Team Member Investigation of the Laser Atmospheric Wind Sounder Facility Instrument

Final Report for NASA Contract # NAS5-30752

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Final Technical Report

This contract, NAS5-30752, was to support my participation on the EOS - LAWS (Earth Observing System - Laser Atmospheric Wind Sounder) Science Team. The principal work prepared under this contract was to prepare for, attend, and contribute to nine LAWS Science Team meetings where I provided input on the design and operations plan for the LAWS instrument. In addition to the Science Team participation, this contract supported studies of digital signal processing techniques of revelance to LAWS. Results of these studies are described in the two attached appendices. The contract work ended with the de-selection by NASA of the LAWS instrument.

Appendix 1

From Technical Digest on Coherent Laser Radar: Technology and Applications, 1991 (Optical Society of America, Washington, D.C., 1991, Vol. 12, pp. 216 - 218.

High Performance Velocity Estimators for Coherent Laser Radars

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The use of high performance Doppler velocity estimators to enhance the performance of laser radars has taken on an added importance in the context of space based wind measurement systems such as the Laser Atmospheric Wind Sounder (LAWS) instrument. The LAWS mission is designed to make wind measurements throughout the troposphere using scattering from atmospheric aerosols at CO_2 laser wavelengths. The need to make velocity estimates in very clean upper tropospheric conditions coupled with the strict limits imposed on the system power and aperture by the orbital platform require that usable velocity measurements be made at the lowest possible signal-to-noise ratios.

The topic of interest here is the problem of estimating the mean frequency of a relatively narrow-band signal hidden by a white noise interference process. This is a problem which has been of significant interest in both the laser and microwave radar communities and has been discussed in some detail by Zrnic'(1979) who analyzed the performance of the traditional estimators for signals of this class.

Zrnic' has also calculated an asymptotic form for the Cramer-Rao (C-R) error bound, the performance of a Maximum Likelihood(ML) velocity estimator. This work is interesting due to the fact that the reported bound significantly outperforms traditional estimators in many cases. In the case of the LAWS instrument, the C-R bound would indicate that performance increases of up to 13 dB over the conventional pulse-pair velocity estimators are possible. There are two caveats to this apparently optimistic result. The first is that the analysis of the C-R bound is asymptotic and is only valid for realizations consisting of a large number of samples of the time series. The second relates to the unfortunate fact that a knowledge of the bound implies that an estimator of the specified performance is possible, it does not guarantee that it will ever be found.

Much of the recent work in velocity estimators has focused on techniques which find the peak value of some sort of a spectral estimator. For example Klostermeyer (1989) has examined the performance of an algorithm which finds the peak of the Burg maximum entropy spectral estimate. He has reported results approaching the Zrnic' C-R bound for some of the points in his parameter space. In an attempt to further explore this approach we have constructed algorithms based on this peak finding methodology for a number of different spectral estimation techniques.

The most promising of these operates by finding the spectral peak of the so called "Maximum Likelihood Spectral Estimate" (MLSE) first described by Capon (1969). A good description of the MLSE technique for spectral estimation and it's relationship to optimal filtering theory is given in Kay and Marple (1981). Although the Capon MLSE estimator is not a true ML estimate of the spectrum of a process, it has several properties to recommend it for our purposes. In particular it produces the best (in the ML sense) velocity estimates of any algorithm which we have tested, and it is very robust, showing only a weak sensitivity to tuning parameters. Another encouraging aspect of the Capon estimator is that is reduces to the ML velocity estimator for the only case where the ML estimator has a known closed form, namely the frequency of a pure sinusoid in noise.

A key element of the Capon estimator involves the inversion of the process autocorrelation matrix which must in itself be estimated. The results presented here are based on performing a forward/backward least squares fit to an AutoRegressive (AR) process model using the Marple technique. The covariance matrix is then taken to be the covariance matrix of the AR model. Other techniques for the estimation of the matrix can be used but care must be taken to ensure that the matrix has positive definite eigenvalues.

We have evaluated the performance of the Capon estimator by making a Monte Carlo simulation of the estimator using a synthetically generated Gaussian signal in white noise. The results presented here are for a typical LAWS parameter choice with 72 samples of a process having a 1 m/s intrinsic spectral width (s) on a ±25 m/s Nyquist interval. A plot of the error probability distribution for the Capon and traditional pulse pair estimators at a signal to noise ratio of -13 dB is given in Figure 1.

As you can see the error character of the Capon estimator is unusual in that it produces a central core of "good" estimates which rises out of a background of essentially random choices for the velocity. This type of behavior has important implications for subsequent processing stages and also poses some difficulty in terms of the choice of an estimator "figure of merit". The standard deviation of the estimate is quite high and is entirely determined by the large error tails. One useful measure of estimator performance is the fraction of the estimates which are in this central core. In Figure 2 we have plotted the fraction of estimates which are within 1 m/s of the correct value for the Capon estimator. We have also included the pulse-pair estimator performance and the C-R lower bound from Zrnic'(1979).

The performance presented in Figure 2 is in fact a substantial improvement on the pulse-pair performance, however it is still about 5 dB short of the C-R bound and an important question is whether any of that potential 5 dB improvement can still be realized.

We have not been able to find a closed form solution to the full ML estimation problem however we have found a way to express this problem as a non-linear optimization problem which can be solved numerically, albeit at enormous computational expense. We have performed this calculation for a few choices of the system parameters and believe that the Zrnic' C-R bound is in fact overoptimistic for typical parameters and that the potential improvement over the Capon estimator errors is only about 1.5 dB . Since the Zrnic' bond is an asymptotic result it is also possible to test the relative performance of the Capon estimator at that asymptotic limit. We have performed this calculation and can confirm that the Capon estimator does come within 1.5 - 2 dB of the C-R bound for large sample lengths however the sample lengths required to reach this convergence are extremely long, on the order of 2000 samples.

