The Determination of Orbit and Attitude Parameters from Landmark Measurements While Including Precession and Higher Order Orbit Perturbation Effects

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I. Introduction

Given here is a description of some of the mathematical algorithms used to compute the orbit and attitude of a geosynchronous satellite while compensating for precession effects and enabling the use of an orbit propagation model more sophisticated than a Keplerian model. The approach keeps the mathematical formulation of the problem as simple as possible and yet enables the inclusion of these higher order perturbation effects. Finally, a discussion of problems which are likely to continue to introduce errors into the gridding of the earth's images is given.

An important observation made here is that once one is in the neighberhood of having orbit and attitude values which enable the accurate gridding of GOES images, then errors in these parameters fall into two natural categories: those which cause line residual discrepencies and those which cause element residual discrepencies. Line residual discrepencies are caused mainly by errors in the attitude, the pitch misalignment parameter and the orbit plane parameters, inclination and ascending node. Element residual discrepencies are mainly caused by errors in the along-track orbit parameters and the roll and yaw misalignment parameters. The advantages afforded by this natural split are taken advantage of. First the parameters which control the line residual discrepencies are computed and then the parameters controling the element residual discrepencies are then computed. This process is repeated iteratively. It appears that one pixel rms errors are achieved when these parameters are found for a sixteen hour time span and these values are applied to the same data.

A strong motivation for carrying out this work has been to make the VIRGS system a stand-alone system for orbit/attitude determination and for gridding the GOES data. Extensive work and study of the software in a quasi-operational setting is now needed to see if this goal has been achieved. Work is also necessary to implement the software into the operational efforts.

II. Orbit Plane and Attitude Determination

Presented here is a method to determine the orbit plane, the attitude and pitch misalignment angle of a geosynchronous satellite. The approach is straightforward. A measure is defined which uses line residual discrepencies to estimate delta incremental steps to improve the current values of these navigational parameters. This incremental improvement steps are repeatedly applied to the navagational parameters until convergence is achieved by the process. The result is a set of navigational parameters which best fit the provided data set of landmark measurements.

The measure using the line residual descrepencies is a least square measure. This measure is minimized by finding the values for the orbit plane, attitude and pitch misalignment parameters at which the first partial derivatives with respect to these parameters are equal to zero. This by itself is not a sufficient condition for a minimum to occur. However, within the usual operational constraints for a geosynchronous satellite and even well beyond these constraints, one's initial starting values for these parameters are sufficiently close to the actual answer that it is gauranteed that the iteration process will converge to the correct values.

This incremental steping for the updating of the values of these navigational parameters is based on Newton's method. A brief description of Newton's method is now given. One has the following equation to solve:

$$f(x)=0 \ .$$

Here x maybe a one dimensional variable and $f(x)=0$ a one dimension equation or x maybe a vector of n variables and $f(x)$ a system of n equations. In either case if one starts out with a good guess x_0, i. e. a value x_0 such that $f(x_0)$ is small and $f'(x_0)$ has an inverse that is bounded in a neighborhood of x_0, then Newton's method will converge to a solution x such that $f(x)=0$. Newton's method uses a first order expansion to find an approximation to Δx which solves

$$f(x + \Delta x) = 0.$$

The first, first order expansion is

$$f(x_0) + f'(x_0)\Delta x = 0.$$

Assuming that the Frechet derivative $f'(x_0)$ has an inverse and applying this inverse to the left-hand side and solving for Δx yields $\Delta x = -f'(x_0)^{-1}f(x_0)$ and the initial guess for x is updated with $x_1 = x_0 + \Delta x$. Applying this idea iteratively yields $x_n = x_{n-1} + \Delta x_{n-1}$ where $\Delta x_{n-1} = -f'(x_{n-1})^{-1}f(x_{n-1})$. Although the bookkeeping for keeping track of the terms which evolve from applying this method to the determination of the orbit plane, attitude and pitch misalignment parameters becomes quite involved, the conceptual frame remains identical to the above described process throughout.

