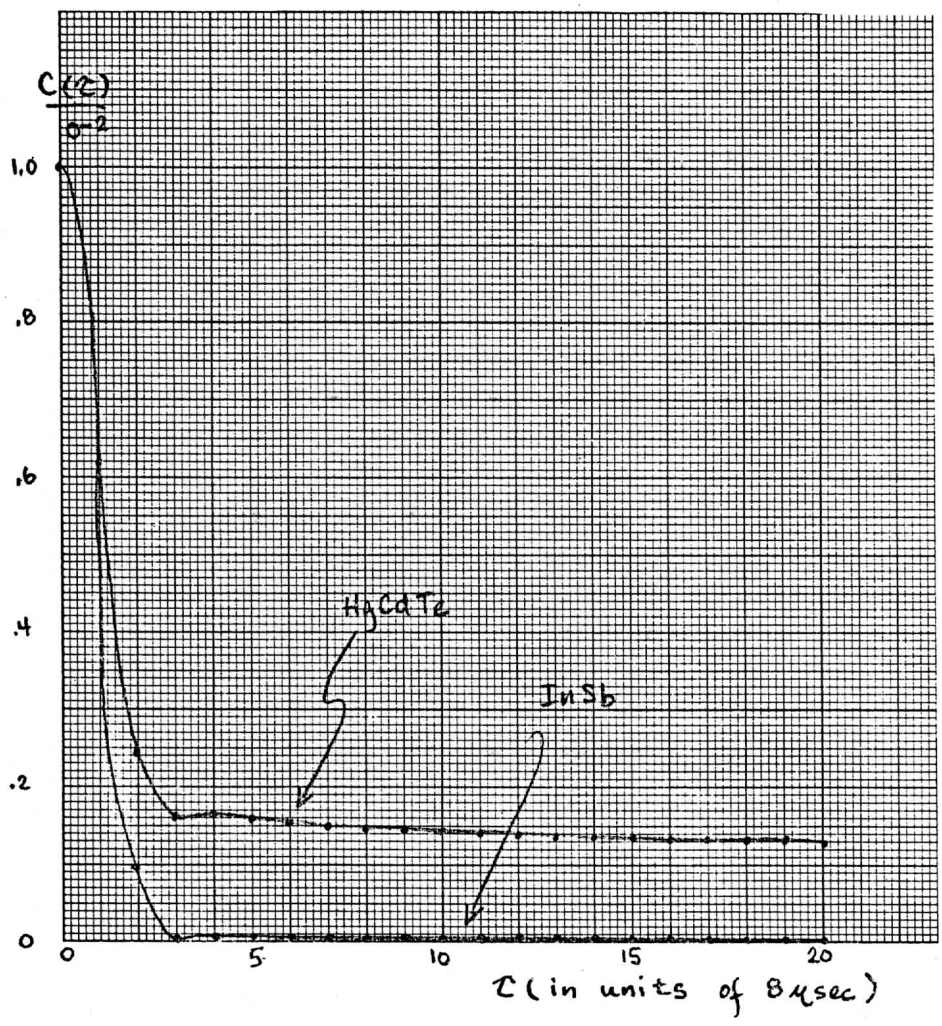

Figure 2. Plot of Autocovariance

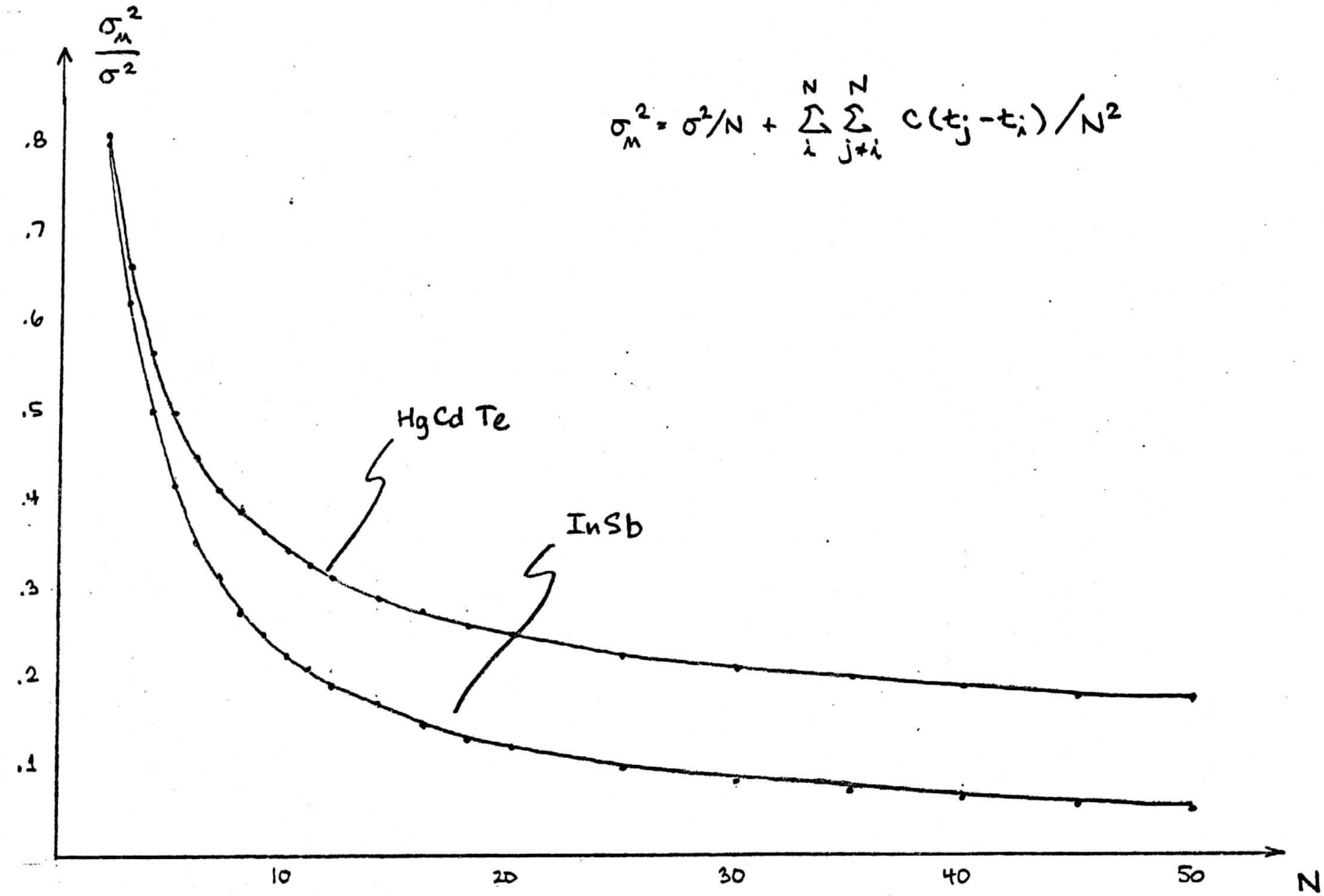

Figure 3. σ_M^2/σ^2 as a function of N.

It is useful to evaluate improvement factors that translate σ/IGFOV to σ/area. The expression for this improvement factor, I, is straightforward

$$I = \left[n \; \sigma^2 / \sigma_M^2 \right]^{1/2} ,$$

where n = number of IGFOV swaths needed to cover the north south displacement in the area. These improvement factors have been evaluated for the VAS HgCdTe and InSb detectors for sounding areas of approximately 30x30 Km^2 and 150x150 Km^2. The results are listed in Table 1. A spin budget evaluated using these improvement factors is listed in Table 2.