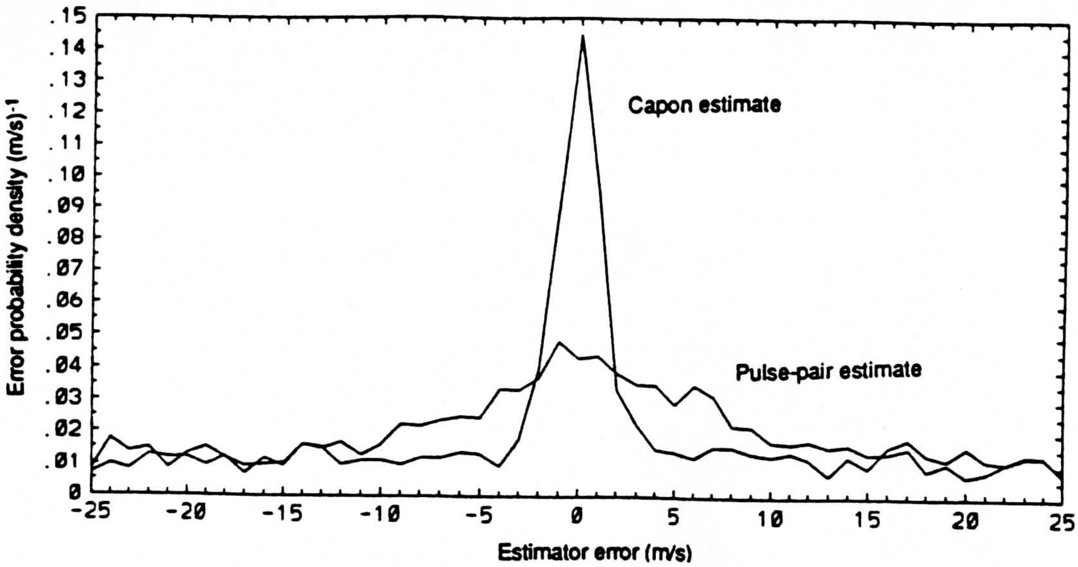

Figure 1. Error distributions for Capon and pulse pair estimators.
SNR = -13 dB.

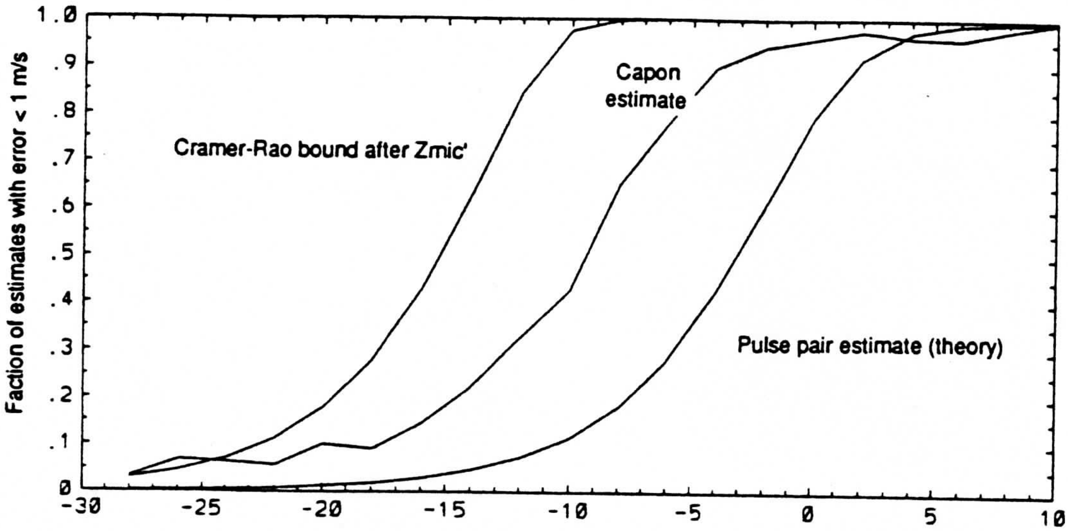

Figure 2. Relative estimator performance vs. SNR

In conclusion we believe that high performance techniques can indeed play an important role in improving lidar velocity measurements. The estimators such as the one described here and others such as the "poly-pulse-pair" method can substantially improve on the pulse-pair technique. These improvements are likely to be on the order of 8-10 dB for typical laser radar parameters, not the 13 dB that one might have expected from earlier estimates based on the Cramer-Rao bound.

Acknowledgements:

This work was supported by NASA and benefited from many helpful conversations with Bob Lee, Mike Hardesty, and other members of the LAWS team.

References:

Capon, 1969: Proc. of the IEEE, v. 57, no. 8.

Kay and Marple, 1981: Proc. of the IEEE, v. 69, no. 11.

Klostermeyer, 1989: Radio Science, v. 24, no. 1.

Zrnic', 1979: IEEE Trans. on Geoscience Electronics, v. GE-17 October 1979.

Appendix 2

An Initial Study of the Use of High Performance Signal Processing Algorithms for the LAWS Instrument

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Summary

In this study I have attempted to assess the potential improvement in low signal to noise ratio LAWS wind sounding performance that can be achieved through the use of high performance velocity estimation techniques. Although an optimum algorithm for the LAWS velocity estimation problem is not known I have investigated a number of potential estimators based on three basic approaches. The first kind of estimator is a multiple filter bank scheme based on the optimum estimator for the detection of a sinusoid in noise, which is known. The second set of estimators is based on AutoRegressive (AR) spectral estimation techniques that have been claimed to have near optimum performance in some of the literature. The third class of estimators appears to be new to the problem of velocity estimation and is based on the Capon spectral estimation procedure which is sometimes referred to as Maximum Likelihood(ML) spectral estimation.

Each of the estimators were evaluated in terms of the signal to noise ratios required to achieve a velocity error of less than 1 m/s with a probability of 50% and 75% for nominal LAWS parameters. All of the estimators performed significantly better than the Pulse Pair(PP) autocorrelation based estimator at low and moderate signal to noise ratios. The best performance was achieved by one of the Capon based estimators and exhibited a performance improvement of 6.6 dB over the PP estimate for 50% probability of less than 1 m/s error, and required 8.4 dB less signal than the PP estimator to achieve a 75% probability.

An important question for the design of the LAWS system is whether this performance is in fact the best that can be achieved or if there is some as yet unknown algorithm waiting to be discovered which is significantly better. Zrnic' has derived an information theory based Cramer - Rao bound which in principle defines the performance of the optimum (true maximum likelihood) estimator. His derivation of the bound is valid only for long data samples and indicates that it would seem to be possible to improve on the performance of the Capon

estimator by another 5dB. I have performed some numerical calculations with long data samples which indicate that the use of this asymptotic bound may be somewhat overoptimistic for the LAWS parameter range and the actual potential performance improvement is probably only about 2.5 dB. About 1dB of the 2.5 dB difference between the two bound estimates is known to result form data end effects which can be recovered in LAWS processing by using overlapping data blocks.

In summary it seems that we are now in possession of a velocity estimator whose performance is within about 2.5 dB of the best possible for the LAWS parameter range. Future work should confirm this conclusion by extending the Zrnic' analysis to a finite length time series. In addition it must be understood that in the low S/N regime the nature of the estimation error of these algorithms is far from Gaussian and some serious thought needs to be given to the combination of multiple estimates into the level-2a wind products since any kind of simple averaging will yield substantially suboptimal performance.