The orbit plane parameters are represented by the variables a and b, the spin axis parameters are represented by the variables d and e and the pitch misalignment parameter is represented by the variable p. In this representation $(a, b, (1. - (a^2+b^2))^{\frac{1}{2}}$ is the orbit plane perpendicular, $(-d, -e, -(1. -(d^2+e^2))^{\frac{1}{2}}$ is the spin axis, both representations are in inertial space, and $(p, (1.-p^2)^{\frac{1}{2}})$ are respectively the sine and cosine of the pitch angle misalignment. These representations were selected to avoid mathematical singularities and ambiguities and to simplify the algebraic rats' nest which would otherwise be encountered if one chose the more conventional parameters inclination, ascending node, declination and right ascension for these quanities. Furthermore these quanities were selected for variables instead of angular quanities since it is simpler and more efficient to take derivatives of square root quanities instead of quanities involving the product of several sines and cosines.

The approach for establishing constraints to determine these parameters is simple and straightforward. The cosine of the angle between the satellite's spin axis and a unit vector pointing from the satellite towards the landmark is found by taking the dot product between the spin axis vector and this unit vector pointing towards the landmark. This same cosine is also determined by the image line number of the landmark measurement and the pitch misalignment parameter. By taking the sum of the squares of the differences of these cosines one creates a measure which can be used for detecting discrepencies in the orbit plane, attitude and pitch misalignment parameters.

The mathematics of the process described in the above paragraph are now given. Noting from the NOAA Technical Memorandum "Earth Locating Image Data of Spin-Stabilized Geosynchronous Satellites" that from a given landmark measurement and a nominal spin axis pointing vector an approximate satellite position can be found. From this type of computation the x and y coordinates of the satellite position are truly the prized coordinates sought for further computation. Hence for each i'th measurement it is assumed that the following triple will be formed (x_i, y_i, h_i) where $ATAN2(y_i, x_i)$ = the celestial longitude of the approximate satellite position, $x_i^2 + y_i^2 = 1$ and h_i is the approximate height of the satellite. If the orbit plane perpendicular is $(a, b, (1. - (a^2 + b^2))^{\frac{1}{2}})$, then these constraints necessarily imply that the satellite is approximately located at $h_i(x_i\text{-}cs{*}a,\ y_i\text{-}cs{*}b,\ \text{-}cs{*}(1.\text{-}(a^2+b^2))^{\frac{1}{2}})/xn$ where $cs = a{*}x_i+b{*}y_i$ and $xn = (1. - cs^2)^{\frac{1}{2}}$. Assuming the landmark being measured at this time has inertial coordinates (u_i, v_i, w_i), then the vector from the satellite to the landmark is approximated by $(u_i\text{-}h_i{*}x_i/xn+h_i{*}cs{*}a/xn,\ v_i\text{-}h_i{*}y_i/xn+h_i{*}cs{*}a/xn,\ w_i+h_i{*}cs{*}(1.\text{-}(a^2+b^2))^{\frac{1}{2}}/x$ This vector is transformed into a unit vector pointing at the landmark when it is divided by the vector's norm. This result is designated as $(xt_i/yn, yt_i/yn, zt_i/yn)$. Note that the notation used in the program ATORPL is being approximated here. The first cosine desired is found by taking the dot product of the resultant unit vector with the spin axis vector represented by $(-d, -e, -(1.-(d^2+e^2)))$.

The second cosine is determined by the line number of the landmark measurement, the picture center line and the pitch angle ζ. The expression for this cosine is $\cos(\pi/2+\zeta+(l-c_l)*r_l)$ where r_l is the radians per line and c_l is the picture center line. Using sum angle formulae and substituting p for $\sin(\zeta)$ and $(1.-p^2)^{\frac{1}{2}}$ for $\cos(\zeta)$ completes the transcription of the expression of the difference of cosines into the variables a, b, d, e and p. This term is squared and then summed over i where each i represents a different landmark measurement.

The sum of squares of differences of cosines is taken as the measure of the model's consistency. This expression is designated as g(a, b, d, e, p) and now it is described how to minimize this measure with respect to the variables a, b, d, e and p. It is known that a necessary condition for the minimum to occur is that the first partial derivatives with respect to these variables be equal to zero. Taking these partial derivatives yields five equations with five unknowns. In addition it is known that an answer is sought close to a, b, d, e and p all being equal to zero. This is true since this represents an equatorial orbit, a spin axis parallel to the earth's spin axis and a zero pitch angle. Taking the origin as an initial guess, Newton's method is applied to this system of equations. Because the iteration method is always started close to the desired unique solution convergence to the desired solution is always achieved. The application of Newton's method necessarily requires the computation of second partial derivatives of the above expressions for the respective cosines.