Table 1. Improvement Factors for VAS Detectors

area/detector	HgCdTe	InSb
30x30 Km^2	2.21	2.38
150x150 Km^2	7.45	13.62

Table 2. VAS Spin Budget

Channel	ν (cm^{-1}	Δν)	NER/IGFOV^{z} (ergs/etc	NER/Area)	Required NER	# Spins
1^{y}	680	10	4.94	.66	.25	7
2	692	16	1.99	.90	.25	13
3	703	16	1.82	.82	.25	11
4	715	20	1.39	.63	.25	7
5	745	20	1.53	.69	.25	8
6	760	20	1.50	.68	.25	8
7	790	20	1.43	.65	.25	7
8	895	140	.16	.07	.25	1
9	1380	40	1.32	.60	.15	16
10	1490	150	.33	.15	.10	3
11^{xy}	2335	50	.05	.0037	.002	4
12^{x}	2680	440	.006	.0025	.002	2 = 87

x InSb detector, all others are HgCdTe

y 150x150 Km^2 area, all others are 30x30 Km^2

z NER is evaluated from

$$NER = \frac{\gamma\sqrt{A_d \Delta f_n} \times 10^7}{A_0 \tau_{\Delta\nu} \Omega D^* \Delta\nu}$$

where

γ = preamplifier degradation factor

A_d = detector area (cm^2)

Δf_n = effective noise bandwidth

A_0 = effective entrance aperture area (cm^2)

$\tau_{\Delta\nu}$ = effective transmission factor in spectral interval $\Delta\nu$

Ω = solid angle IGFOV (ster)

D^* = specific spectral detectivity (cm $\sqrt{Hz}$/W)

$\Delta\nu$ = spectral bandwidth (cm^{-1})

```
'RUN,/ER MENZEL,4623,2154429787,$20.00
'FOR,SZI SIGMCL
      DIMENSION C(100)
      EXTERNAL P
      COMMON FC,F1,F2,F0,DELTN,T1,T2,TAU,TWOPI
      TWOPI=2.*3.14159265
      READ(-,-)  FC,F1,F2,T1,T2,T,N
      WRITE(-,-) FC,F1,F2,T1,T2,T,N
      F0=SQRT(F1*F2)
      DELTN=F2-F1+FC*ALOG(F2/F1)
      EPSIL=.000001
      DO 20 I=1,N
      TAU=I*T
      CALL NIROMB(P,.001,200.,EPSIL,10,5,C1,SIMP,2,IE,NS)
      CALL NIROMB(P,200.,2000.,EPSIL,10,5,C2,SIMP,2,IE,NS)
      CALL NIROMB(P,2000.,100000.,EPSIL,10,5,C3,SIMP,2,IE,NS)
      C(I)=C1+C2+C3
      WRITE(-,-) I,C(I)
   20 CONTINUE
      DO 40 N=2,50
      SIGMSQ=0.
      DO 30 I=1,N
      DO 30 J=I,N
      IF (I.EQ.J) GO TO 30
      SIGMSQ=SIGMSQ+2.*C(J-I)
   30 CONTINUE
      SIGMSQ=SIGMSQ/N**2+1./N
      WRITE(-,-) N,SIGMSQ
   40 CONTINUE
      END
'FOR,SZI P
      FUNCTION P(F)
      COMMON FC,F1,F2,F0,DELTN,T1,T2,TAU,TWOPI
      FL=F0*(F/F0-F0/F)/(F2-F1)
      FL=ABS(FL)*2.42
      TSAMP=945./SQRT((945-420*FL**2+15*FL**4)**2+(945*FL-105*FL**3
     T+FL**5)**2)
      A=TWOPI*T1*F
      B=TWOPI*T2*F
      C=1+B*B
      E=-ATAN(B)
      D=1.-2.*(SIN(A-E)+SIN(E))/(A*SQRT(C))+1./C
      TNOIS=SQRT(D)
      PFT=TWOPI*F*TAU
      P=TSAMP**2*TNOIS**2*(1+FC/F)*COS(PFT)/DELTN
      RETURN
      END
'XQT
750. .026 26000. .03 .01 .000016 50
'FIN
```