Introduction

A critical issue in the design of the LAWS instrument is the ability of the sensor to make wind soundings in clean upper troposphere and lower stratosphere regions. Due to the large costs of increasing either the laser power or the telescope size it is important to ensure that the signal processing algorithms used in the velocity estimation process work as well as possible in these marginal signal to noise regions which are expected to occur frequently during the instrument operation. The purpose of this study is to make a preliminary assessment of the role high performance signal processing algorithms can play in enhancing the LAWS performance. No analytical expressions for the variance of the candidate algorithms are known so I will be performing a series of Monte Carlo simulations to evaluate the performance of the estimators.

Before beginning to perform the evaluations we must first decide what performance criteria are appropriate to characterize estimator performance. Traditionally estimators have been evaluated by considering the variance of the estimator error. For situations with large numbers of samples and high signal to noise where the estimator statistics are normal this has proven to be an acceptable statistic however for our case where the time series are rather short and signal to noise ratios(SNR) are not high it is very difficult to get a stable estimate of the error variance. In this study I have chosen instead to rate the estimators in terms of the fraction of the estimates which can be considered "good", that is they lie within some error tolerance of the actual mean velocity of the process which generated the time series. I have arbitrarily chosen to defines estimates which lie within 1 m/s of the true mean velocity as "good" however as long as this number is a small fraction of the Nyquist interval the comparative results are not a strong function of this particular choice. It should be noted that this criteria is quite similar to evaluating the error probability density function of the estimator at zero error.

The estimator which maximizes the error probability distribution at zero error is referred to as the Maximum Likelihood (ML) estimator. Although an ML algorithm is not known for signal models applicable to LAWS there are a number of useful results which arise from information theory concerning ML estimators. The most important of these is the existence of an achievable bound, known as the Cramer - Rao bound which defines the performance of the ML estimator even when no explicit form of the estimator is known.

The problem of velocity estimation from a coherent Doppler radar has been examined in some detail by Dusan Zrnic' in a paper which appeared in IEEE transactions on Geoscience Electronics (GE-17, October 1979). In this paper Zrnic' presents two important results: the first is an analytic expression for the error of the Pulse Pair (PP) autocovariance velocity estimator (his eq 4.14) which frequently appears in the LAWS literature. The second is an expression for the Cramer - Rao bound for the problem of a random Gaussian signal in white noise (his eq A.29). It should be noted that both expressions are asymptotic approximations with respect to the number of samples and are strictly valid only for a "large" number of samples. For the case of the PP estimator we can easily evaluate the accuracy of the approximation using Monte Carlo simulations, the evaluation of the accuracy of the C-R bound is more problematic and will be considered in the final section.

The Design of the Monte Carlo Simulations

In order to perform a Monte Carlo study we will have to chose a synthetic signal model. The one which I have chosen consists of starting with two time series of Gaussian white noise. One of the series is then filtered using a long (100 point) Gaussian impulse response Finite Impulse Response(FIR) filter which is constructed to have the appropriate spectral width and a randomly selected mean velocity, uniformly distributed across the Nyquist interval. The filtered series, which represents the Doppler signal, is then scaled to the appropriate mean Signal to Noise Ratio and added to the white noise sequence. To produce the results presented in this report this process is repeated 250 times for each of a number of SNR values and the fraction of good estimates at each SNR is tabulated. A diagram showing the general processing geometry appears in Figure 1. All simulations have been performed on the baseline LAWS configuration described in the memo from Emmitt which assumes a 9.11 micron laser wavelength, a ±25 m/s Nyquist processing interval, 1 km slant range resolution consisting of 72 time series samples, and a 1 m/s intrinsic signal spectrum width due to wind shear and turbulence effects.

In order to present the theoretical estimator performances from Zrnic' which are derived in terms of error variances in our "good estimate fraction" form we will assume that the theoretical estimates are representative of a Gaussian error distribution so that the probability of a good estimate is given by the error function, erf(Δf), of the 1 m/s threshold normalized by the theoretical estimator standard deviation.

In Figure 2 the results of a Monte Carlo simulation of the PP estimator are shown on a plot that also includes the theoretical PP estimator performance and the C-R bound as presented by Zrnic'. The general agreement between the PP estimator performance and the theoretical curve is quite good. At high SNR the theoretical curve tends to be somewhat optimistic, a property noted by Zrnic' while at very low signal to noise the PP estimator outperforms expectations due to the fact that in the finite 50 m/s Nyquist interval there is a probability of 0.04 of getting a "good" estimate with no information whatsoever.

One can also see form Figure 2 that there is substantial room for improvement between the performance of the PP algorithm and the C-R bound. In fact if the C-R bound in this form is applicable to these parameters a performance improvement over the PP estimator of 11.3 dB for 50% probability of a good estimate and 13.6 dB for a 75% probability should be possible.

Candidate Estimator Performances

The first estimator which we will consider is one that is based on the known ML estimator for a signal model consisting of a sinusoid in white noise. The ML estimator for this case consists of a filter bank of an infinite number of the narrowest possible Doppler filters which is applied to the time series. The frequency corresponding to the filter with the largest output amplitude provides a ML estimate of the sinusoid frequency. The estimator described here which I will refer to as a Block Filter Bank (BFB) estimator is based on the observation that over a short enough time interval the signal from our signal model appears very much like a sinusoid. The estimator then consists of dividing the time series into a number of blocks, in this case 12 points long, and computing a bank of 256 filters for each block using a zero padded FFT and then averaging the output powers for each block. The BFB estimate is defined as the center frequency of the filter with the peak average amplitude. This estimator bears an obvious relationship to the Welsh averaged periodogram spectral estimator.

The performance of the BFB estimator is presented in Figure 3. Note that in spite of the simplicity of this estimator, for a large range of signal to noise ratios it significantly outperforms the PP estimator. There are many ways to refine this particular sort of estimator and I have tried a number of them of which none provide more than about 1 dB of further improvement in spite of a significant increase in processing cost. It should also be noted that the computation requirements of the BFB estimator are by no means small, they are in fact larger than the higher performance estimators which follow.

The second family of estimators which I will consider is based on AutoRegressive(AR) spectral estimation strategies. These estimators, sometimes called maximum entropy, are known for producing high spectral resolution representations of narrow band processes. A good discussion of various spectral estimators including the AR and Capon techniques appears in the 1981 article by Kay and Marple (Proc. of the IEEE, Vol 69. no. 11). The use of an AR based velocity estimator has been recently advocated by Klostermeyer (Radio Science, Vol 24, no. 1, 1989) where he explores the use of an AR based spectral peak velocity estimator for VHF wind profiling radars. In this paper he reports performance near the C-R bound for a limited range of signal to noise values.