Although the computations of the partial derivatives for the above procedure are simply based on the chain rule and the rule for taking derivatives of exponential powers from basic calculus, some illustrative examples would be helpful to one without the time to recall all these details. To start the partial derivative with respect to a of $xt_i = u_i - h_i*x_i/xn + h_i*(a*x_i + b*y_i)*a/xn$ is found where $xn = (1. - (a*x_i + b*y_i)^2)^{\frac{1}{2}}$. Note first that the partial with respect to a of $cs = a*x + b*y_i$ is simply x_i. It follows that the partial with respect to a of cs^2 is $cs*x_i*2$. Hence the partial with respect to a of 1/xn is $-\frac{1}{2}*(1.-(a*x_i+b*y_i)^2)^{-\frac{3}{2}}*(-2)*cs*x_i$ or $cs*x_i/xn^3$. Using this and the chain rule yields that the first partial with respect to a of xt_i equals

$$-h_i*cs*x_i{}^2/xn^3 + h_i*(2*a*x_i+b*y_i)/xn + h_i*(a*x_i+b*y_i)*cs*x_i/xn^3.$$

Partials with respect to the other variables and higher order partials are handled in the same manner. Note that in SUBROUTINE ATORPL the beginning letters on variables PA, PAA, PAB, PD and etc. stand for, respectively, the first partial with respect to a, the second partial with respect to a, the second partial with respect to a and b, the first partial with respect to d and likewise for representing partial derivatives with respect to other variables.

When one wishes to solve for all five parameters, one takes the first partial derivative with respect to each of these parameters and sets the equations for these partial derivatives equal to zero. This yields five equations with five unknowns. Newton's method is then applied. The Frechet derivative of this expression comes from the second partial derivatives of the original expression g(a, b, d, e, f) for the sum of the squares of the differences of the cosines. The iteration step of Newton's method is repeated until convergence is achieved. Following the residuals which result from this newly computed set of parameters are outputted for review. This same process is operative when one requires a smaller number of these parameters. In this case the first partials with respect to this smaller set of parameters are set equal to zero and these equations are solved using Newton's method while varying the smaller set of parameters under consideration.

III. Changing the Orbit Model on the VIRGS System

Presently the orbit model being used on the VIRGS System is a simple two-body Keplerian model. Since this model omits several important perturbative effects, it is important to upgrade this model to include these effects. At the same time it is important to preserve the simplicity, efficiency and ease of implementation of the original Keplerian model. Presented here is an approach which retains this computational simplicity while accounting for these perturbative effects.

As before, once the attitude is known, the orbit plane perpendicular can be found by the same, simple nonlinear regression scheme used to find the attitude of the satellite and the along-track orbit parameters are found through the application of a linear regression scheme which is applied iteratively until convergence is achieved. The major difference is that these methods account for the higher order perturbations because of the use of a mathematical strategem. The strategem is the observation that the peturbations can be applied to the landmark observations while at the same time achieving the same mathematical result. Additionally it is mentioned that this new approach applies equally well to the algorithms used to extract both the attitude and orbit parameters from landmark measurements alone.

The mathematical strategem for including the perturbation effects is now described. For this discussion three vectors are used all of which occur at the instant a landmark is measured: S_1 is the inertial vector from the center of the earth to the satellite assuming a Keplerian orbit propagation is being used from an orbit's initial state, S_2 is the inertial vector from the center of the earth to the satellite assuming a better orbit model is being used to propagate the satellite's position from the same initial state for the orbit and E is the inertial vector from the center of the earth to the landmark which has been measured. To meet the constraints that the landmark measurement imposes it is necessary that the inertial vector from the satellite to the landmark, $E-S_2$, point in the same direction as the spin scan camera when the landmark was observed. The pointing direction of the spin scan camera is determined by the image coordinates of the landmark measurement, line number and element number, the beta count for that scan line, the pointing direction of the satellite's attitude and the misalignment parameters of the spin axis. It is important to note that the above vector difference can be written in another manner. The following expression represents the same difference: $(E+(S_1-S_2))-S_1$. Using this expression for this vector difference enables one to apply the perturbative effects of a better orbit propagator to the landmark measurements themselves and at the same time retain the simplicity of determining the orbit's initial state with Keplerian parameters.

This approach has the flexibility of permitting any orbit propagator to be used without changing the algorithms which determine the orbit's initial state. Such an approach has a wide range of application for satellites whose orbits are determined by ranging data or other earth based measurements for determining the satellites' positions.

Some notation from functional analysis is now introduced to justify the above iteration scheme. The problem is to determine a solution to the following equation:

$$x = x_0 + K(x) + P(x)$$

where x_0 is the initial state vector at time t_0, $K(x)$ represents the Keplerian effects of a point centered mass and $P(x)$ contains the perturbation effects. Note that in an orbit determination x_0 is found by reducing the constraints introduced from actual measurements. The initial problem that is solved at the beginning of the iteration is

$$y = y_0 + K(y)$$

where y_0 is the initial state vector found by fitting Keplerian orbit parameters to the measurement constraints. The next equation solved is

$$z = z_0 + K(z) + P(y).$$

Here z_0 is the initial state vector found by fitting a Keplerian orbit to the measurement constraints with the perturbation effects P(y) added to the landmarks. This is a variation on Picard's method or the method of successive approximations, an approach often used to establish the existence of solutions to initial boundary problems in ordinary differential equations. The same argument for convergence applies here also. Only one detail has to be taken care of. That detail is that the sequence of initial state vectors converge to a solution. This convergence is established using detailed arguments using Newton's method.

IV. Compensating for Precession in the Attitude Computation

The precession model from the NOAA Technical Memorandum "An Attitude Predictor/Target Selector by Bruce M. Sharts has been implemented in the software to model the precession. The implementation mainly uses the details given on pages 4 and 8 of the article. If W is a unit vector parallel to the spin axis direction and S is the vector from the satellite to the sun, the torque vector produced by the radiation pressure acting on the satellite is parallel to the direction WxS (here x between the two vectors signifies a cross product). From a given precession rate, theoretical and empirical values are available from the above mentioned NOAA Technical Memorandum on page 8, one can simply step the spin axis in the torque vector direction using appropiately small time intervals. The precession rate is a parameter to be set by operational use.

The incorporation of these precession effects into the attitude computation is straightforward. On the first iteration of an iteration loop in the driver program of the UPGORB software package, no precession effects are considered. Following the precession effects on the line number observations are computed and these adjustments are fedback to the attitude computation. Since the newly computed attitude at an epoch in the time interval under consideration is nearly parallel to the original attitude, the precession effects theirselves are also nearly the same. One iteration would be sufficient to account for these effects. However, more iterations are done to insure that convergence for other parameters is achieved.

V. Error Analysis of the Operational Navigation for GOES Satellites

Currently navigational parameters for the GOES East and West satellites are routinely generated on a daily basis on NOAA/NESS's VIRGS system. Over the past several years the installment, modifications and improvements of the VIRGS system have substantially improved the accuracy of the daily navigation generated by NOAA/NESS. However, an assessment of the accuracy of the navigation will be useful not only to substantiate the recognized improvements but also to provide guidance and direction for future improvements. The main focus here are the orbit and attitude models, the mathematics and software used to generate parameters for these models and some known errors within the system for which a resolution has not yet been adopted. Admittedly this approach limits discussion about operational and engineering aspects of the GOES data system which substantially affect the computation of the navigational parameters. Hopefully, these aspects will be covered by personel with more qualifications to cover these aspects than the current author.

A simple, direct way to assess the accuracy of the navigation is to measure the image position of a recognizable, earth landmark, transform the earth coordinates to satellite image coordinates using the navigation model, take the differences and consider the residuals. It is useful to divide these residuals into the categories of line and element residuals. Measurable line residuals are mainly caused by errors in the determination of the satellite's attitude, the satellite's orbit plane parameters, inclination and ascending node,and the satellite's pitch misalignment parameter. The element residuals are caused mainly by the along-track orbit parameters, semimajor axis, eccentricity, mean anomoly and argument of perigee, the roll and yaw misalignment parameters and apparent rotation of the earth in the image frame caused by errors in the attitude parameters.