One problem with using AR estimators is related to the difficulty of choosing the correct order for the AR process model which one is going to fit to the data. This is illustrated in Figure 4 where I present the results for an AR estimator using a Least Squares prediction filter form(see Kay and Marple) with a fixed order of 10 which is near optimal for low s/n cases. As one can see this estimator provides a substantial improvement over the PP estimates at low SNR but disappointing results at higher SNR. As explained by Klostermeyer this degradation of performance is due to the tendency of the AR estimator to split the signal into several peaks none of which is aligned with the true signal center frequency. This problem can be controlled by limiting the estimator order at high SNR using some form of "order trimming" procedure. In Figure 5 the results of applying the Marple order trimming procedure where the order is limited to that which explains 55% of the total signal energy are presented. This result is somewhat more encouraging in that it exhibits an improvement of about 2 db over the BFB estimator, about 6 dB better than PP, at low SNR while remaining superior to the PP estimator at higher SNR although the high SNR performance is still perhaps somewhat disappointing.

The final class of estimators which we will consider is based on a finding the spectral peak of a Capon power spectrum estimate. The Capon spectral estimate is sometimes called the maximum likelihood spectral estimator however it is not the ML estimate of the power spectral density. A discussion of Capon spectral estimators appears in Kay and Marple.

The Capon estimator of order P is based on an optimum filtering theory derivation where for each frequency a filter of length P is constructed which maximizes the ratio of output power at the particular frequency to sum of the powers passed at all other frequencies. Once these filters are computed it is much like the ML estimator for sinusoidal signals except that the filter bank is computed from the above constraint using an estimate of the autocorrelation function to lag P rather than assuming that the appropriate filters are sinusoids. It can be shown that for a signal consisting of a long time series of a sinusiod in noise it will reproduce the ML estimator for for that signal model. A plot of the velocity estimation performance of a P=12 Capon based velocity estimator appears as Figure 6. It can be seen that the low signal to noise performance of the Capon estimate is slightly better than the AR algorithm and in the -5 to 0 dB s/n range the estimator is from 1 to 2 dB better than the AR values. At high signal to noise ratios there is still some evidence of the a problem caused by the use of too high an estimator order. Although the problem is much less serious than for the AR system it possible to exploit some of the relationships between the Capon and AR estimates to perform an order trimming operation and improve the high SNR estimates. The results of this refinement are shown in Figure 7 which at the present time is the best performance I have found for any estimator operating in the parameter regime defined by the LAWS baseline.

<u>Discussion</u>

A summary of the results presented above appear in Table 1 where I have listed the SNR values required to achieve either 50% or 75% "good" estimates based on the 1 m/s error threshold. It can be seen that the order trimmed Capon estimator exhibits a substantial performance improvement over the traditional pulse pair velocity estimator for nearly all signal to noise values however it is appears to still be 5dB away from the best possible performance as indicated by the Cramer - Rao bound. It is important to determine if the C-R bound is in fact achievable for this set of operating parameters or if the Capon estimator is already near optimum in performance.

About 1 dB of the 5 dB difference results from the fact that the Capon estimator underutilizes the P points on either end of the 72 point long segment due to the edge effects on the autocorrelation estimates. In a real LAWS implementation this 1 dB can be recovered by overlapping the segments by P points. In order to understand the remaining 4 dB difference we will need to go back and examine the assumptions behind the C-R bound derivation.

The weakest assumption for our part of the parameter space is the fact that the estimator is assumed to operate on a long time series. It is not immediately obvious if the 72 point sample used in this study is long enough to satisfy this condition. One experiment that we can perform is to look at the performance of the Capon estimator on a substantially longer series. the results of repeating the Monte Carlo calculations for a 2000 point series is shown as Figure 8. It can be seen in this figure that the performance of the Capon estimator is now substantially closer to the C-R bound being only 2-3 dB lower in performance than the bound. Based on this result it seems that some of the missing 4 dB may in fact be due to a limitation of the C-R bound analysis itself. There is however probably the potential to achieve another 2 dB performance improvement over the order trimmed Capon algorithm. It seems that a reasonable future effort would be to extend the Zrnic' analysis to the case of a shorter time series. Although algebraically tedious the procedure is straightforward and should be tractable through the use of symbolic computer algebra systems or by numerical evaluation of the likelihood functions.

In conclusion we have been able to substantially narrow the range of possible LAWS velocity estimation performance. We now have an estimation strategy whose performance is probably within 2-3 dB of the best possible algorithm for the LAWS parameters. This performance of this estimator can be used to provide a useful input for the lidar system design studies. It should be emphasized that the later processing stages where the radial estimates are combined into wind estimates are far from trivial. At low SNR the velocity error distribution is far from normal and it not yet known if we will be able to assign a confidence indication to the estimate or if that will have to be derived in the context of the other observations. It is important to address these issues as soon as possible since they may also have a significant impact on the overall system design.

Table 1

Estimator Performance Summary

All SNR values are for ±25 m/s Nyquist bandwidth

Estimator	SNR Required for 50% errors < 1m/s	SNR Required for 75% errors < 1m/s
Pulse Pair (actual)	-3.9	+0.7
Cramer - Rao bound after Zrnic'	-15.2	-12.9
Block Matched Filter	-8.2	-4.8
Autoregressive least squares form order 10	-9.7	-6.0
Autoregressive least squares form order trimmed	-10.0	-6.4
Capon order = 12	-10.5	-7.7
Capon AR autocorrelations with order trimming	-10.3	-7.7