The relative disjointness of the categories of line and element residuals simplifies both the error analysis and the computation of the orbit and attitude parameters. First the orbit plane parameters, the attitude parameters and the pitch mialignment parameter are computed. This computation is almost wholely based on the line residuals. The accuracy of this computation is assessed by computing a bound for the norm of the inverse of a matrix used in the last iteration step in the convergent process that employs Newton's method to compute these parameters. Some rules of the thumb will have to be developed to use this bound for a matrix norm effectively. One can first generate an artificial, over abundant set of landmark measurements to find a bound for the best case. In this case the process for finding the inverse of the matrix should be well conditioned. For regular, good operational cases the norm of the matrix inverse should be compared to the norm of the matrix inverse in this best case. With sufficient operational experience one will develop a measure for how large this norm can be and still have good results generated.

The norm of the inverse of this matrix can be used in a second way. One may have a sufficient landmark measurement set to determine the attitude and orbit plane parameters but this set may not be sufficient to also determine the pitch misalignment parameter accurately. Threshholds for assessing this problem by considering the bound for the norm of the inverse of the matrix can be developed with further operational experience.

A specific different problem may occur, if for each landmark, the errors are consistently off in the same direction and have the same magnitude. Part of this problem in the past was caused by the satellite or the earth in the image frame being mislocated due to errors in linking the earth's physically tied coordinate system to the celestial coordinate system or errors in locating the sun's position. Ben Remondi, Jim Ellickson and Kathy Kelly have worked to eliminate these problems and discrepencies. One notable impact from their efforts is that landmarks closer to the limbs of the image can now be used in the process of determining orbital and attitude elements. The only reasons remaining for observable constant offsets would be a misplacement of a landmark's image position because of confusion caused by shoal waters or a misassignment of the satellite's height due to an overly simple model.

Briefly described now are some of the causes of element residuals. It has been noted consistently that whenever one uses the same set of orbital elements which were used to generate the betas for the time arc in which one is making landmark measurements, the landmarks have always been displaced in the same direction by an average amount which lies between two or three elements. The reason for this displacement is also known. It simply happens because the beta counts and time information are stored in the wrong scan lines at the SDB. If one could confirm that this error occurs consistently and is always of the same magnitude and in the same direction, then with appropiate software changes on the VIRGS System the impact of this error could be eliminated from the residuals being observed. Additionally a careful write-up of this error could be provided to researchers using the navigational parameters from the VIRGS so that they could also bypass this problem.

A second, potential cause of an element bias in a predict maybe caused by a miscalculated semimajor axis due to the time arc of the measurements being to short or because the Keplerian model simply averages out too many perturbative effects. The implementation of a better model should reduce or eliminate both of these problems. First a shorter time arc maybe used for estimating the orbital elements because it will no longer be necessary to average out the perturbation effects. Finally, the better orbit model should give better predicts over longer time spans.

The observable effects of the attitude, orbit plane and pitch misalignment parameters will now be discussed. The motion of the subsatellite point in the line direction in a time sequence of images is determined by the relative orientation between the spin axis and the orbit plane perpendicular. If the spin axis and the orbit plane perpendicular are pointing in opposite directions, the subsatellite point will not move in the line direction. If the orbit plane perpendicular and the spin axis form an angle of 179° between themselves, the subpoint of the satellite will move up and down in the frame with an amplitude of 1°. Similarly a separation of 178° gives a 2° amplitude. The phase of this sine wave motion is largely determined by the right ascension of the spin axis. When the top of the spin axis is pointing as close as possible to the earth, i. e. the spin axis is tilted up towards the south pole of the earth, the earth will be at its lowest point in the image frame, i. e. the line number of the subsatellite point will have its highest value for a twenty-four hour span. Twelve hours later the earth will be at its highest point in the line direction.

At this time, June, 1981, the orbit plane parameters are determined on the IBM 360 and transferred to the VIRGS. These parameters are then used along with landmark measurements to determine the attitude of the satellite. This procedure is in use because current software on the VIRGS system only uses the sinusoidal motion of landmarks in the line direction to compute the attitude of the satellite. New code to determine the orbit and attitude parameters from landmark measurements alone will shortly be available on the VIRGS system. Part of this sinusoidal motion of landmarks is caused by the sinusoidal motion of the subpoint of the satellite in the line direction. The second major component of this motion occurs when the spin axis of the satellite is not parallel to the spin axis of the earth. This causes the earth to appear to rotate back and forth around the subpoint of the satellite in the image frame.

The extent of this rotation is determined by the declination of the spin axis. If the declination equals 89^{o}, the largest rotation observed will be 1^{o}. Correspondingly, if the declination equals 88^{o}, the largest rotation observed will be 2^{o}. The phase of this rotation is determined by the right ascension of the spin axis, the longitude of the satellite's subpoint and the time of the year.

The length of the lever arm for determining this rotation is determined by the line-element separation of landmarks measured for the orbit plane and attitude determination. This separation does not have to occur in the same image frame. For example landmarks could be alternately extracted from the left side and right side of the image frames. However, such separations do need some time proximity. One's results will not be as good if one measures on the right side of image frames four four hours and then switches and starts measuring on the left side of image frames. These are not hardfast rules. Before one commits himself to an operational schedule of landmark ingests, one should run through a set of artificial landmark measurements generated by the navigational model and check the norm of the above discussed inverse matrix to see if it is within previously established operational bounds.

The pitch misalignment parameter affects the line residuals in a constant manner, i. e. if the other parameters are correct, an error in the pitch misalignment parameter will cause a constant offset in the line residuals. However, in computations this will not occur. If landmarks are measured over a ten hour or less period, an error in the pitch misalignment parameter will cause a shift in the phase and amplitude in the sinusoidal wave tracking the motion of the subsatellite point of the satellite. This phase and amplitude shift can be avoided by having the pitch misalignment parameter computed at the same time but again care is necessary to establish that one's landmark data base is adequate for this task.

The last thing to discuss about problems affecting the line residuals is the effect of nutation and precession on the spin axis pointing direction. The effects of nutation are not accounted for either on the current navigational system or with the updates included with this report. The effect of the nutation should be to degrade the line residuals. If this degradation is relatively random with mean equal to zero, its effects may be averaged out with additional measurements. Also, one might use the phase and amplitude of the errors in the time of the sun pulse detection to measure nutation and subsequently remove its impact.

The precession will be handled empirically in the new update. The approach is based on the technical memo of NOAA by Bruce Sharks. The spin axis is precessed away from the sun in discrete steps with the precession rate to be imputed by the user. There is a place for the precession rate in the common block NAVCOM and from heretofor it will be assummed that PRERAT will be set as 100*samples/day. The effect of this precession is included in the attitude and orbit plane computation.

Even if one had a perfect set of orbit and attitude parameters for the satellite, there would still exist discrepencies causing registration and gridding errors. The fact that the beta counts and scan line timing information are stored on wrong scan lines with no certainty whether this displacement of the information is by one or by two scan lines creates uncertainties in the system amounting to two and a fraction pixel in the element direction. Although it is widely speculated that nutation can cause displacements up to two and three visible samples, this number is speculative and has not been subject to a close study. Finally, it has not been ascertained whether the simple model of role, pitch and yaw accounts for all the nuances of the stepping of the spin scan camera with respect to the satellite's principal axis.

For many years the effect of along-track errors in the satellite's position have been ignored on image alignment systems like the VIRGS system at the World Weather Building or the McIDAS system at the University of Wisconsin. These errors were ignored because they did not significantly impact the accuracy in locating landmarks whose longitudes were within forty to fifty degrees of the the longitude of the subsatellite point and hence during standard operations these errors were not observable. An analagous situation is now described. If one is located high above a plane and changes one's position by moving to one's left, the perspective does not change significantly if one looks back and thereby compensates for one's change of position. In fact this perspective does not change unless the distance moved is some significant fraction of one's height above the plane. However, with the GOES satellites one is looking at a spheroid instead of a plane. In this case moving to the left, although it does not change one's perspective of the landmarks at the subsatellite point, does cause these landmarks to move to the right with respect to landmarks located to the apparent rim of the image. This results because the landmarks close to the rim keep their relative position with respect to the rim, while the landmarks in the center of the image move with respect to this rim. Now that these errors in the along-track position of the satellite are being taken out with the implementation of a better sun model and a more accurate transformation from the earth's coordinate system to an earth centered celestial coordinate system, these discrepencies should be eliminated.

In-track errors have not been so much of a problem on the VIRGS system because their occurances cause attitude adjustments which in turn cause rotational errors in aligning images which are observable. Since these type of errors are observable, they have been avoided.

VI. Acknowledgements

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