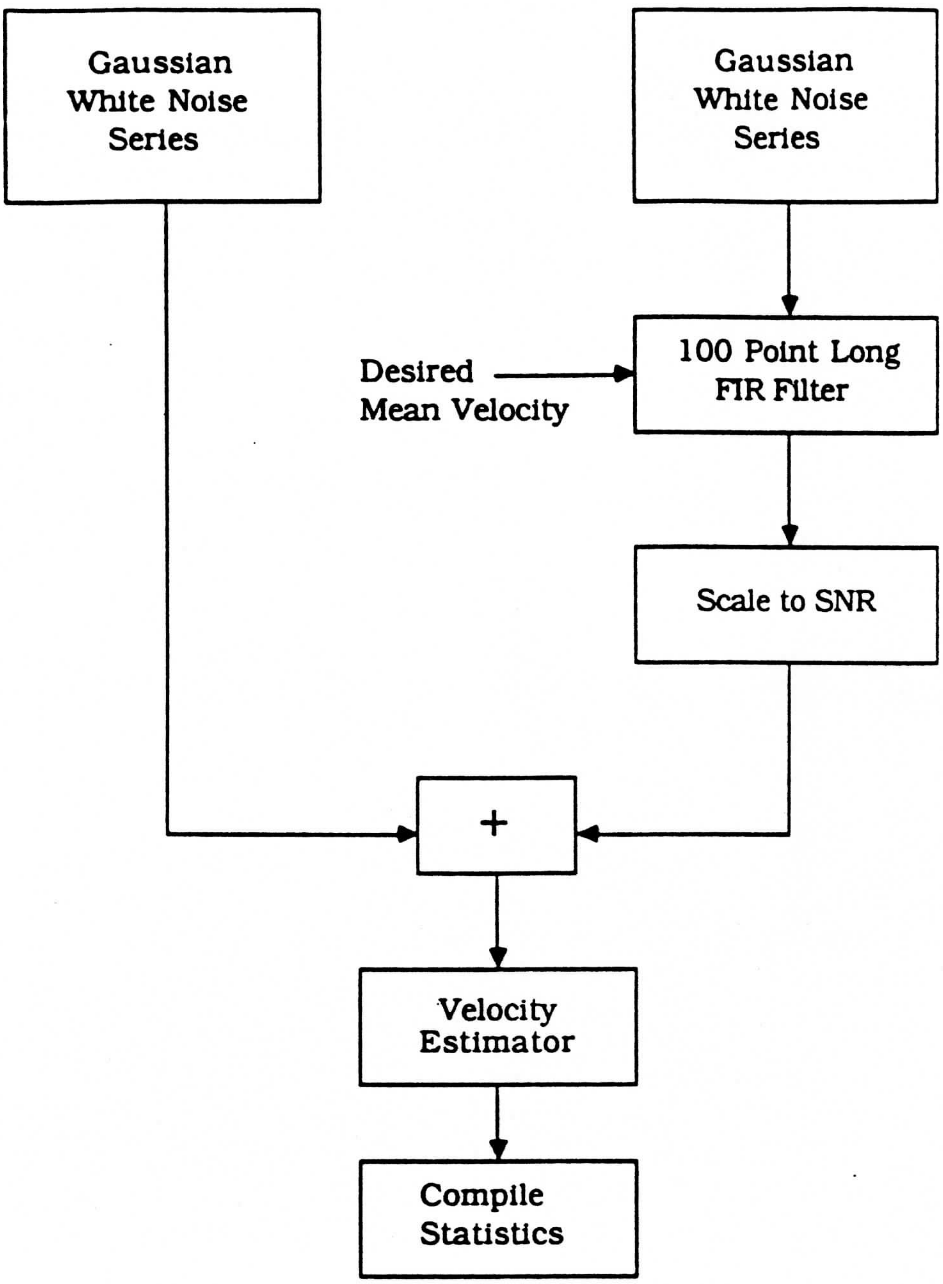

Figure 1. Estimator evaluation processing geometry

Figure 2. Pulse Pair Estimator Performance

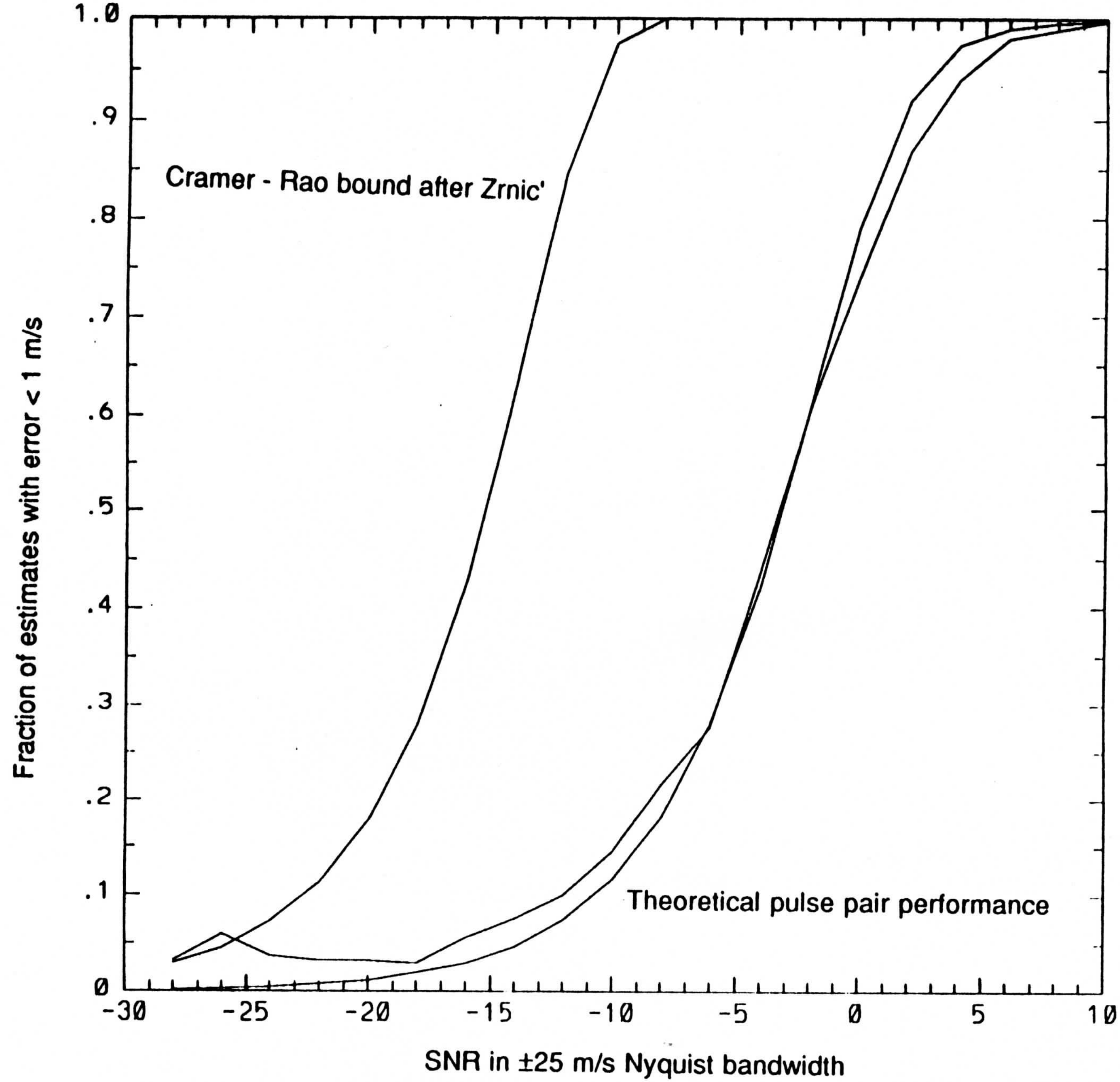

Figure 3. Block Filer Bank Estimator Performance

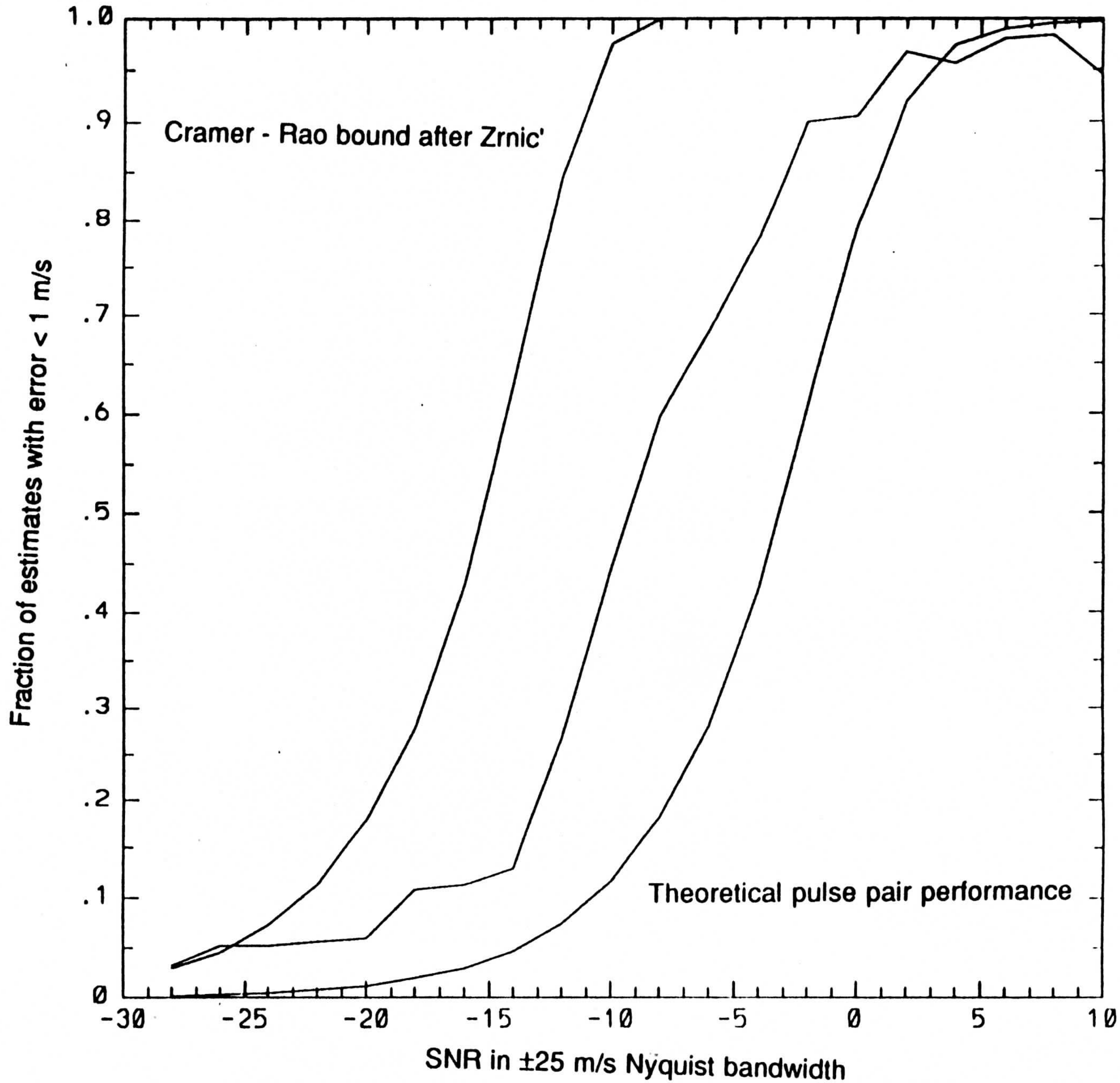

Figure 4. Autoregressive Estimator (Order 10) Performance

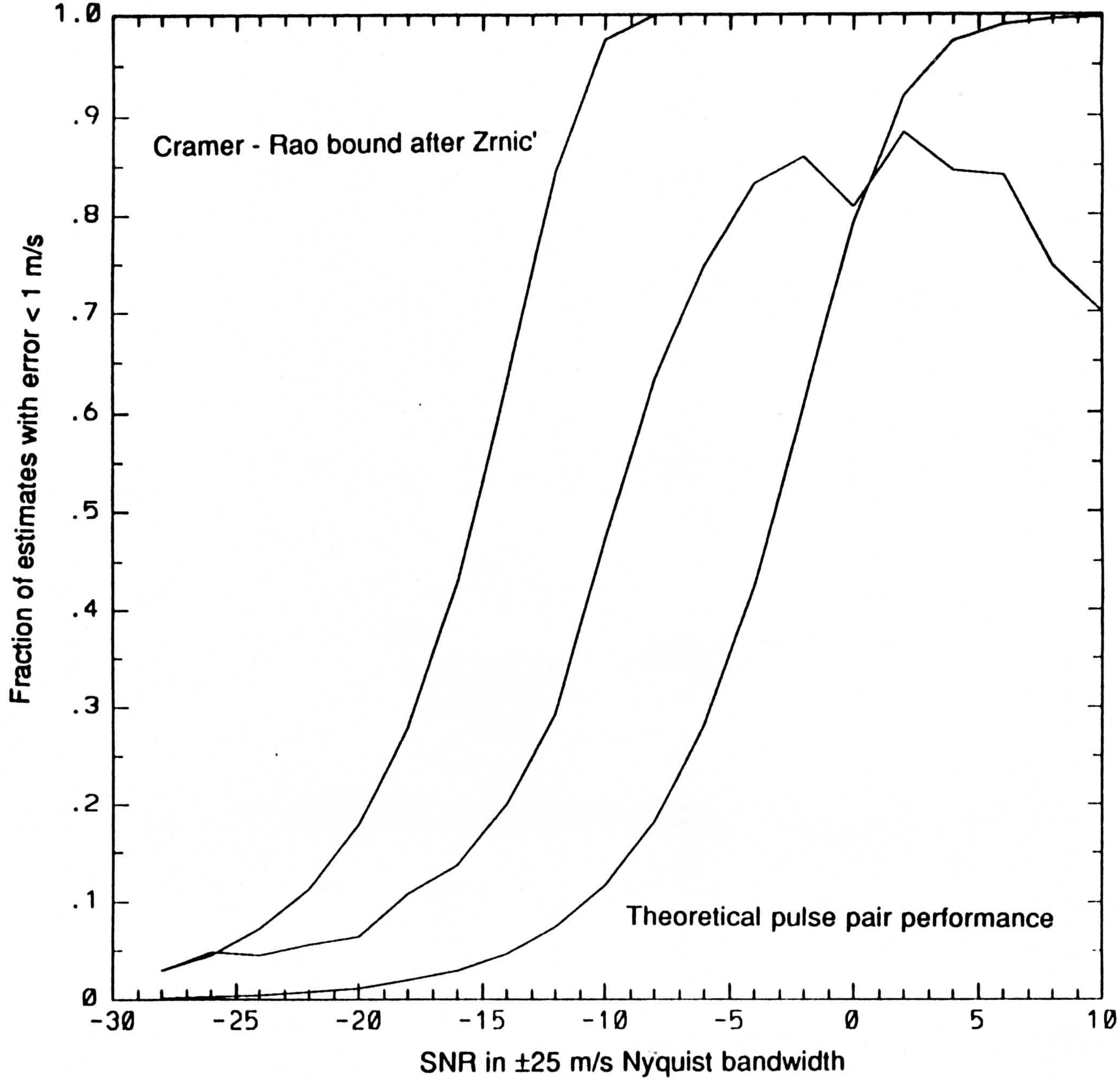

Figure 8. Capon Estimator Performance for 2000 Point Time Series

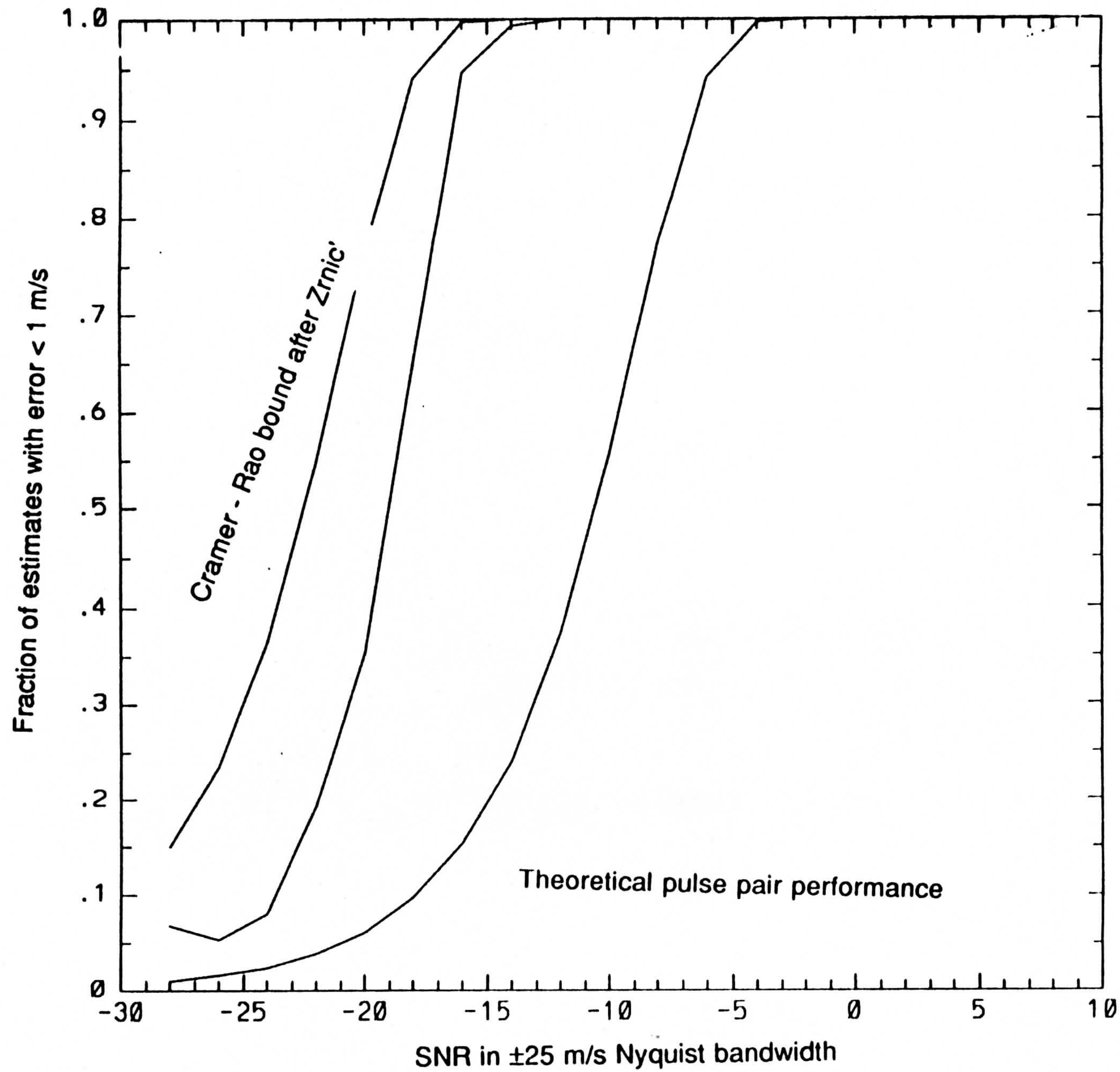

Figure 7. Order Trimmed Capon Estimator Performance

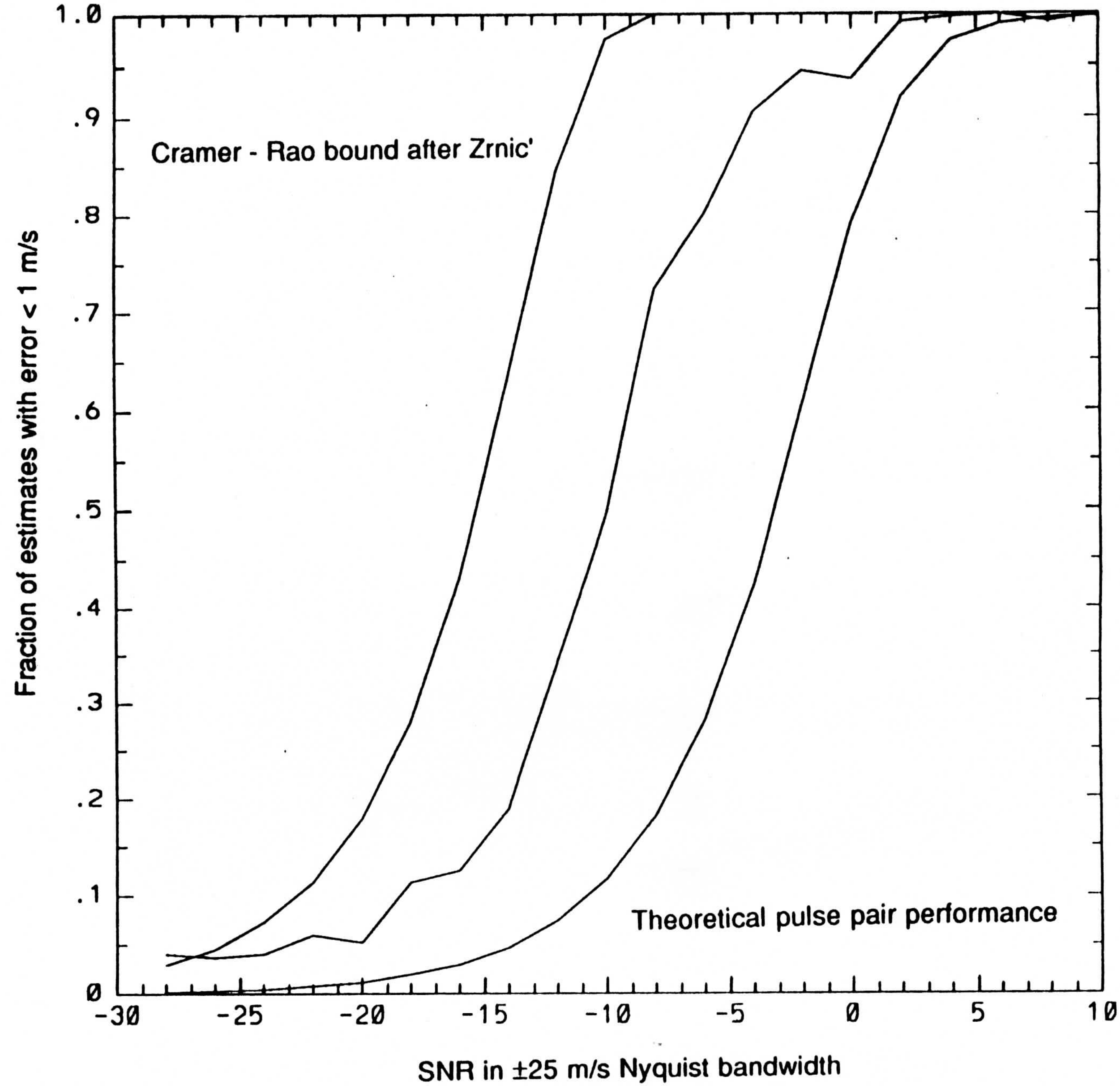

Figure 6. Capon Estimator(Order 12) Performance

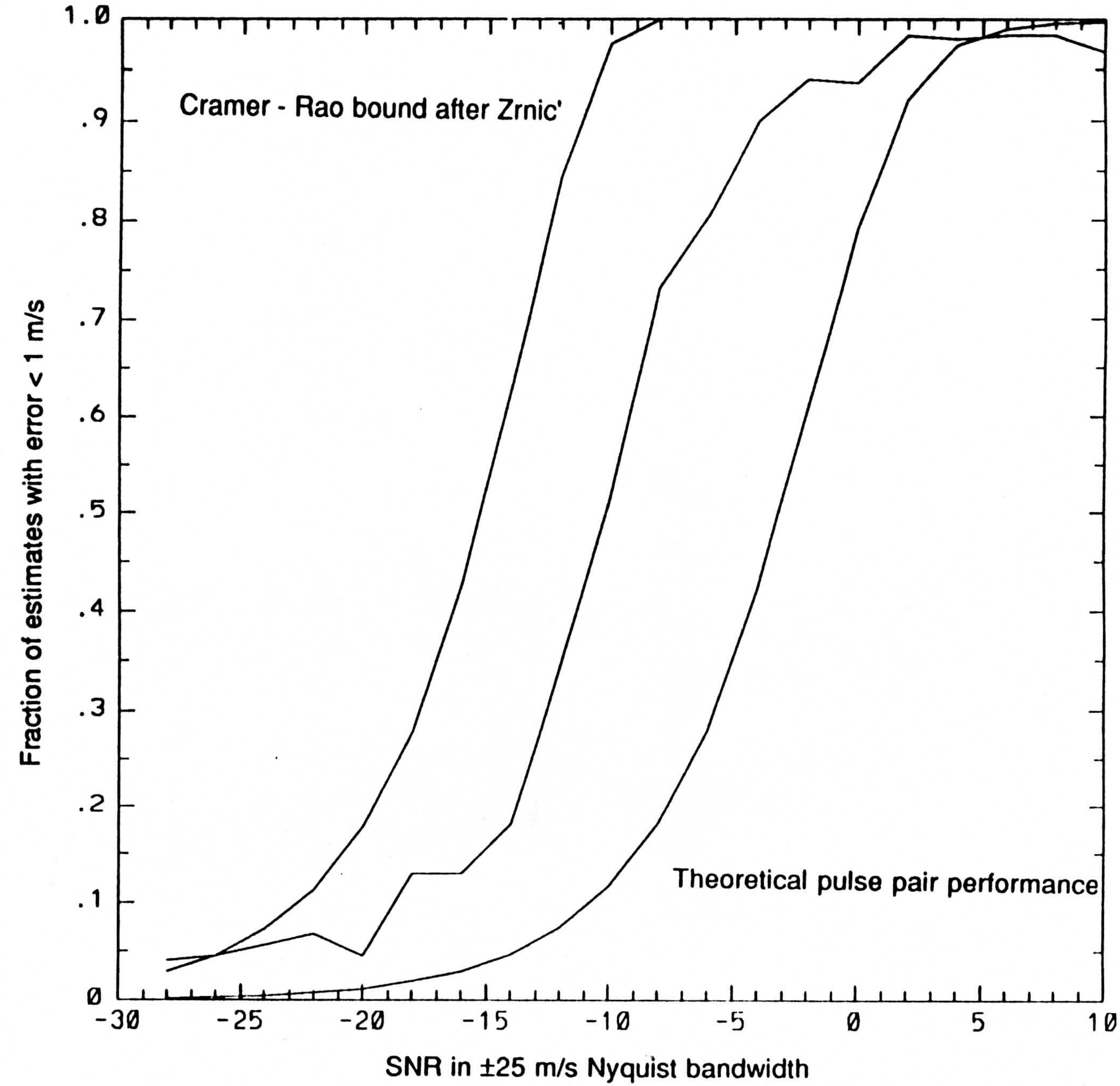

Figure 5. Order Trimmed Autoregressive Estimator Performance